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CSCI 150
Introduction to Digital and Computer System Design
Lecture 2: Combinational Logical Circuits IV

Jetic Gū

Overview

- Focus: Boolean Algebra
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch2 2.4, 2.5; v5: Ch2 2.4, 2.5
- Core Ideas:
  1. Boolean Algebra III: K-Map

P0
Review

Boolean Algebra I&II

- AND, OR, NOT Operators and Gates
  - Simple digital circuit implementation
  - Algebraic manipulation using Binary Identities
- Standard Forms
  - Minterm & Maxterm
  - Sum of Products & Product of Sums

Review

P1 Optimisation

Boolean Algebra III: K-Map

Cost Criteria;
Map and Map Manipulation

Summary

P1
Optimisation

K-Map

- Karnaugh Map, or just K-Map
  - For optimising 2-4 variable boolean expressions
    - Skip: 5,6 variable K-Maps can also be drawn but are not very intuitive to use

Concept

Two Variable Maps

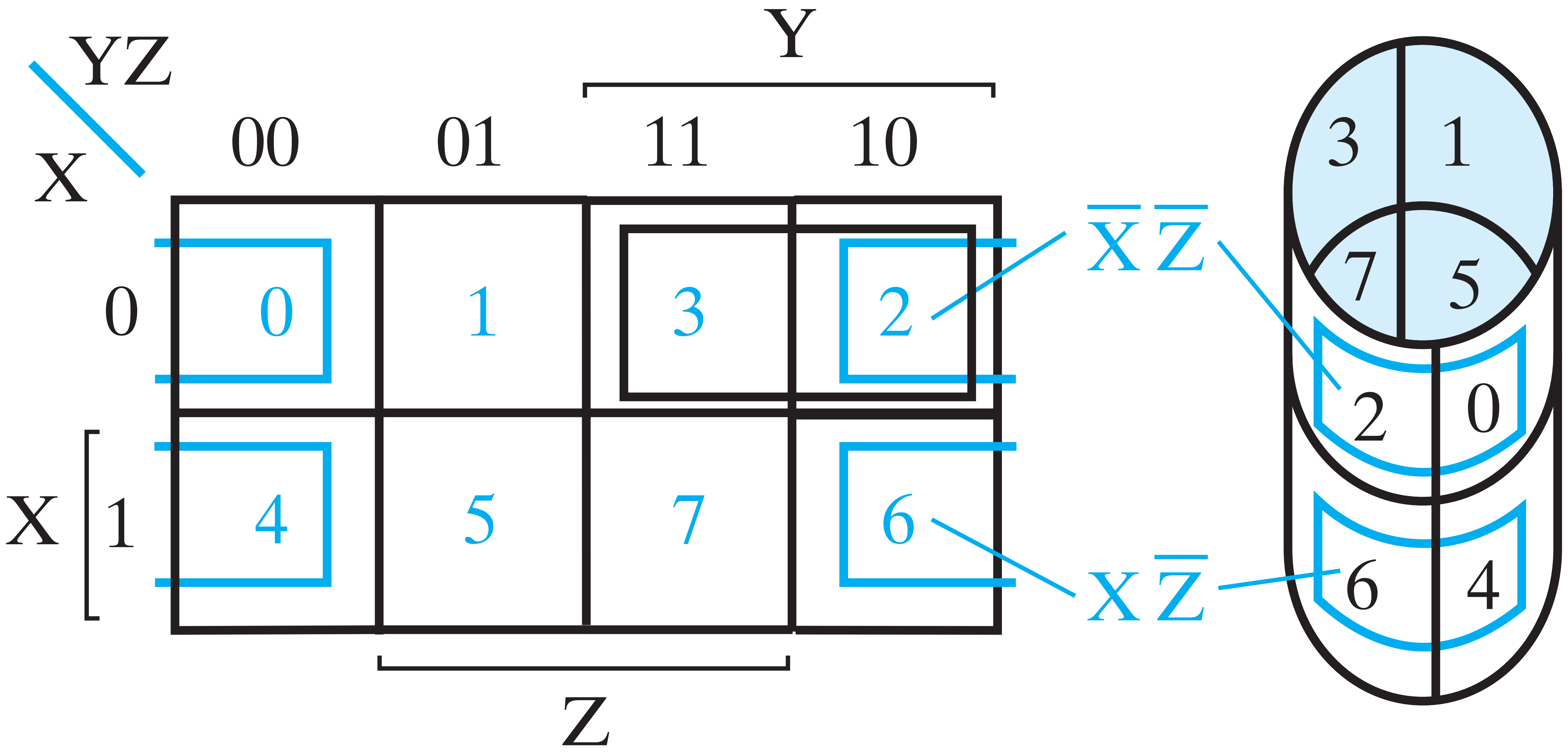
X \ Y	0	1
0	0	1
1	2	3

X \ Y	0	1 (Y)
0	$\overline{X}\,\overline{Y}$	$\overline{X}Y$
1 (X)	$X\overline{Y}$	XY

- Number of squares in each map is equal to the number of minterms for the same number of variables, light blue digit above is the index (of minterm)
- Two squares are adjacent if they only differ in one variable
- Binary value inside at each position indicates the truth table value for that term

P1
Optimisation

Three Variable Maps



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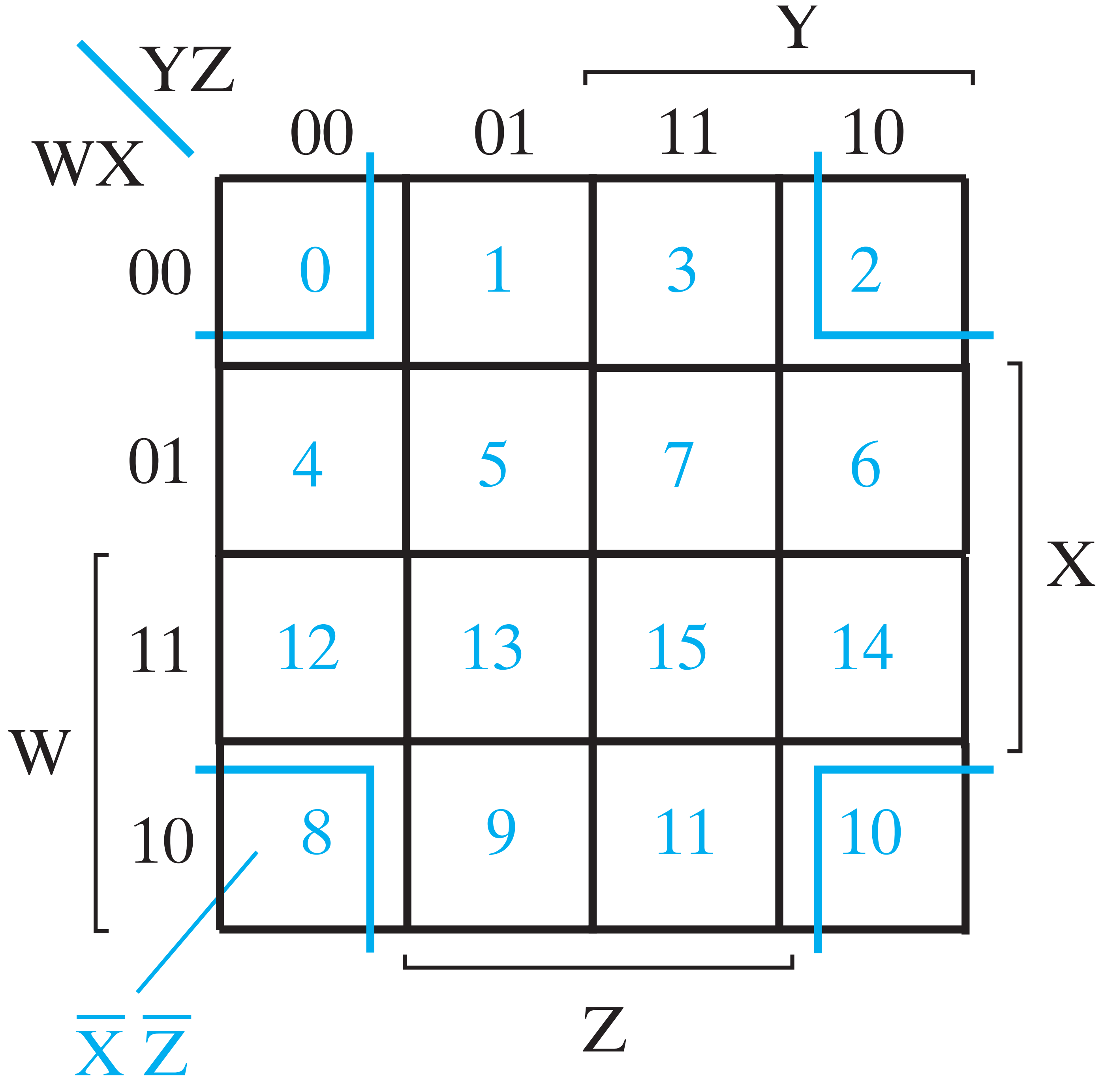
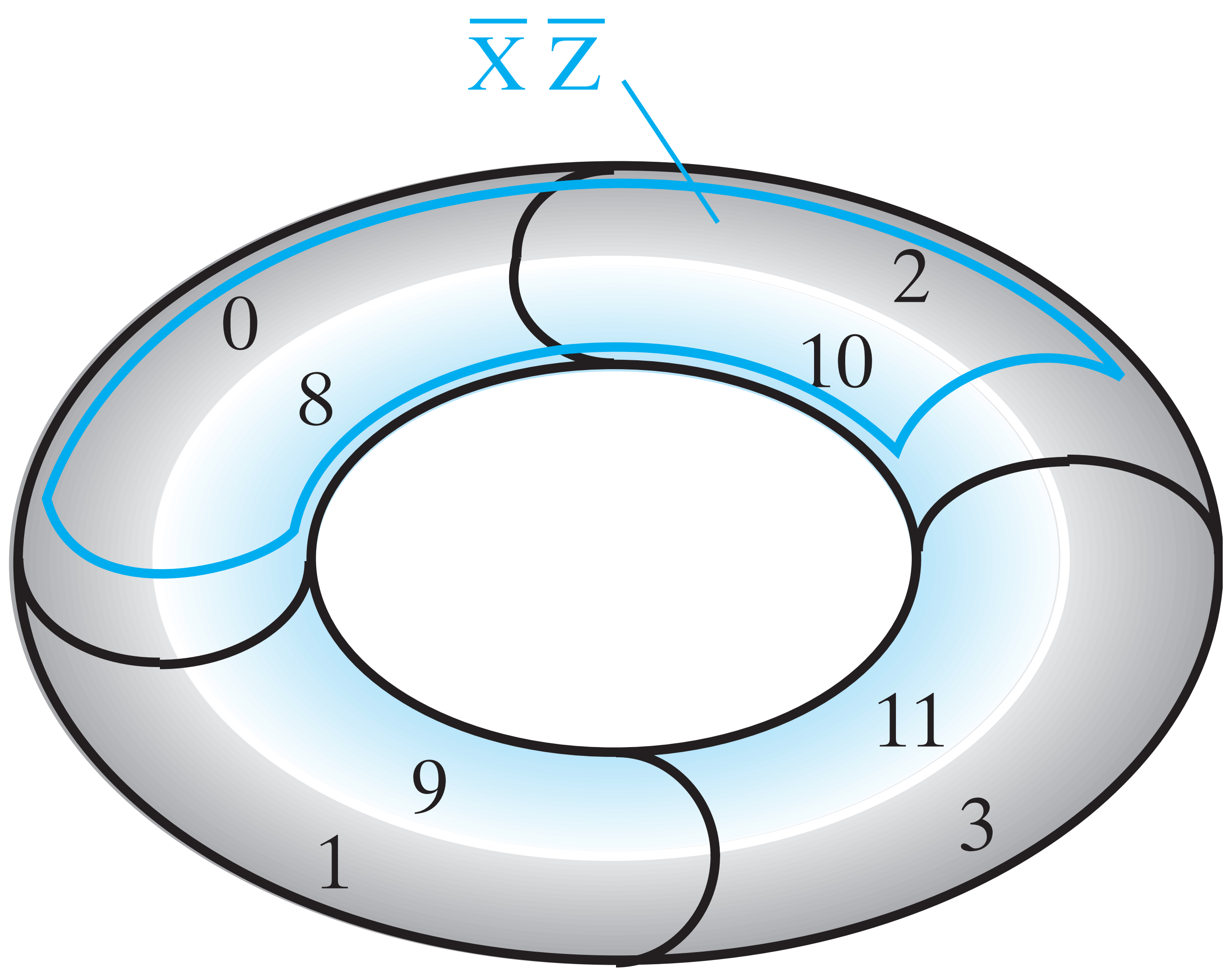
Concept

