



15.10.20 11:55

# CSCI 150

## Introduction to Digital and Computer System Design

### Lecture 3: Combinational Logic Design IV



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2020 Fall Semester (S3)

# Overview

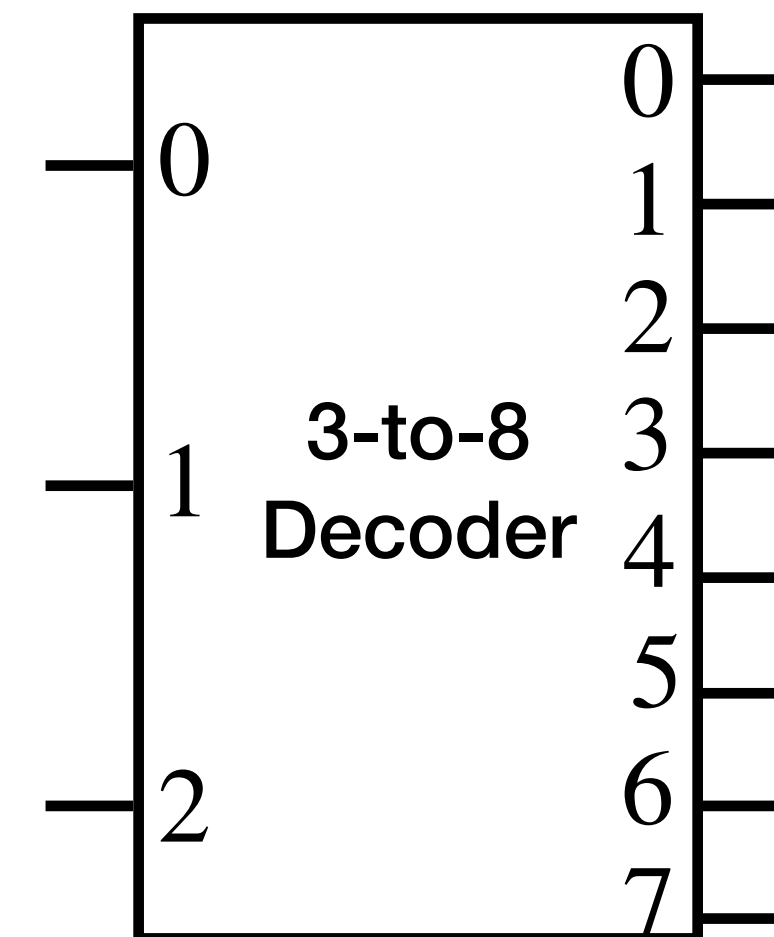
- Focus: Logic Functions
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch3 3.6, 3.7; v5: Ch3 3.6, 3.7
- Core Ideas:
  1. Encoder
  2. Multiplexer

# Systematic Design Procedures

1. **Specification:** Write a specification for the circuit
2. **Formulation:** Derive relationship between inputs and outputs of the system  
e.g. using truth table or Boolean expressions
3. **Optimisation:** Apply optimisation, minimise the number of logic gates and literals required
4. **Technology Mapping:** Transform design to new diagram using available implementation technology
5. **Verification:** Verify the correctness of the final design in meeting the specifications

# Functional Components

- Value-Fixing, Transferring, Inverting, Enabler
- Decoder
  - Input:  $A_0A_1 \dots A_{n-1}$
  - Output:  $D_0D_1 \dots D_{2^n-1}$ ,  $D_i = m_i$

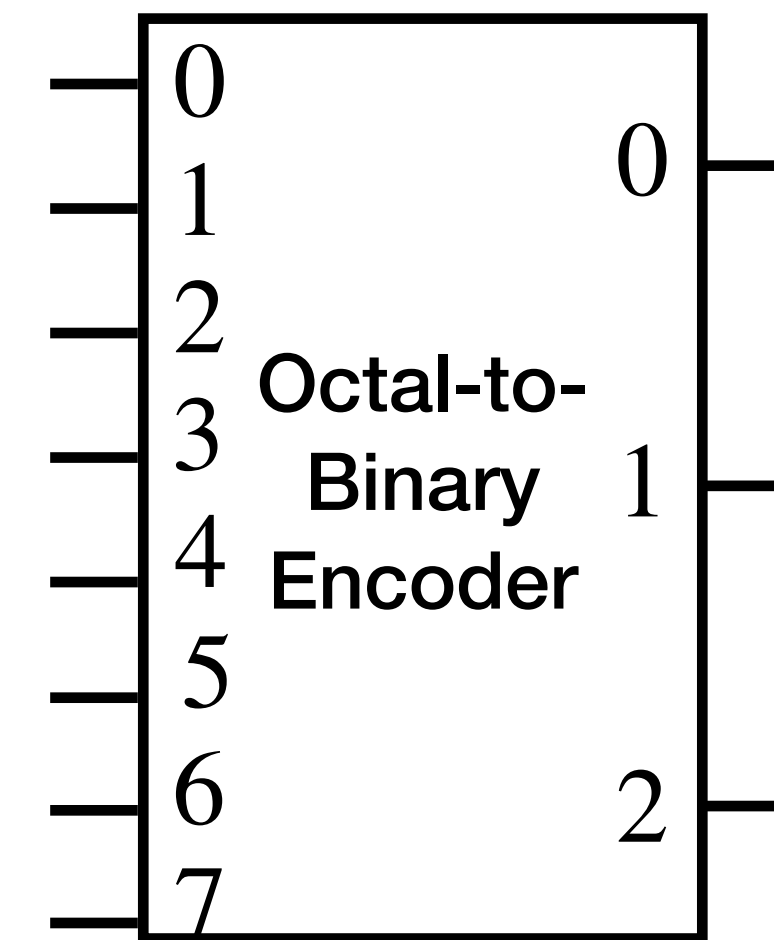
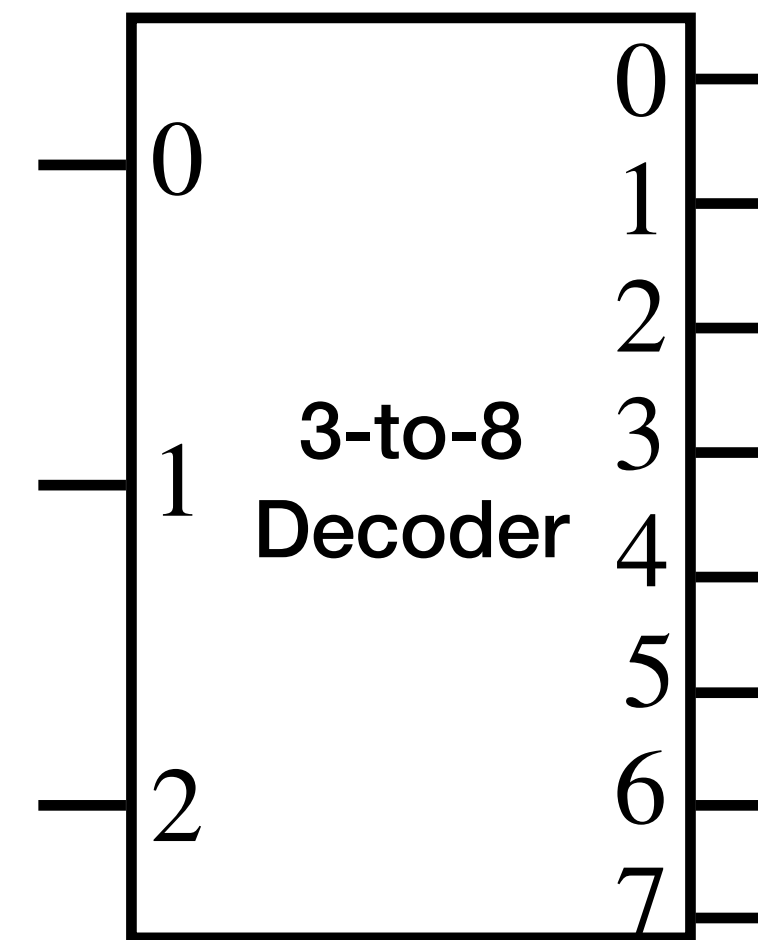