P1
Optimisation

Four Variable Maps





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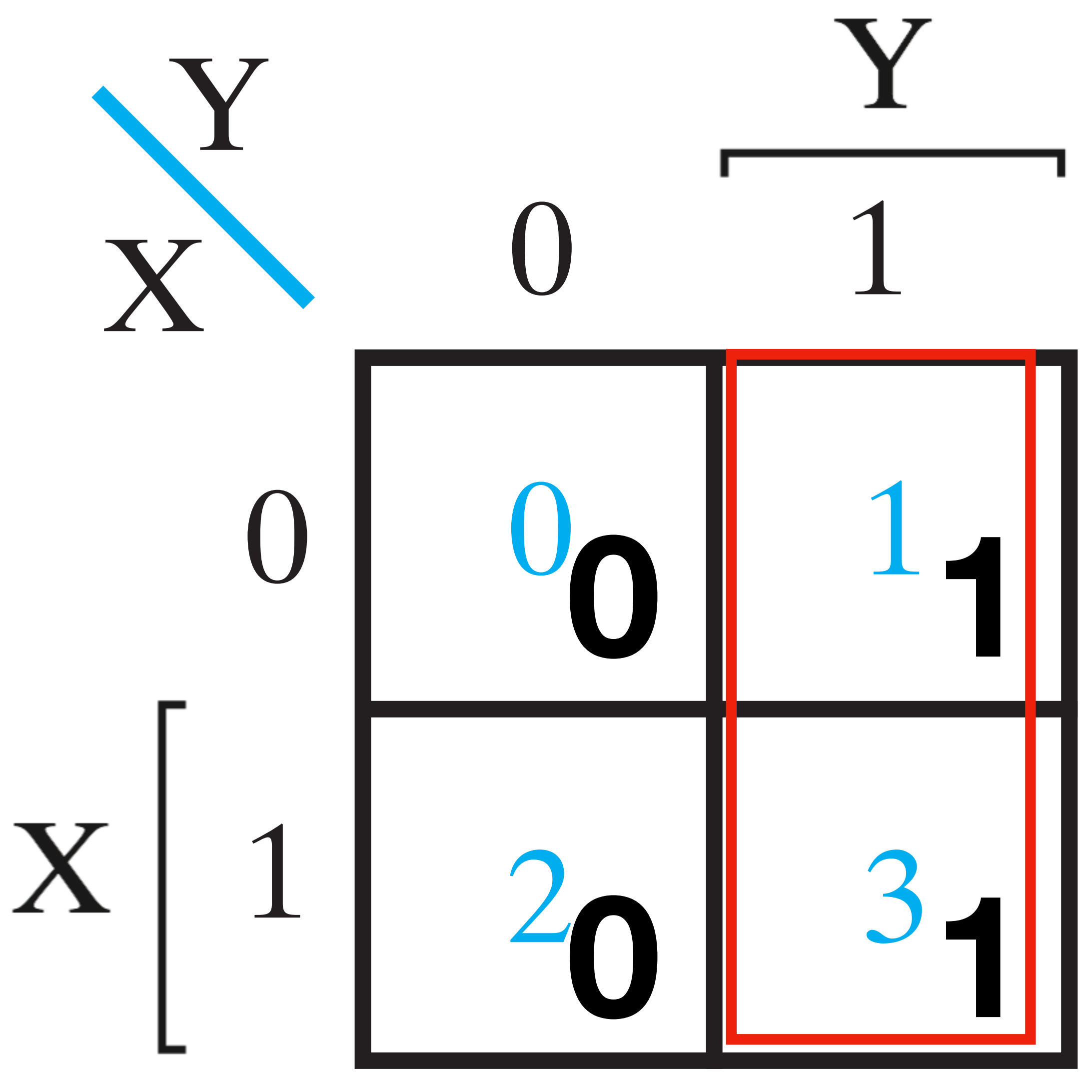
Concept

P1
Optimisation

Two Variable Maps
Optimisation



Truth Table

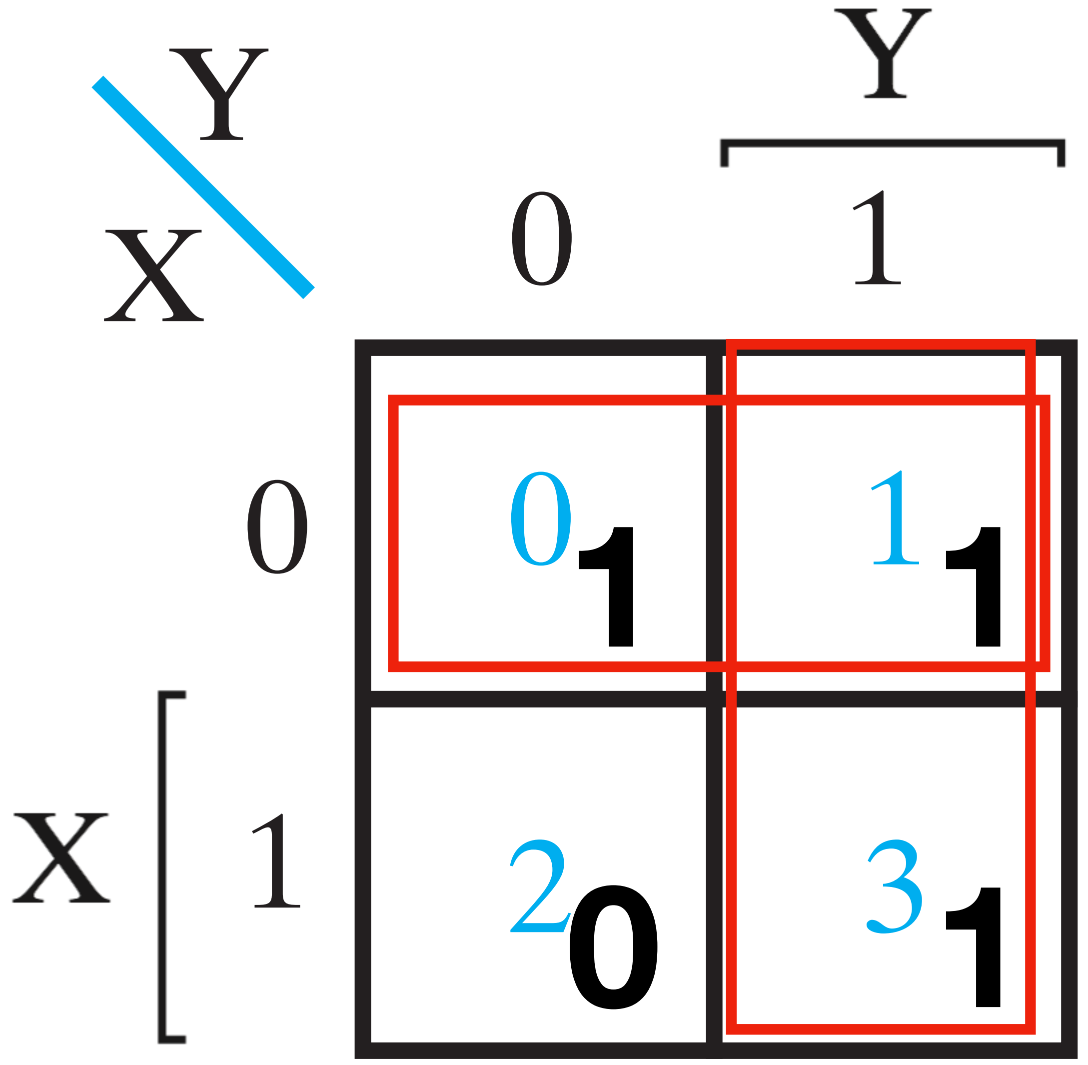
X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s;
  The length of the edge needs to be a power of 2
- Step 3: Read off the selected rectangles, connect with OR

Concept

P1
Optimisation

Two Variable Maps Optimisation



$$\overline{X} + Y$$

Truth Table

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s;
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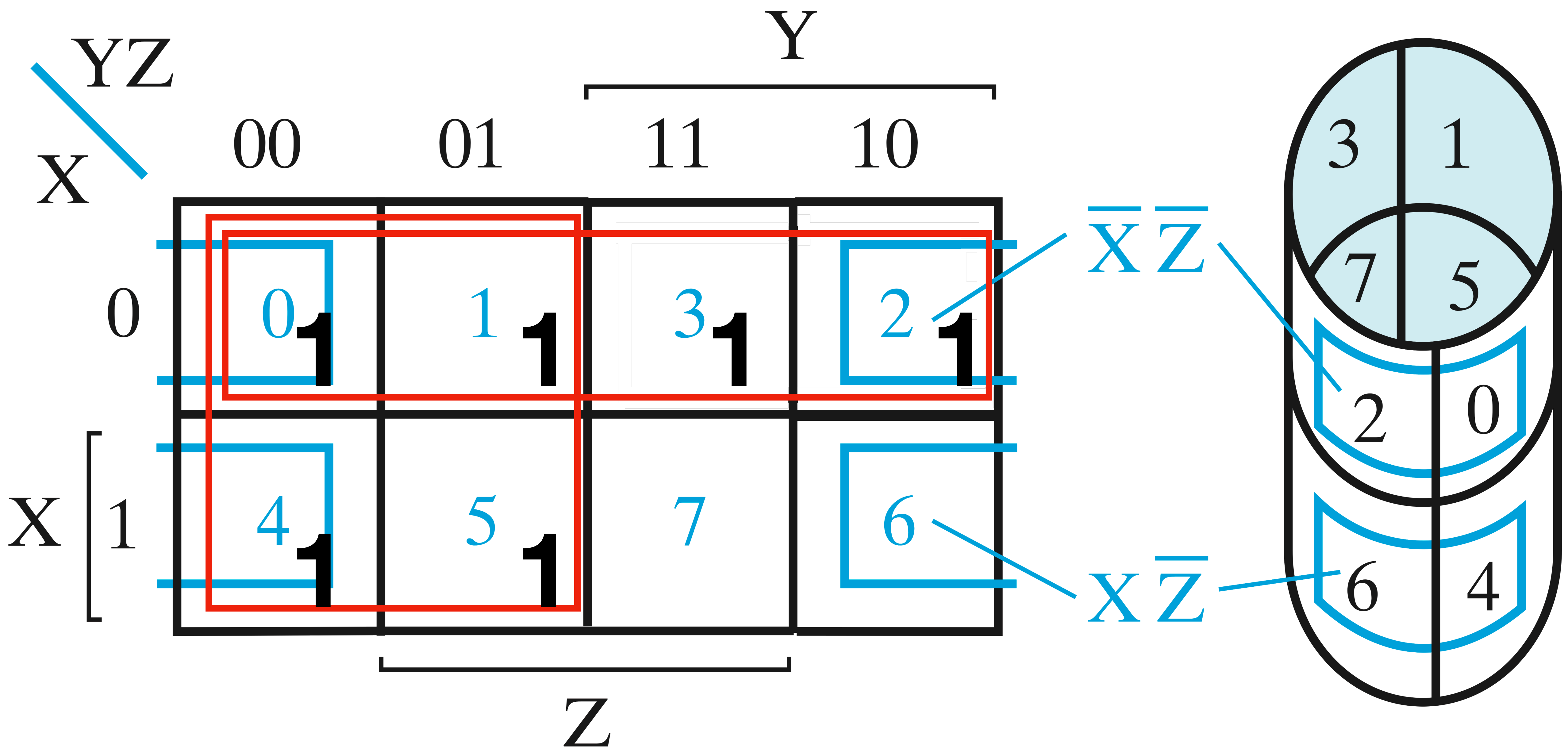
Demo