# Encoder

Wait, didn't we just covered this?  
Oh, that's decoder

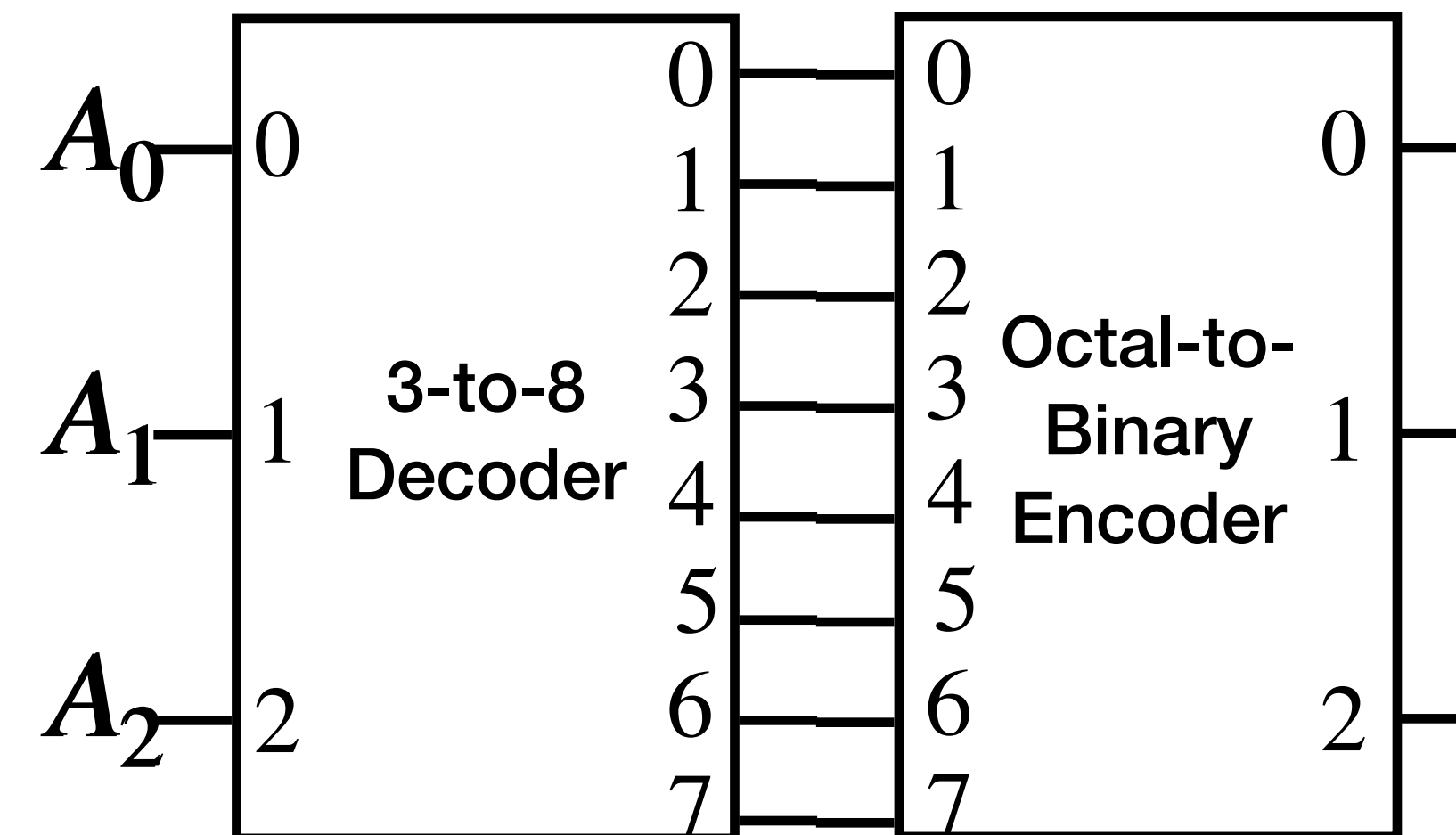
# Encoder

- Inverse operation of a decoder
- $2^n$  inputs, only one is giving positive input<sup>1</sup>
- $n$  outputs