P1
Optimisation

Three Variable Maps Optimisation



$$F(X,Y,Z) = \Sigma m(0,1,2,3,4,5)$$
$$= \overline{X} + \overline{Y}$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s;
The length of the edge needs to be a power of 2
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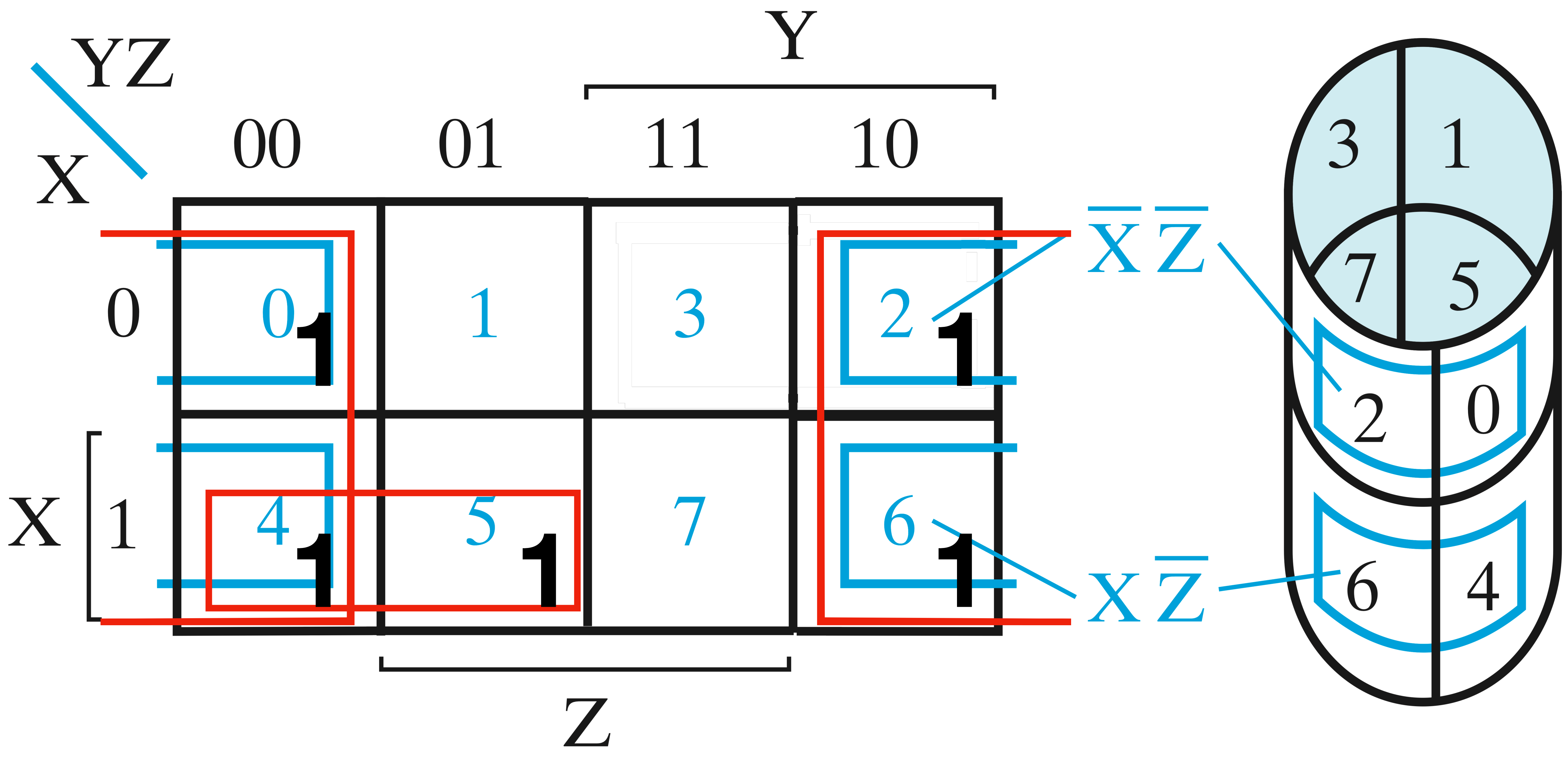
Concept

P1
Optimisation

Three Variable Maps Optimisation

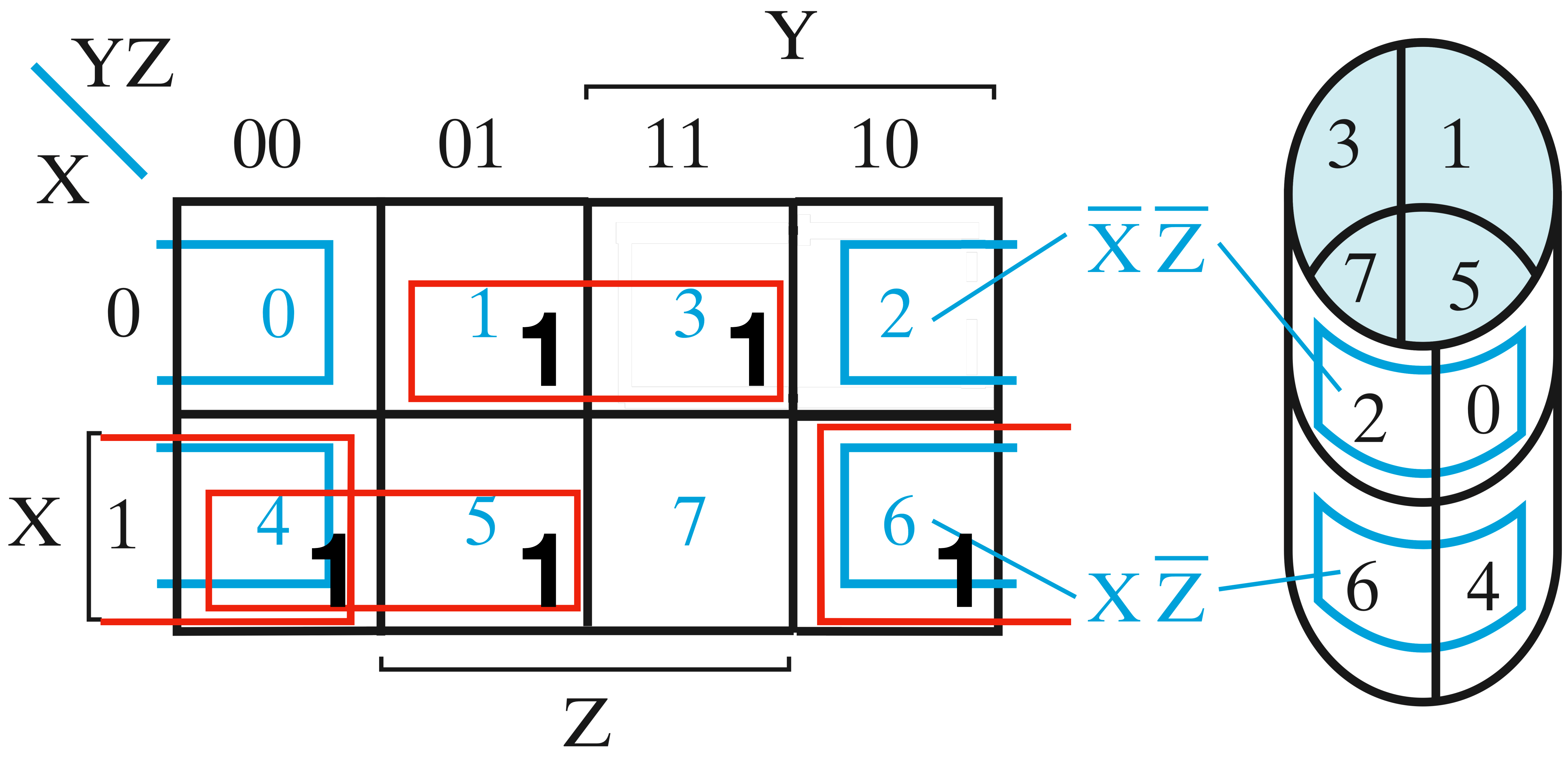


$$F(X, Y, Z) = \Sigma m(0,2,4,5,6)$$
$$= X\overline{Y} + \overline{Z}$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s;
  The length of the edge needs to be a power of 2
- Step 3: Read off the selected rectangles, connect with OR

Demo

Three Variable Maps Optimisation



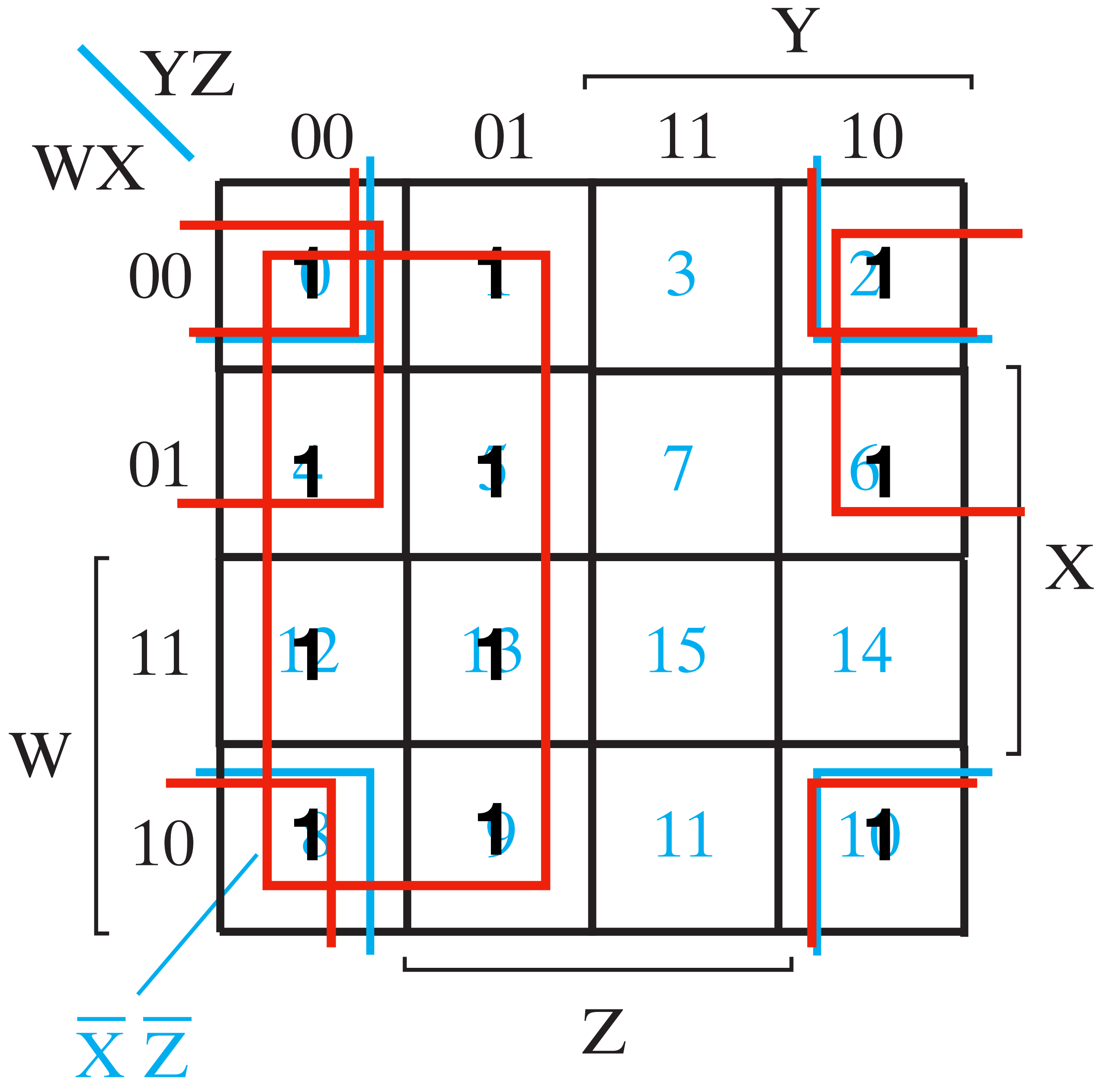
$$F(X,Y,Z) = \Sigma m(1,3,4,5,6)$$
$$= \overline{X}Z + X\overline{Y} + X\overline{Z}$$

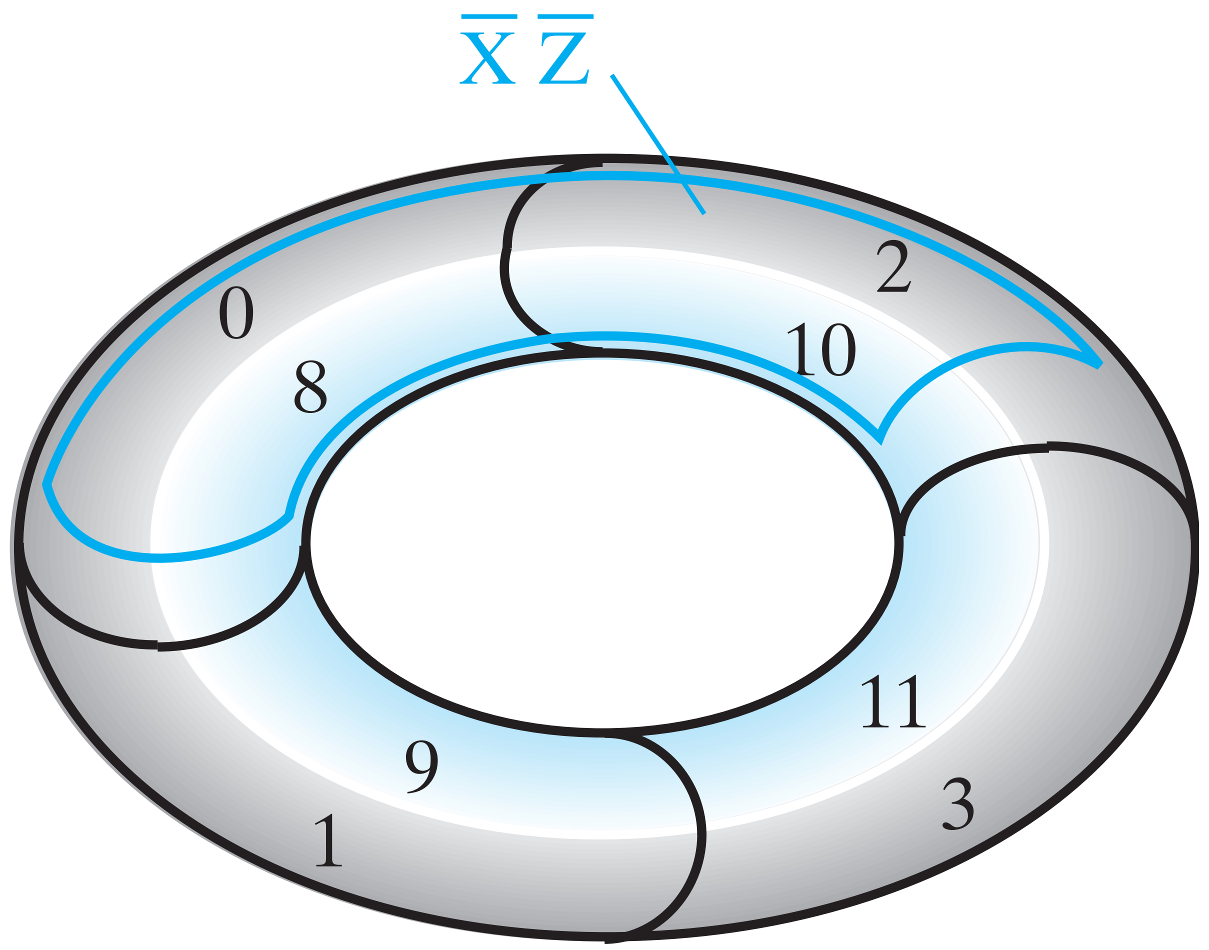
- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s;
  The length of the edge needs to be a power of 2
- Step 3: Read off the selected rectangles, connect with OR

Exercise

P1
Optimisation

Four Variable Maps Optimisation





$$F(W,X,Y,Z) = \Sigma m(0,1,2,4,5,6,8,9,10,12,13)$$
$$= \overline{Y} + \overline{X}\,\overline{Z} + \overline{W}\,\overline{Z}$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s;
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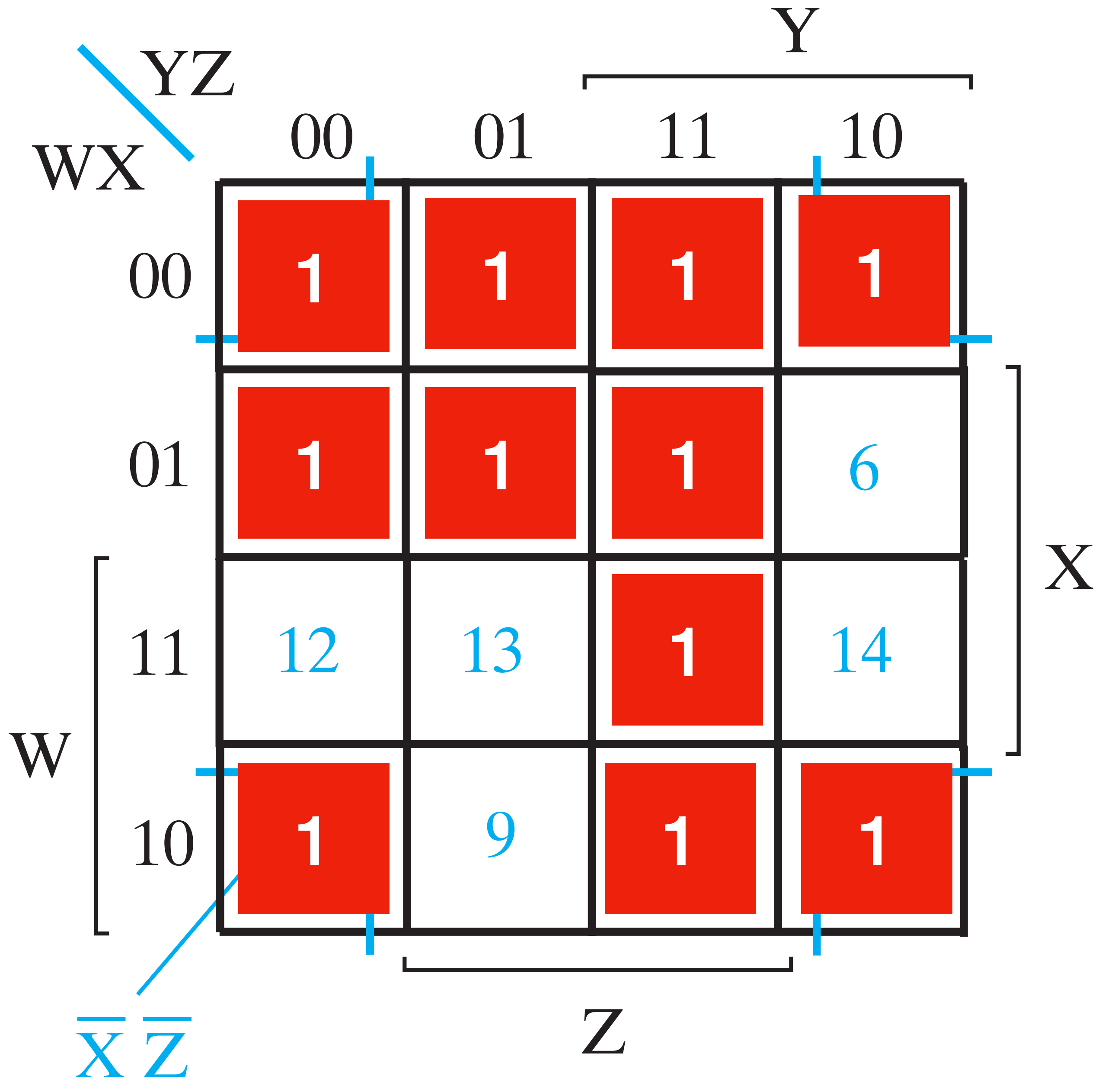
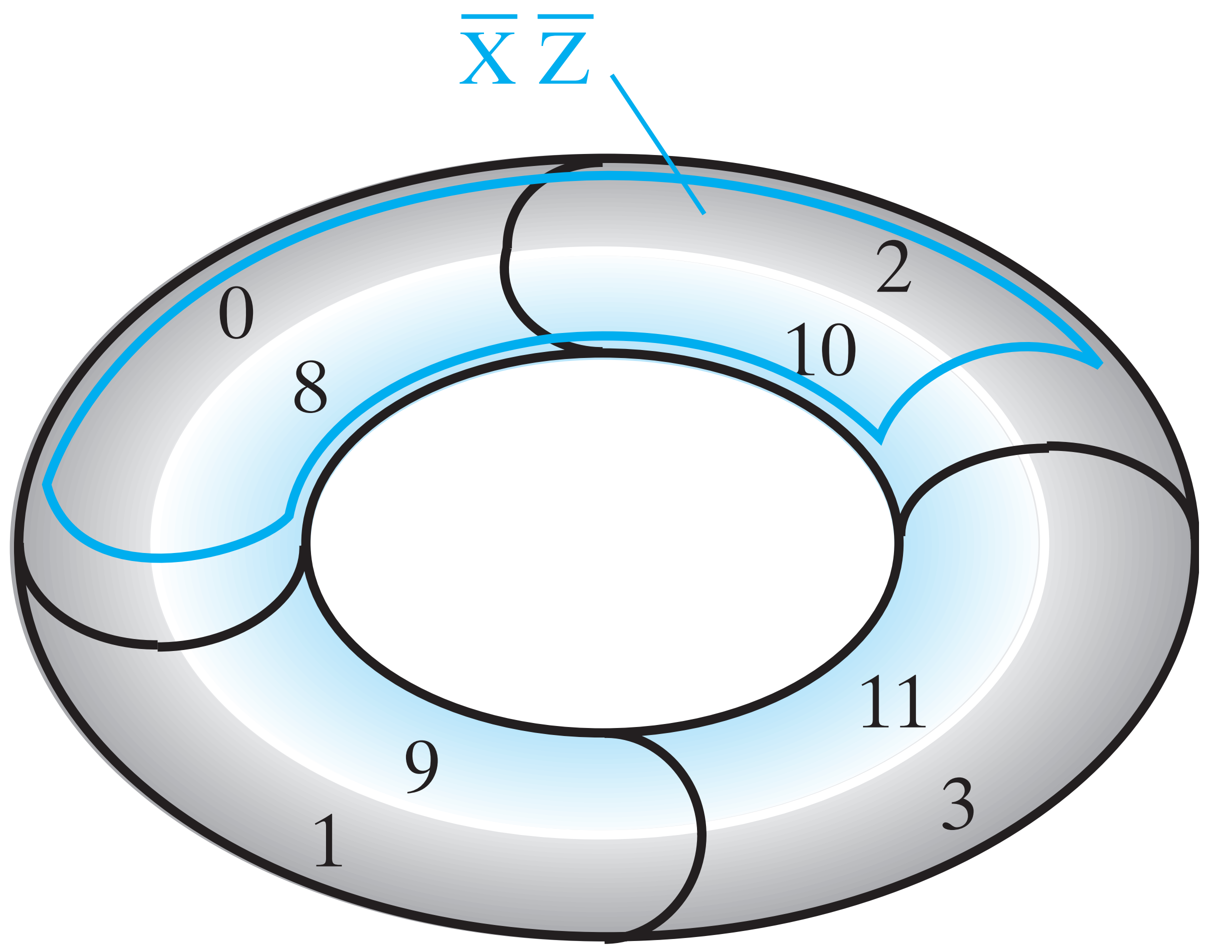
Exercise