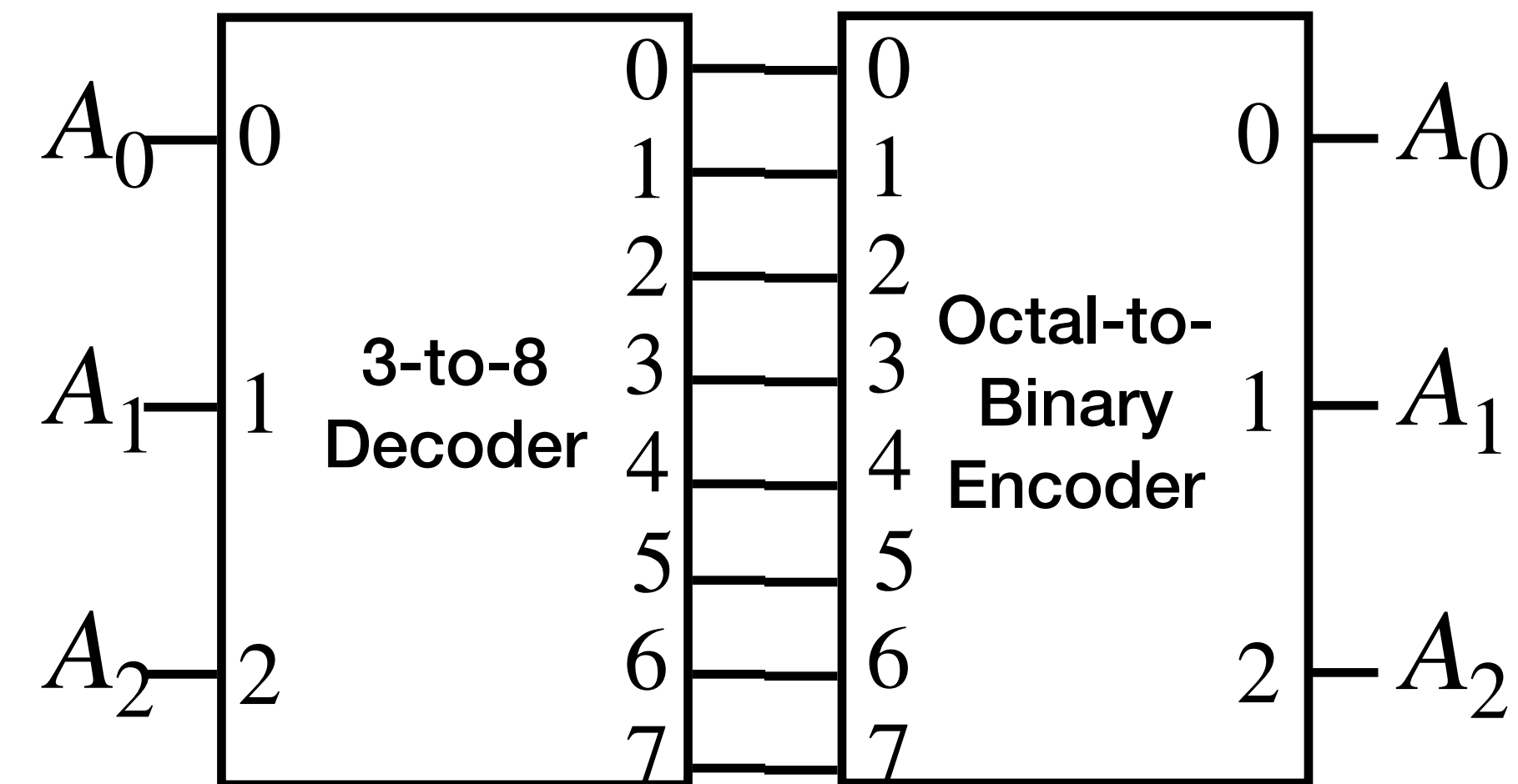
# Encoder

- Inverse operation of a decoder
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- $n$  outputs



# Encoder

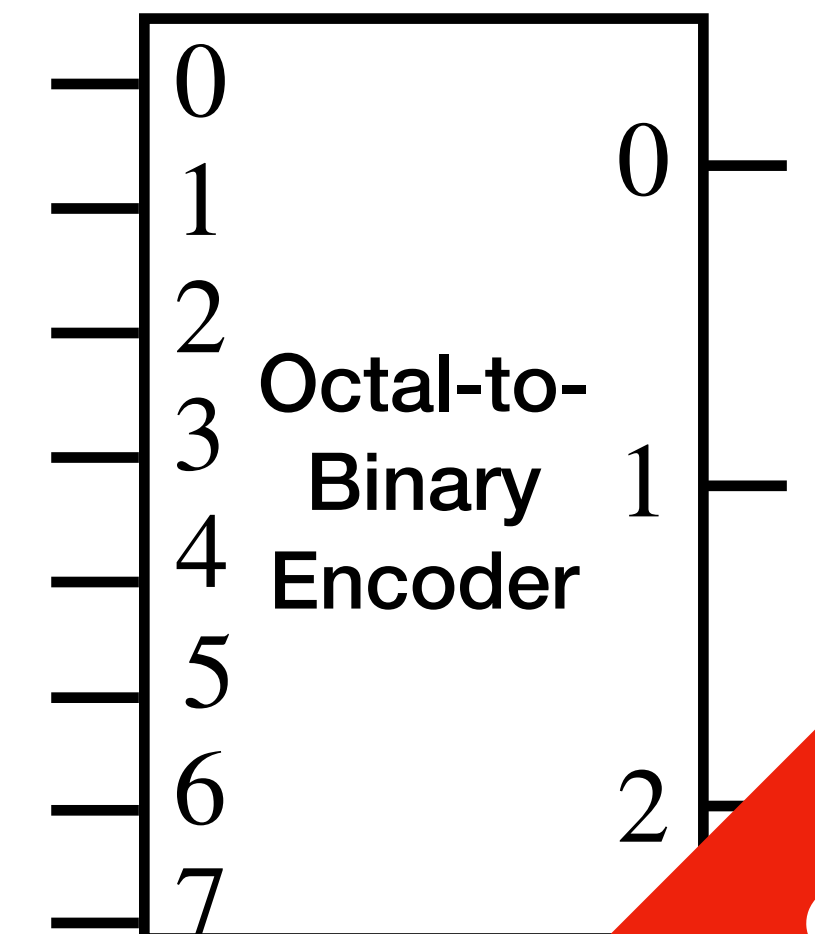
- Inverse operation of a decoder
- $2^n$  inputs, only one is giving positive input<sup>1</sup>
- $n$  outputs