P1
Optimisation

Four Variable Maps Optimisation





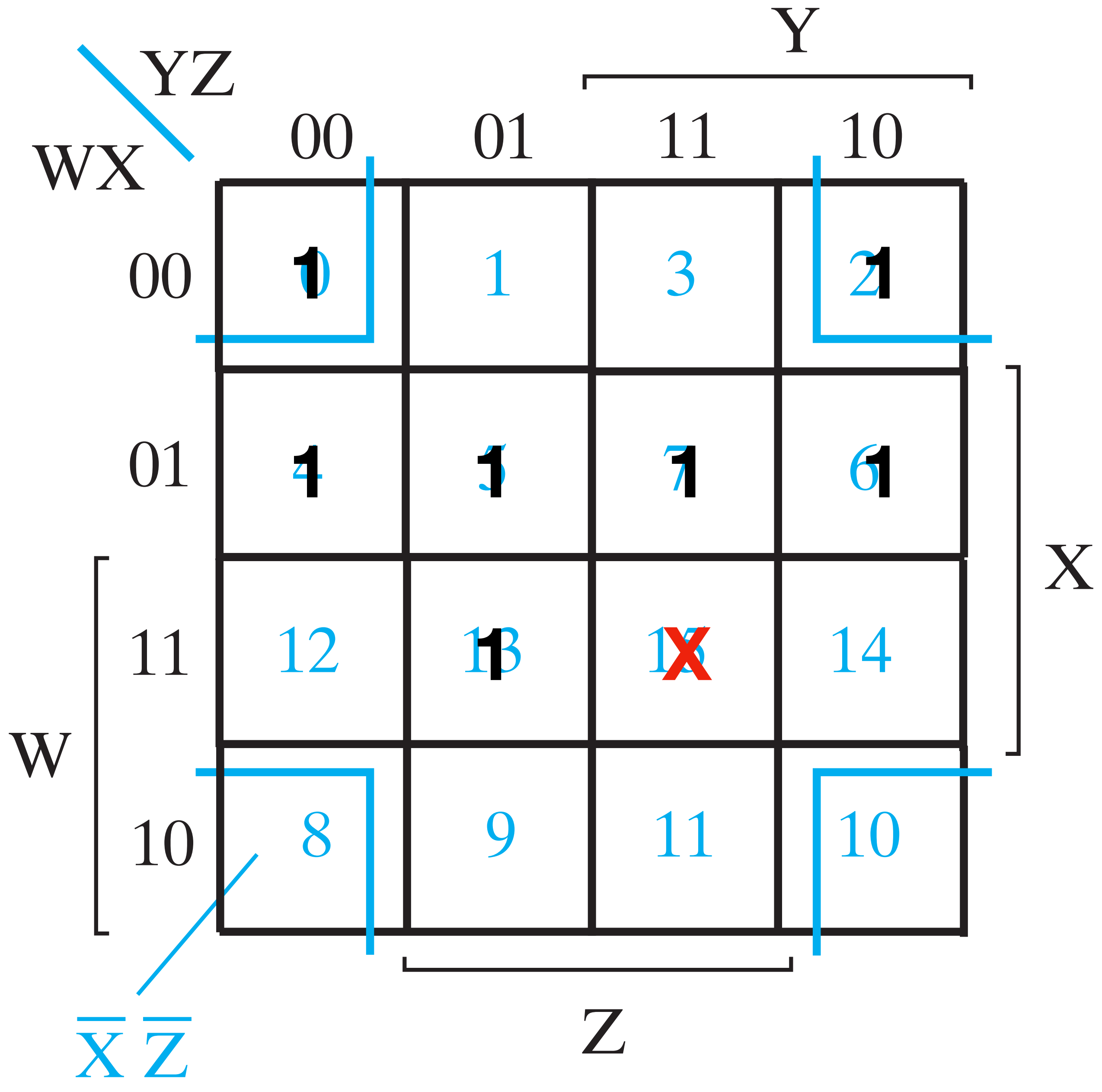
$$F(W,X,Y,Z) = \overline{W}\,\overline{Y}\,\overline{Z} + \overline{W}Z + \overline{X}Y + YZ + W\overline{X}\,\overline{Z}$$

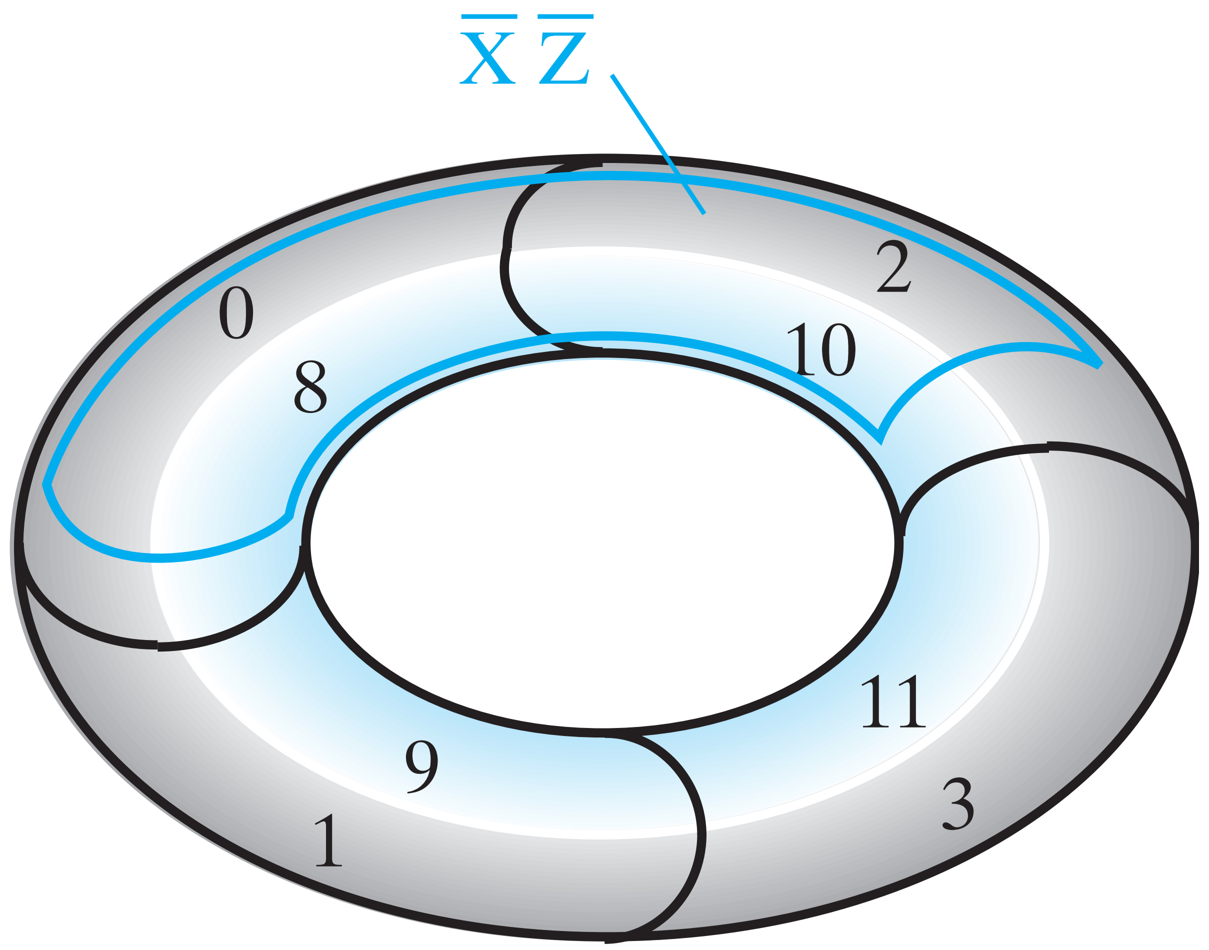
- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s;
  The length of the edge needs to be a power of 2
- Step 3: Read off the selected rectangles, connect with OR

Exercise

P1 Optimisation

Don’t Care Condition



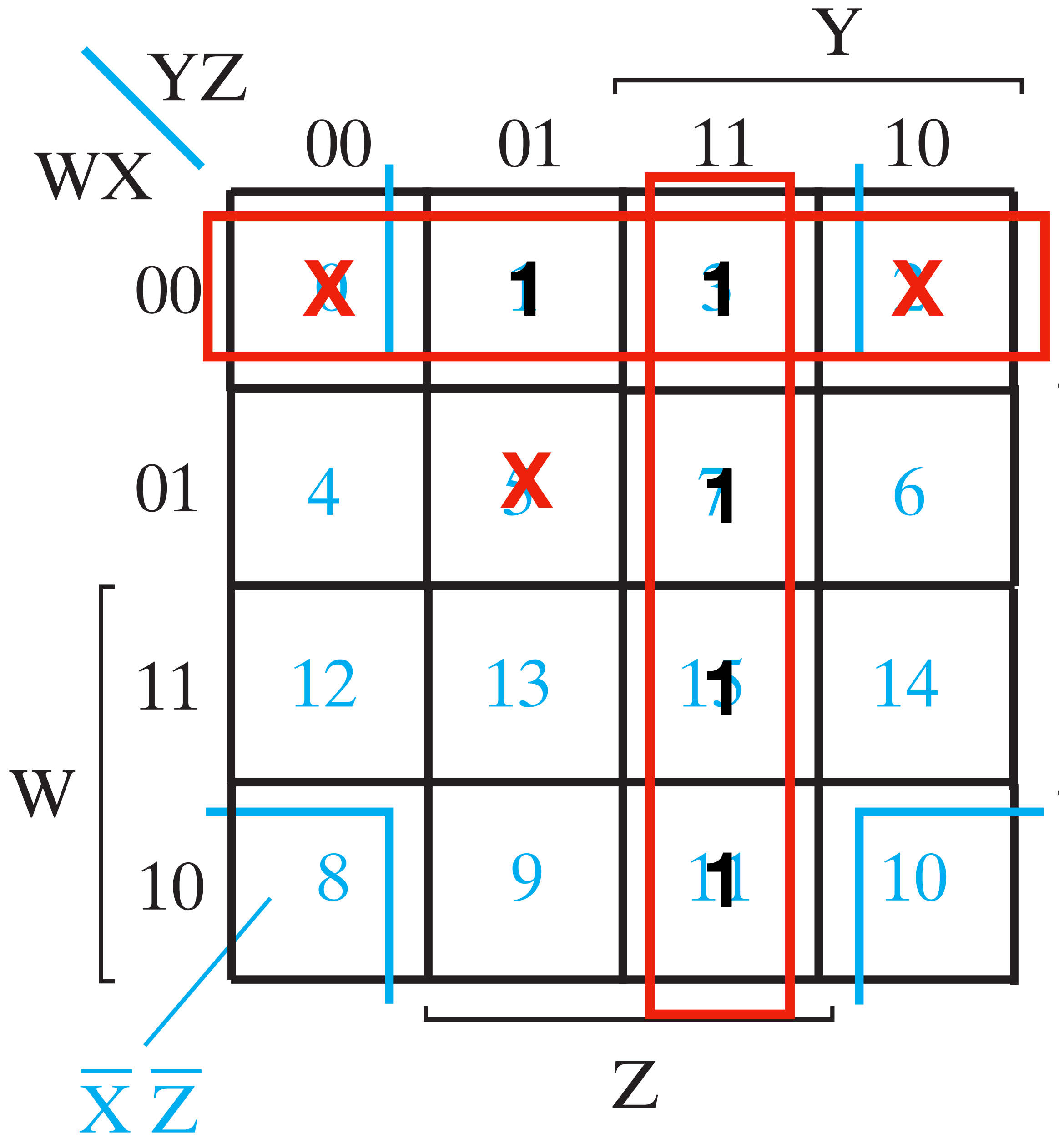


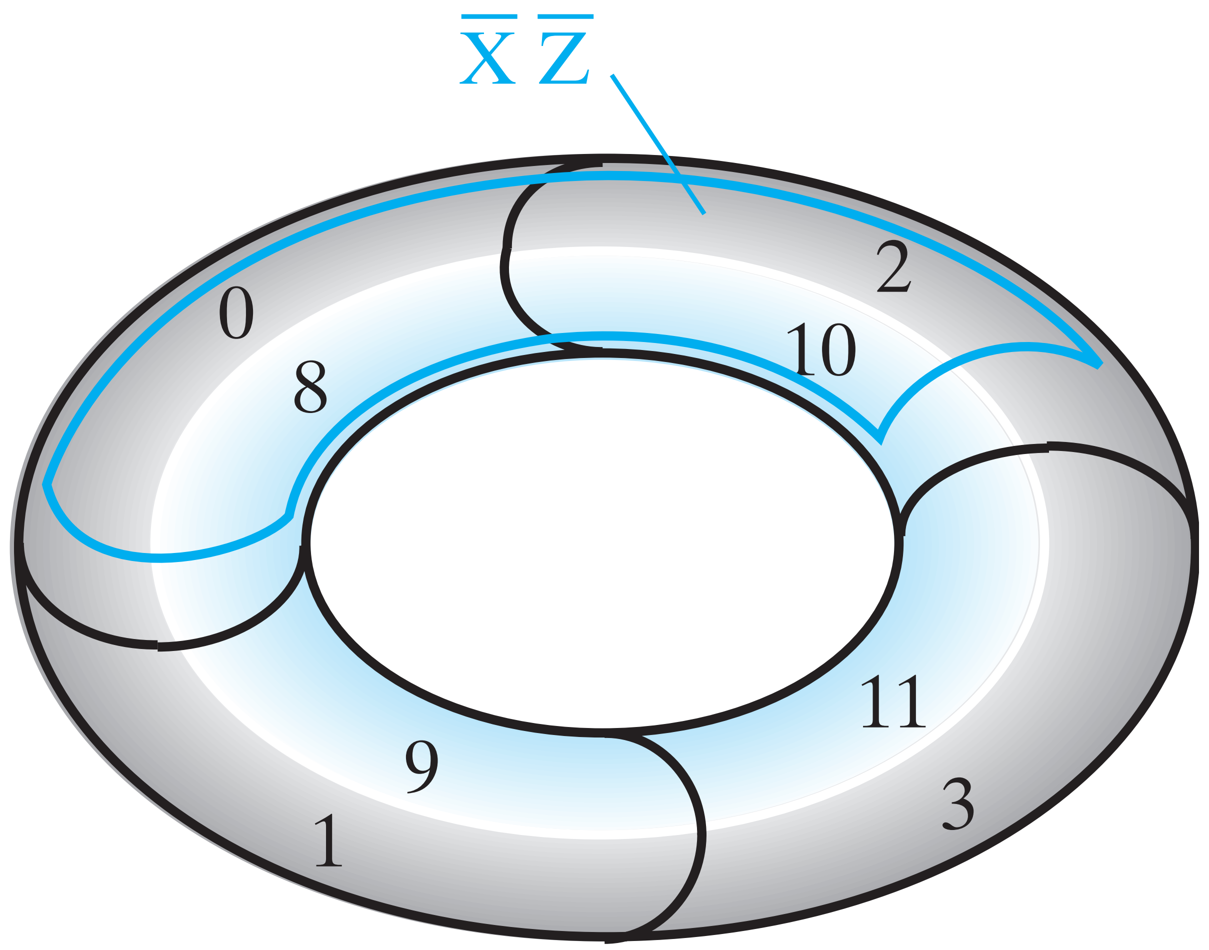
- Sometimes we don’t care what the output is when the inputs are in certain combinations

Concept

P1
Optimisation

Don’t Care Condition





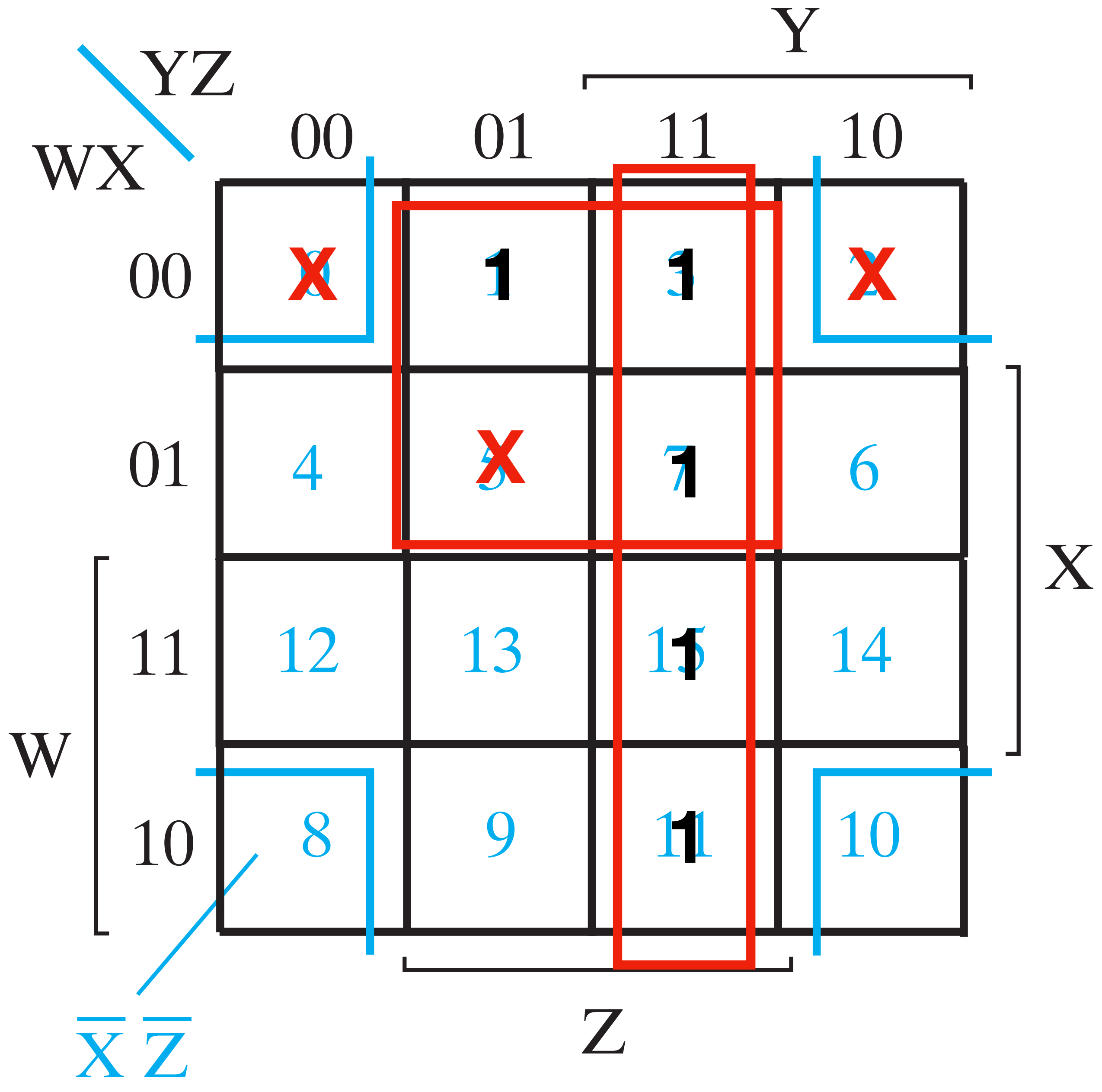
- Sometimes we don’t care what the output is when the inputs are in certain combinations

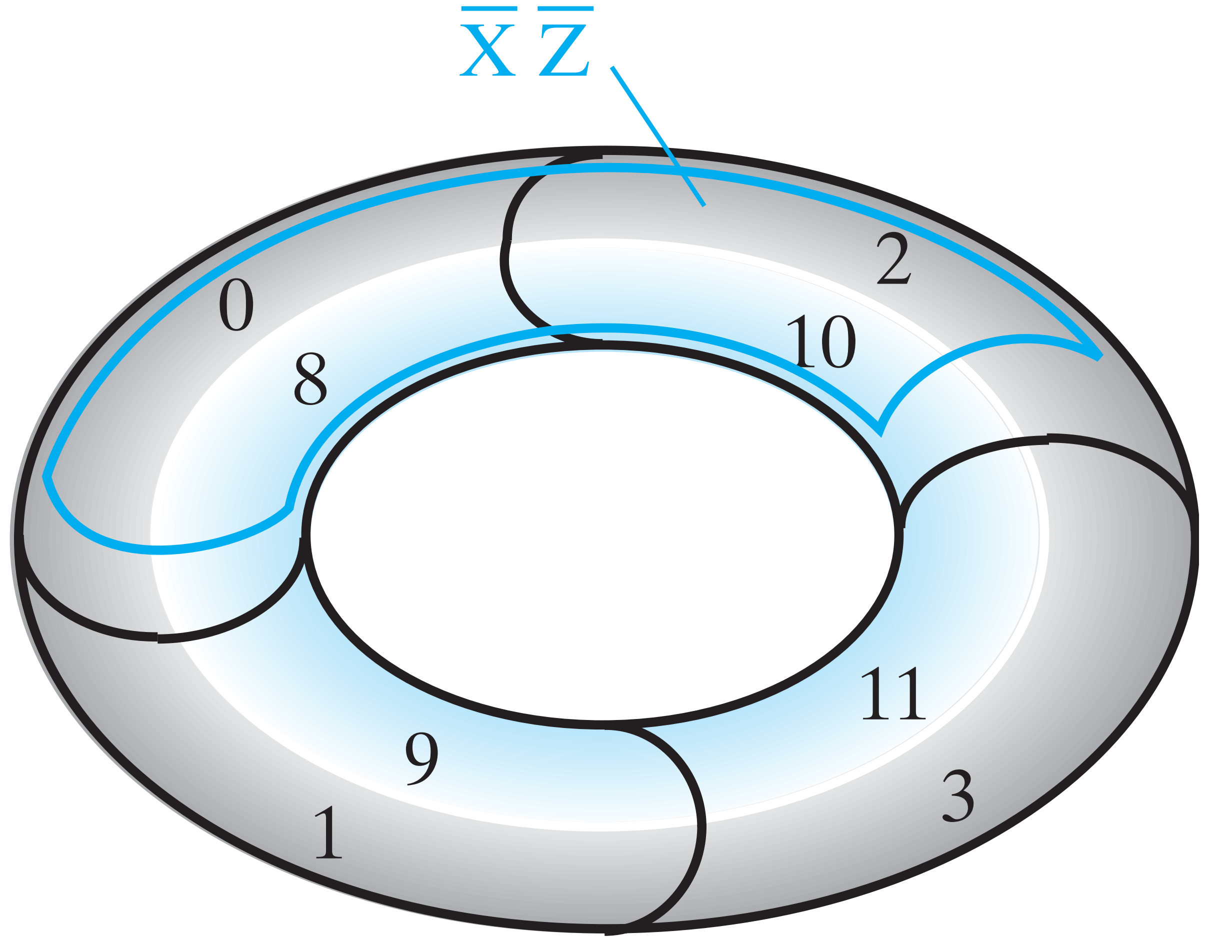
$$F = YZ + \overline{W}\overline{X}$$

Demo

P1
Optimisation

Don’t Care Condition





- Sometimes we don’t care what the output is when the inputs are in certain combinations

$$F = YZ + \overline{W}Z$$

Demo

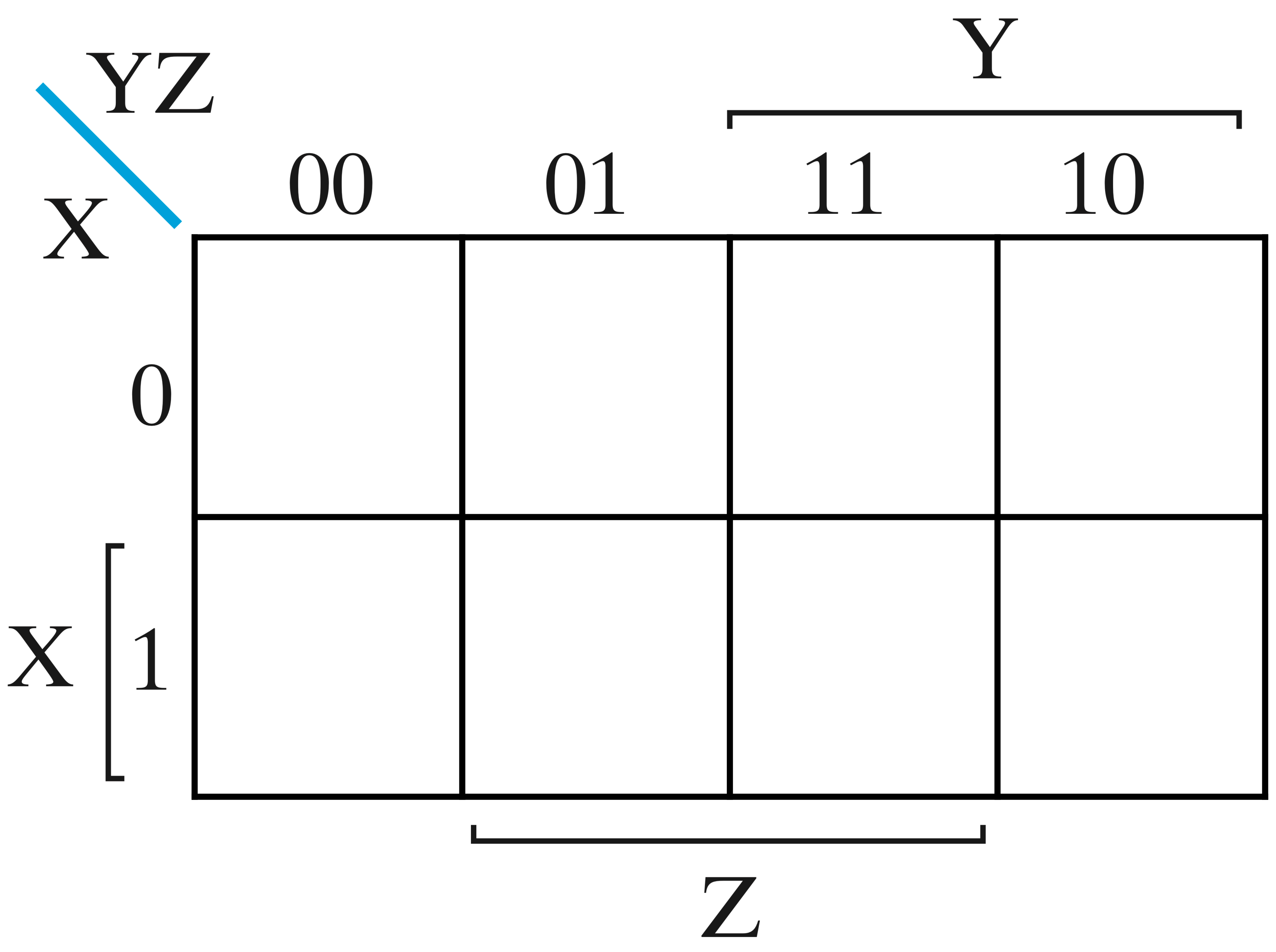
P1
Optimisation

Summary

- Boolean Algebra III: K-Map
  - Two Variable K-Map
  - Three Variable K-Map
  - Four Variable K-Map
  - Don’t care optimisation