# Encoder

D <sub>7</sub>	D <sub>6</sub>	D <sub>5</sub>	D <sub>4</sub>	D <sub>3</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>	A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>
							1	0	0	0
						1		0	0	1
					1			0	1	0
				1				0	1	1
			1					1	0	0
		1						1	0	1
	1							1	1	0
1								1	1	1



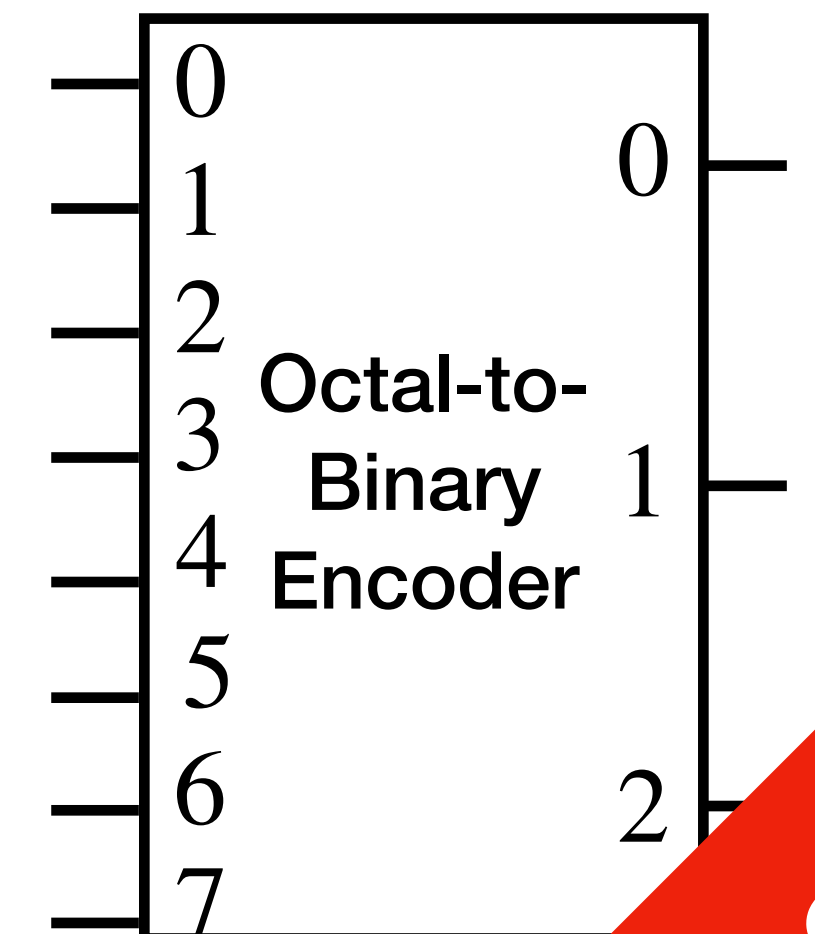
# Encoder

D <sub>7</sub>	D <sub>6</sub>	D <sub>5</sub>	D <sub>4</sub>	D <sub>3</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>	A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>
							1	0	0	0
						1		0	0	1
					1			0	1	0
				1				0	1	1
			1					1	0	0
		1						1	0	1
	1							1	1	0
1								1	1	1