Review

Exercises



$$F(X, Y, Z) = \Sigma m(0,2,6,7)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s;
  The length of the edge needs to be a power of 2
- Step 3: Read off the selected rectangles, connect with OR

Exercises

X \ YZ	00	01	11	10
0	1			1
1			1	1

$$F(X, Y, Z) = \Sigma m(0,2,6,7)$$

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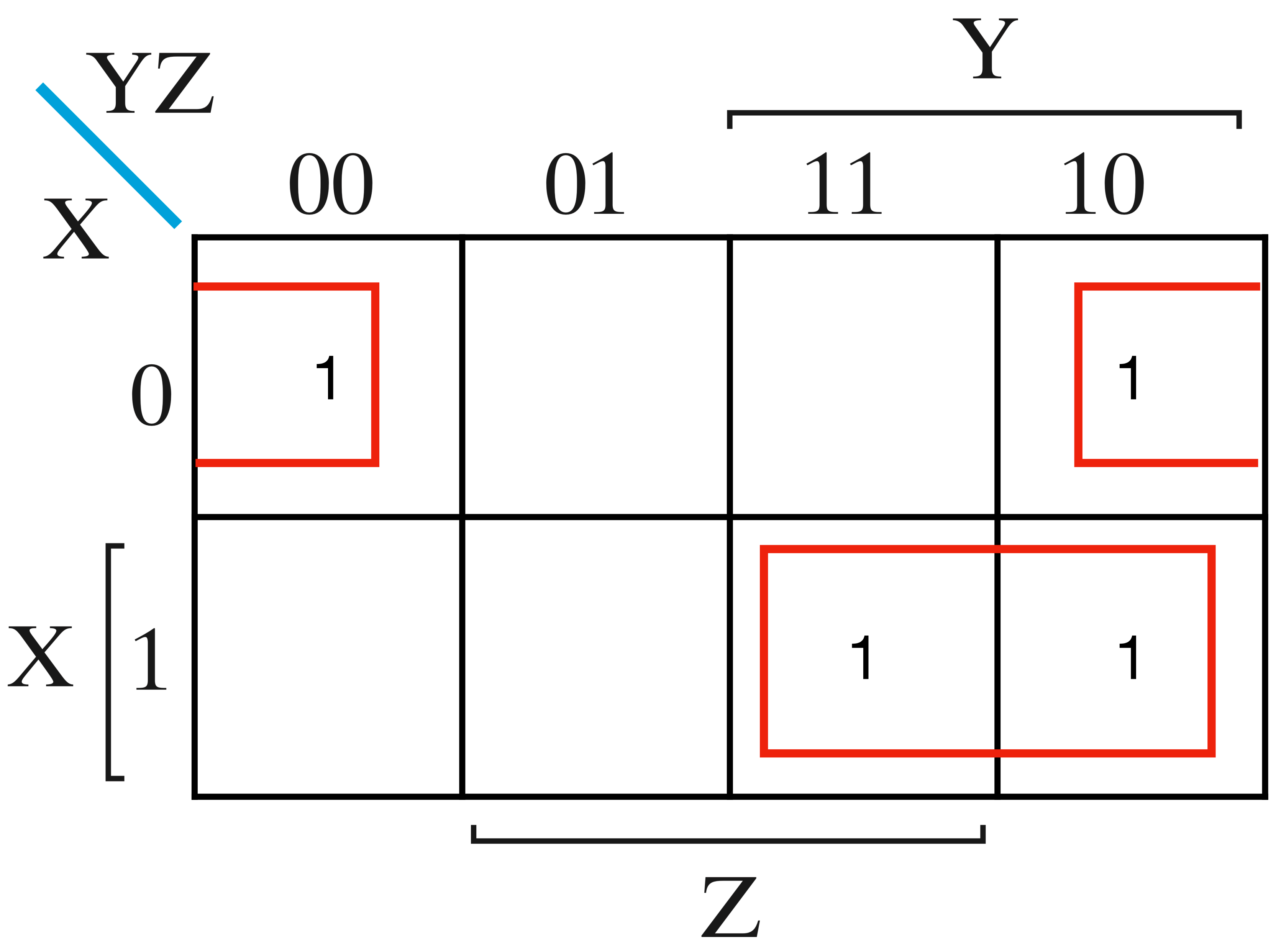
Exercise

Exercises



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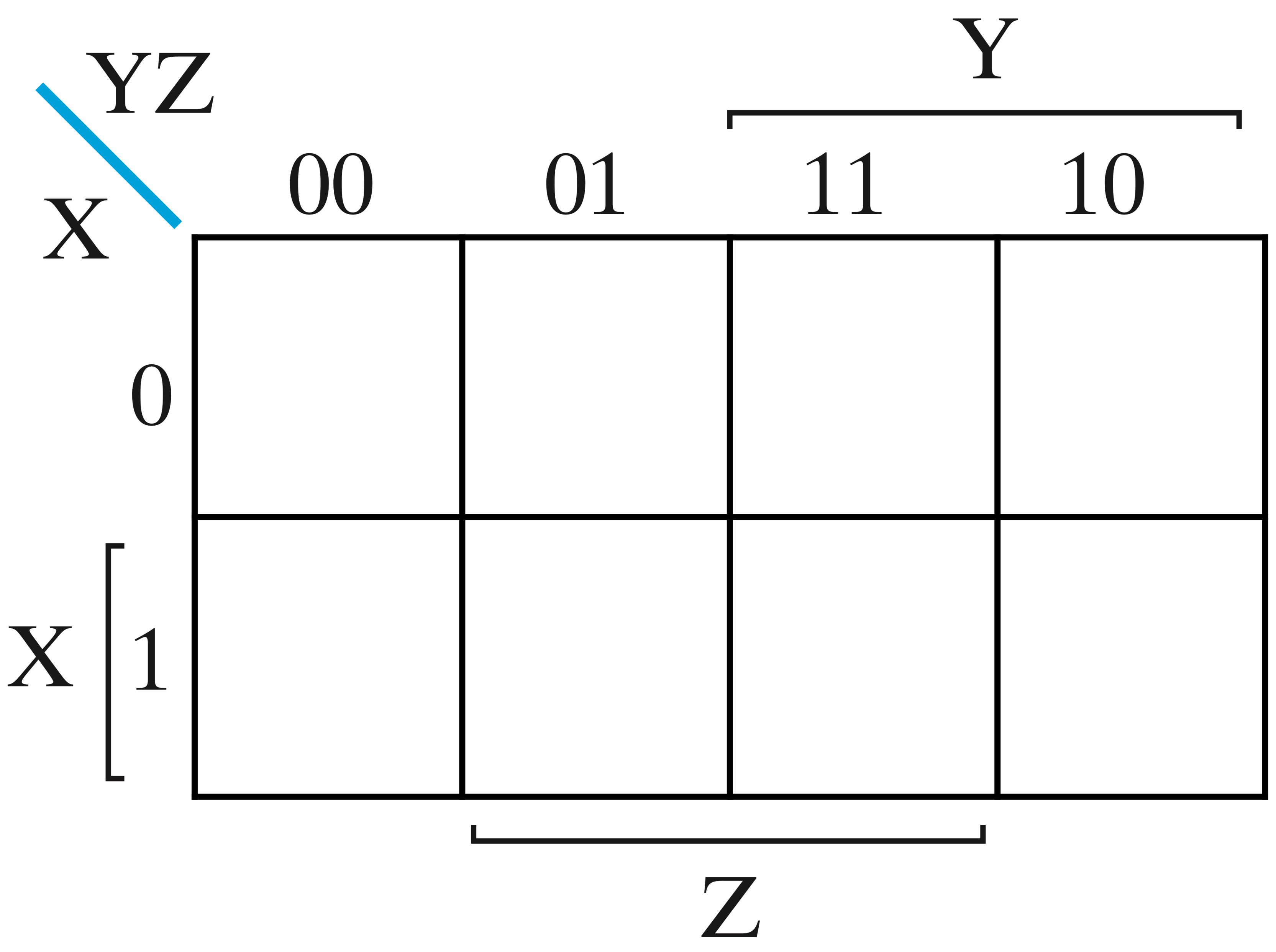
Exercise

Exercises



$$F(X, Y, Z) = \Sigma m(0,1,2,4)$$

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Exercise

Exercises

X \ YZ	00	01	11	10
0	1	1		1
1	1			

$$F(X,Y,Z) = \Sigma m(0,1,2,4)$$

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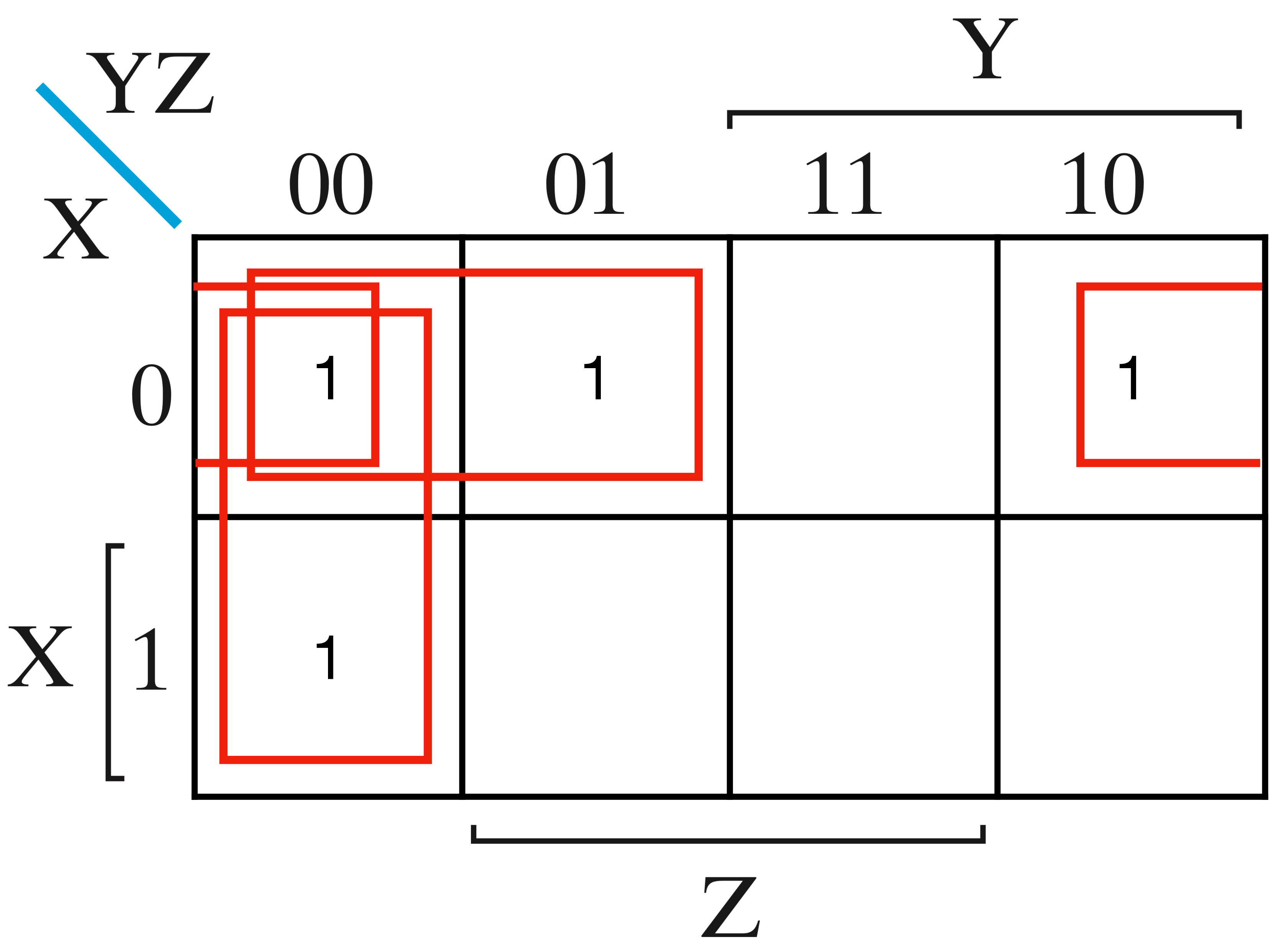
Exercise