$$A_0 = D_1 + D_3 + D_5 + D_7$$

$$A_1 = D_2 + D_3 + D_6 + D_7$$

$$A_2 = D_4 + D_5 + D_6 + D_7$$



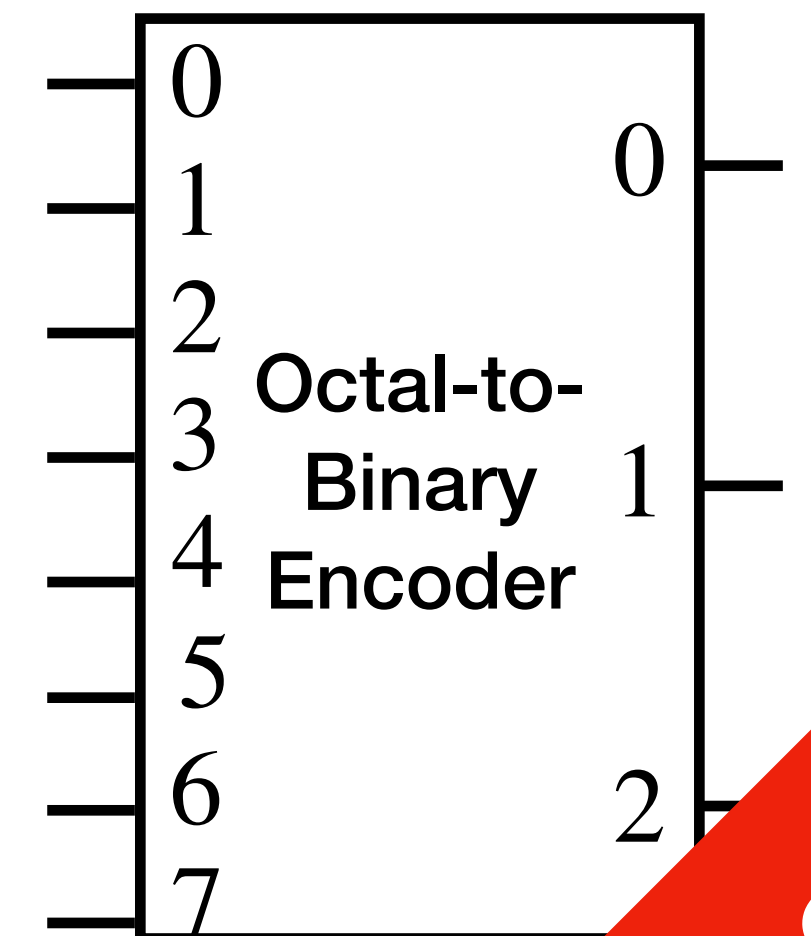
# Encoder

$$A_0 = D_1 + D_3 + D_5 + D_7$$

$$A_1 = D_2 + D_3 + D_6 + D_7$$

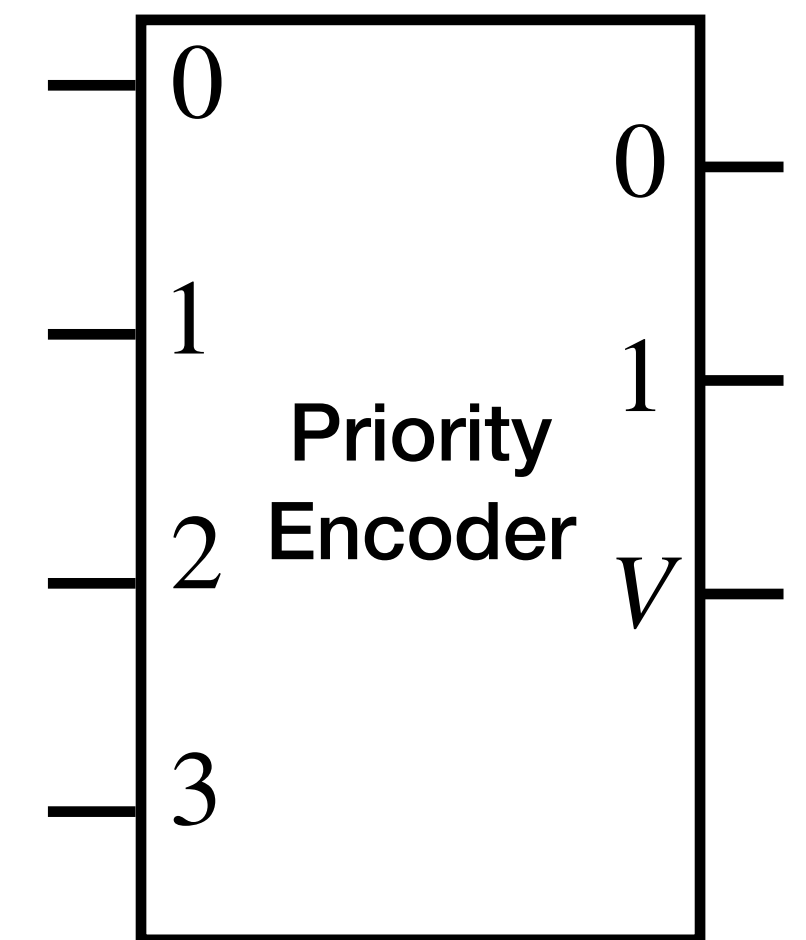
$$A_2 = D_4 + D_5 + D_6 + D_7$$

- What happens if the inputs are all 0s?
- What happens if the inputs include multiple 1s?



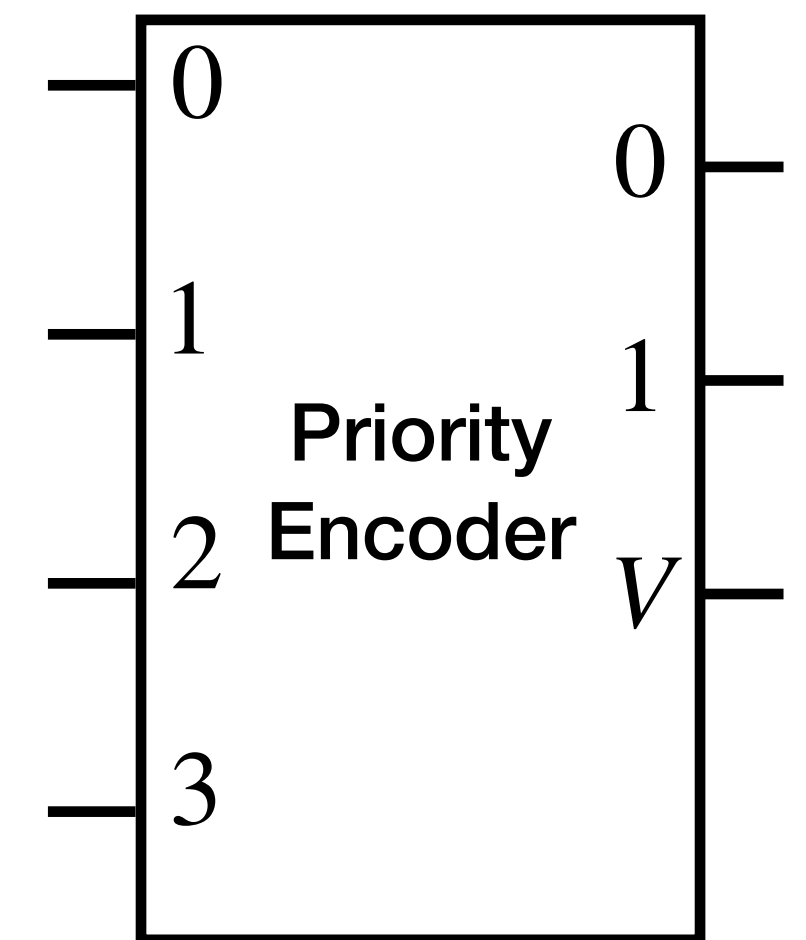
# Priority Encoder

- Additional Validity Output  $V$ 
  - Indicating whether the input is valid (contains 1)
- Priority
  - Ignores  $D_{<i}$  if  $D_i = 1$