Exercises



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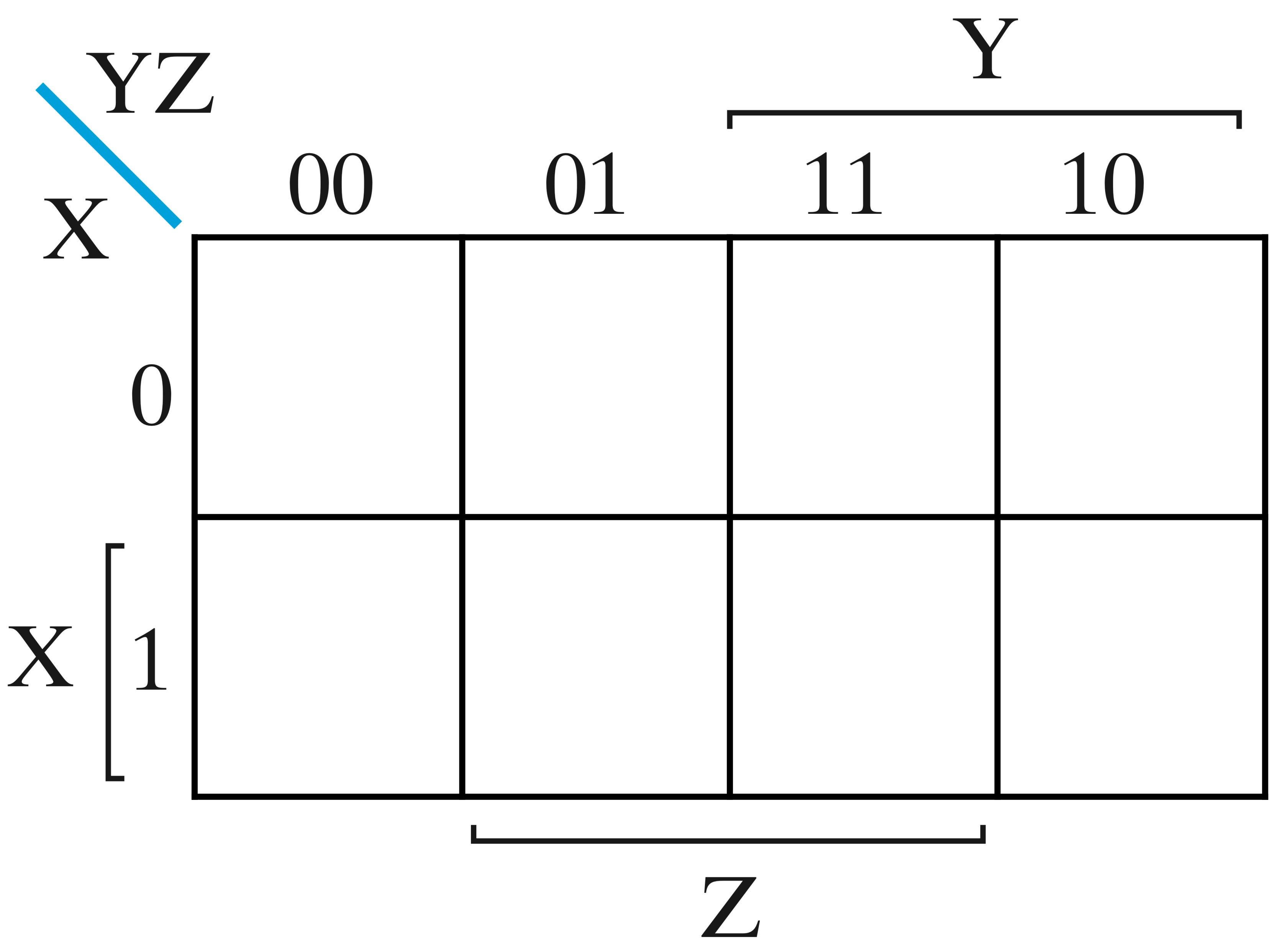
Exercise

Exercises



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Exercise

Exercises

X \ YZ	00	01	11	10
0	1		1	1
1	1			1

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Exercise

Exercises

X \ YZ	00	01	11	10
0	1		1	1
1	1			1

Y
Z
X

$$F(X, Y, Z) = \Sigma m(0,2,3,4,6)$$

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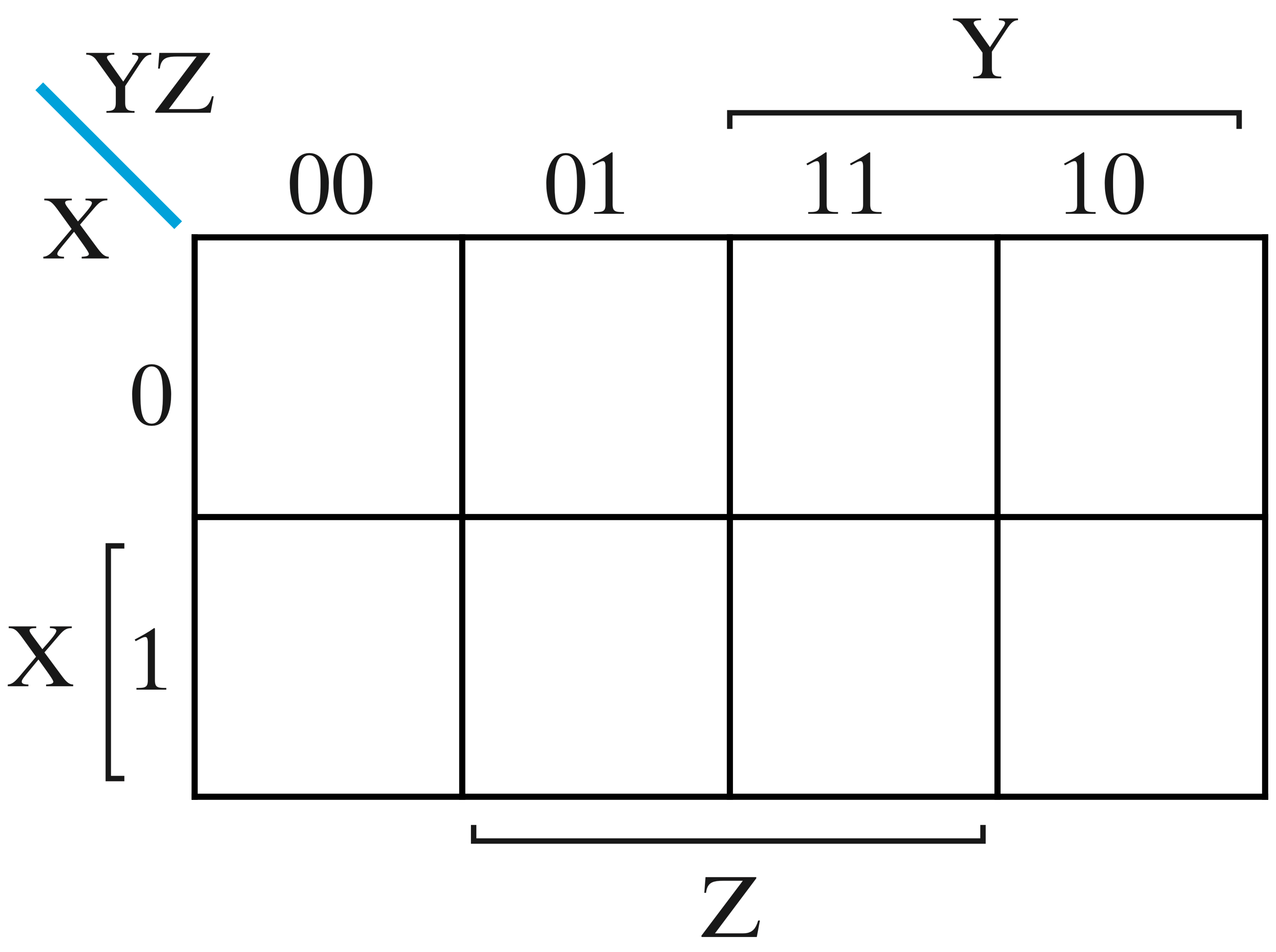
Exercise

Exercises



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Exercises

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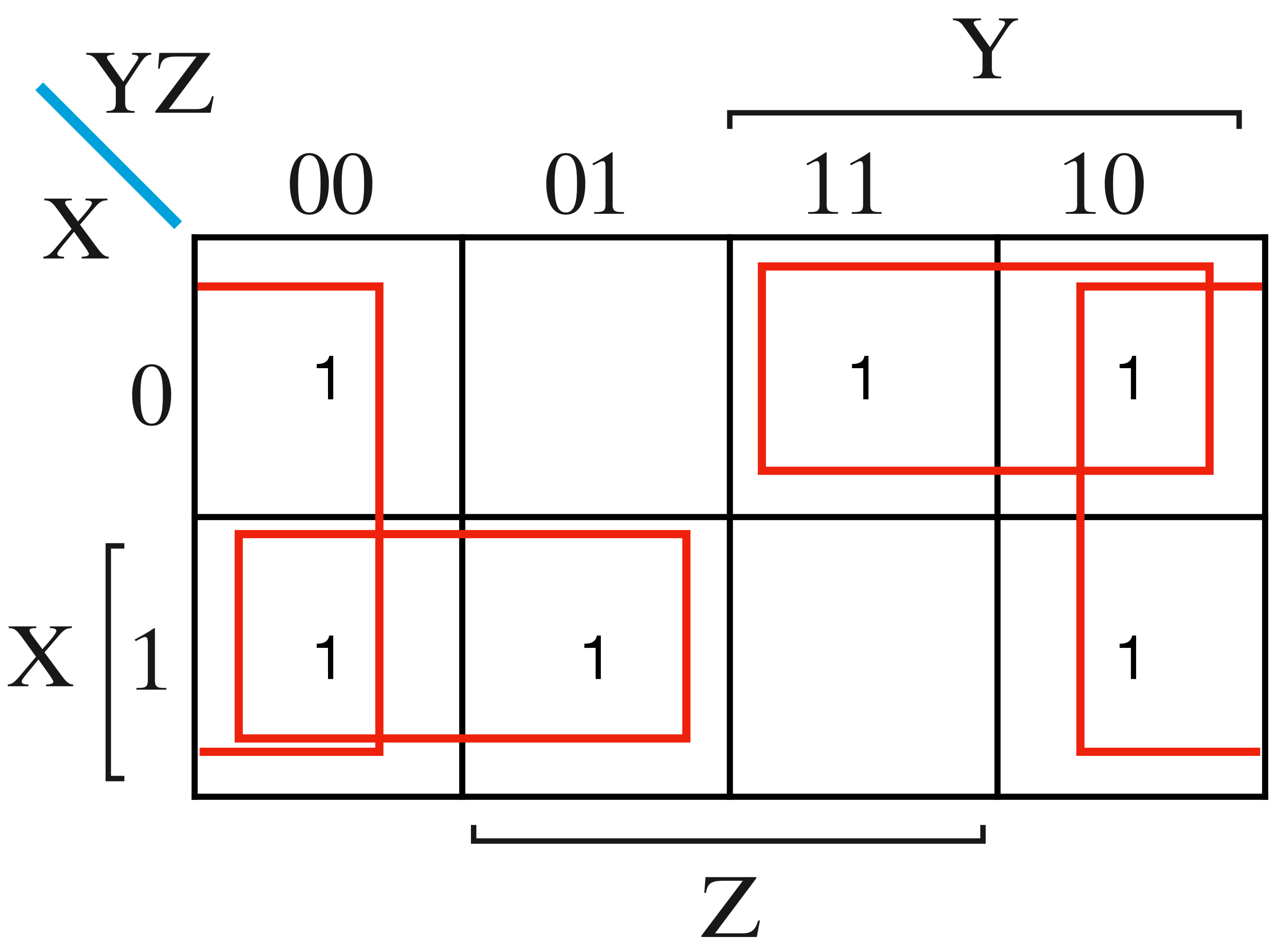
Exercise

P1
Optimisation

Exercises



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