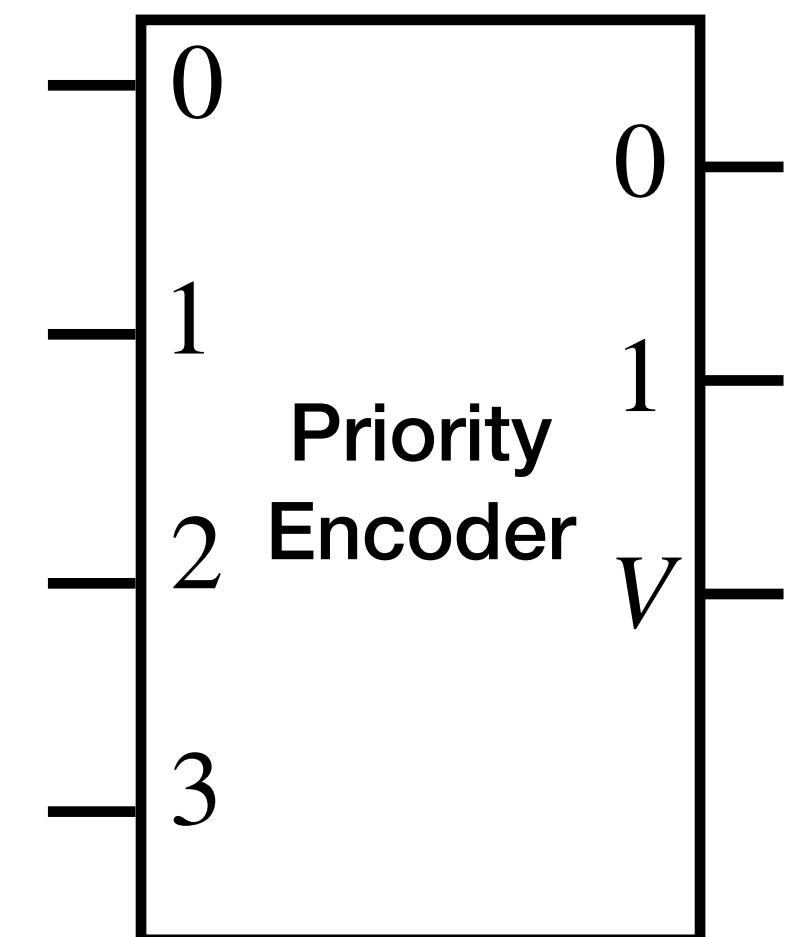
# Priority Encoder

D <sub>3</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>	A <sub>1</sub>	A <sub>0</sub>	V
0	0	0	0	X	X	0
0	0	0	1	0	0	1
0	0	1	X	0	1	1
0	1	X	X	1	0	1
1	X	X	X	1	1	1



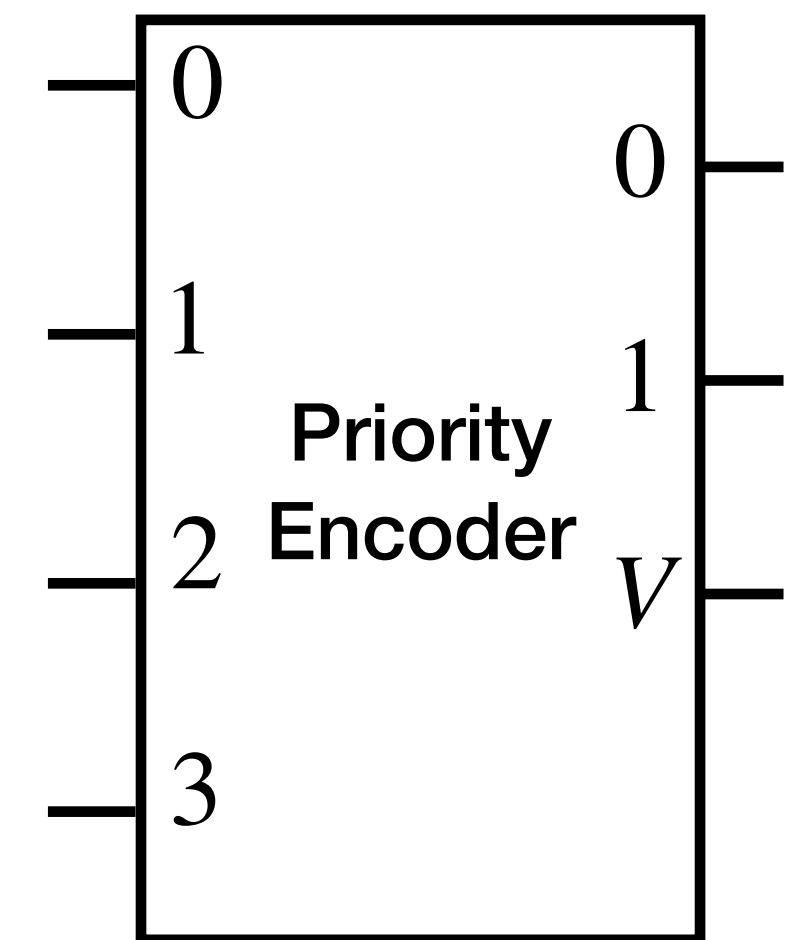
# Priority Encoder

D <sub>3</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>	A <sub>1</sub>	A <sub>0</sub>	V
0	0	0	0	X	X	0
0	0	0	1	0	0	1
0	0	1	X	0	1	1
0	1	X	X	1	0	1
1	X	X	X	1	1	1



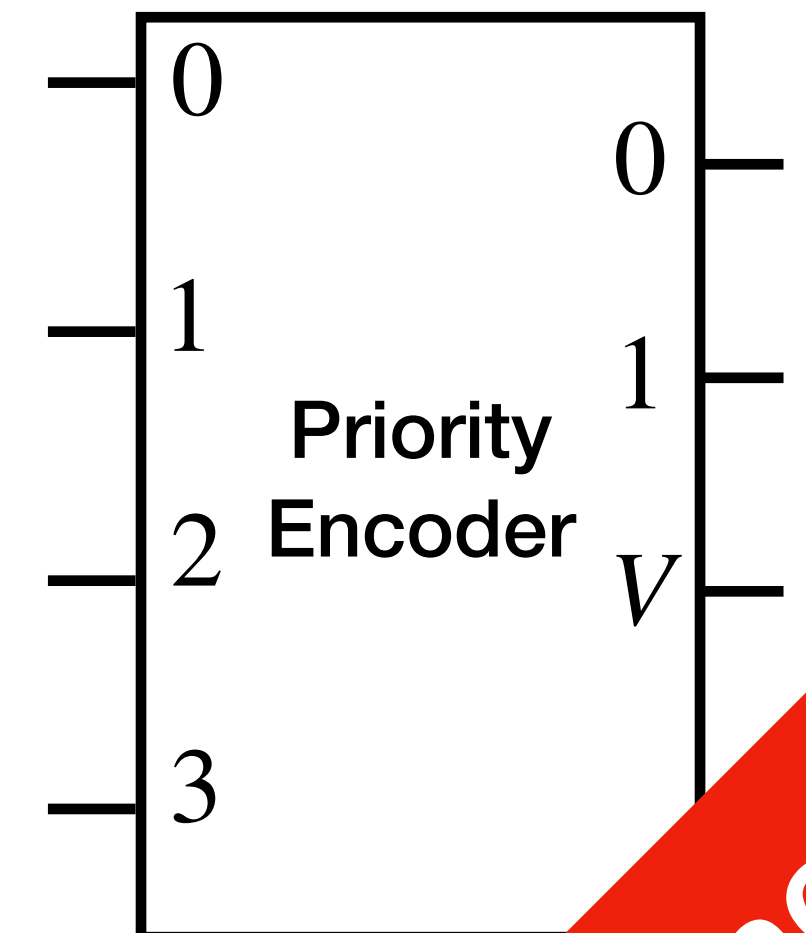
# Priority Encoder

D <sub>3</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>	A <sub>1</sub>	A <sub>0</sub>	V
0	0	0	0	X	X	0
0	0	0	1	0	0	1
0	0	1	X	0	1	1
0	1	X	X	1	0	1
1	X	X	X	1	1	1



# Priority Encoder

D <sub>3</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>	A <sub>1</sub>	A <sub>0</sub>	V
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	X	0	1	1
0	1	X	X	1	0	1
1	X	X	X	1	1	1





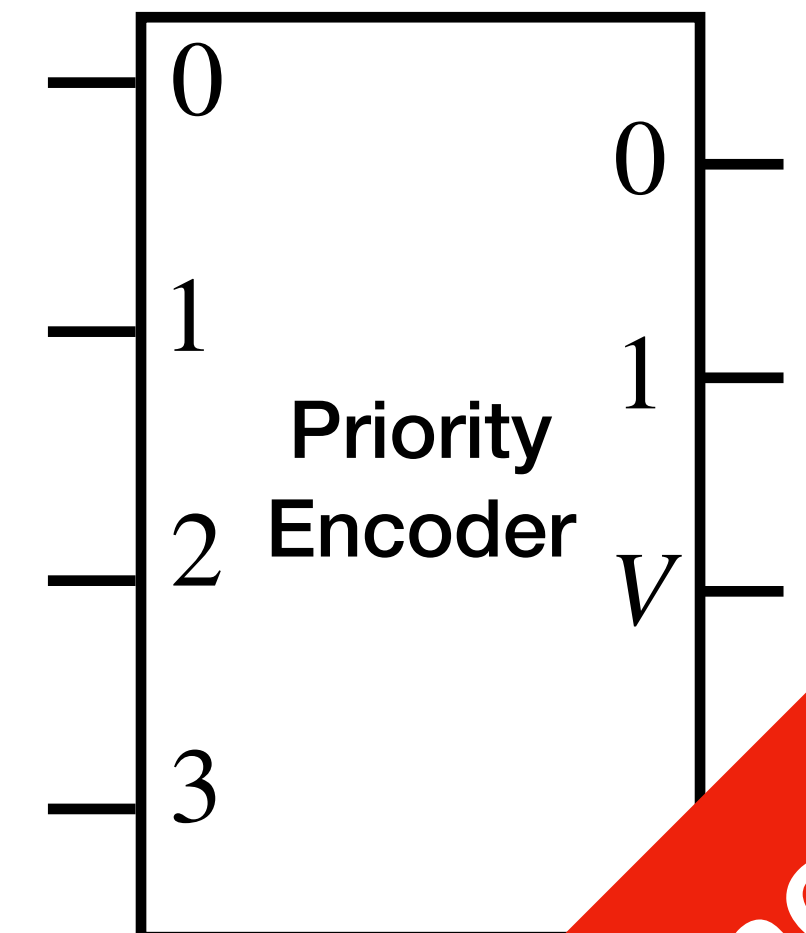
# Priority Encoder

$D_3$	$D_2$	$D_1$	$D_0$	$A_1$	$A_0$	$V$
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	X	0	1	1
0	1	X	X	1	0	1
1	X	X	X	1	1	1

$$V = D_3 + D_2 + D_1 + D_0$$

$$A_1 = D_3 + \overline{D_3}D_2 = D_2 + D_3$$

$$A_0 = \overline{D_3}\overline{D_2}D_1 + D_3 \\ = \overline{D_2}D_1 + D_3$$



Concept

# Multiplexer

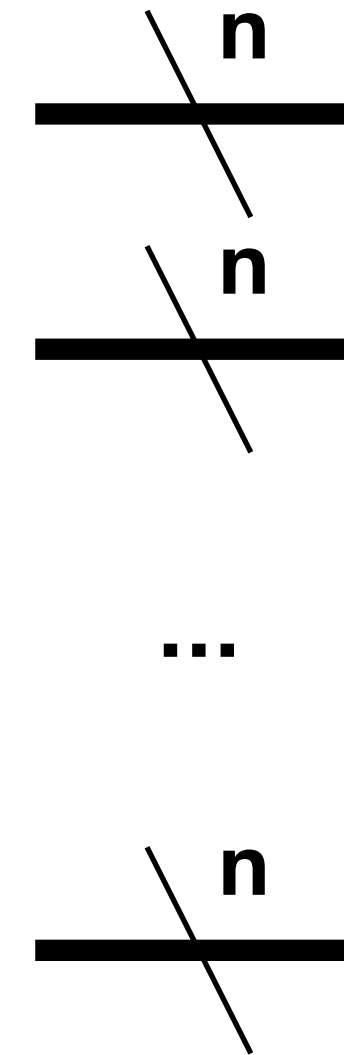
## Switch Modes

# Multiplexer

- Multiple  $n$ -variable input vectors
- Single  $n$ -variable output vector
- Switches: which input vectors to output

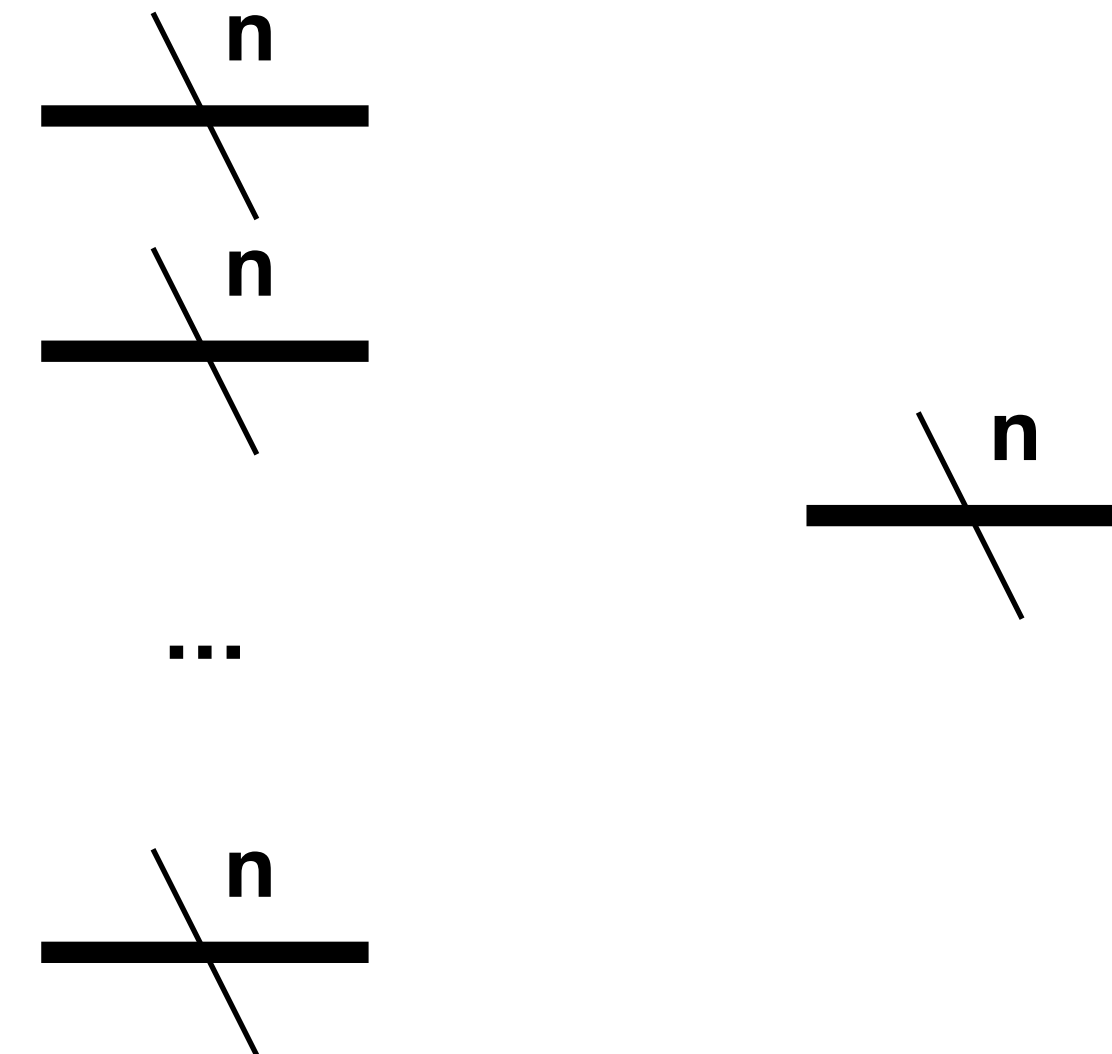
# Multiplexer

- Multiple  $n$ -variable input vectors
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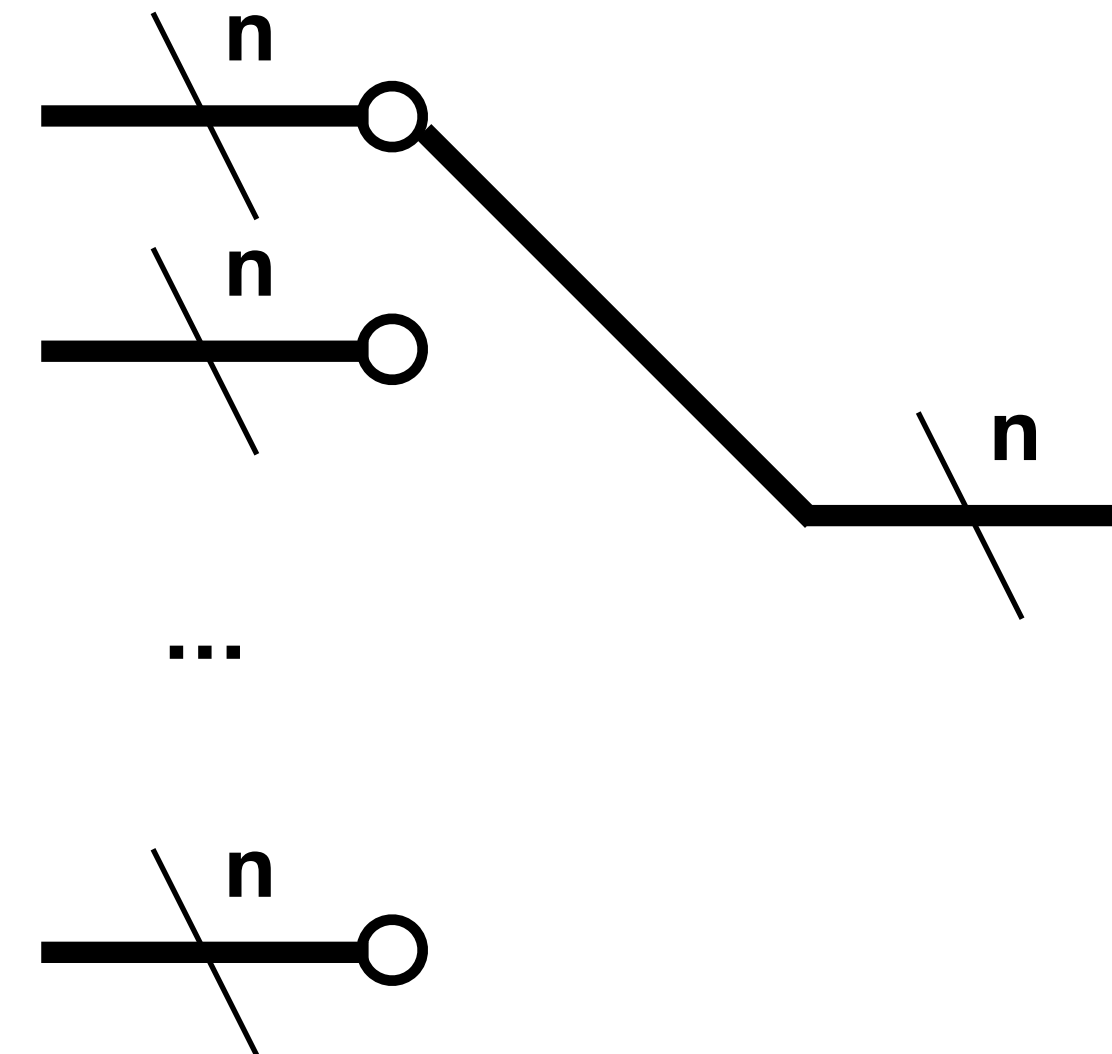
# Multiplexer

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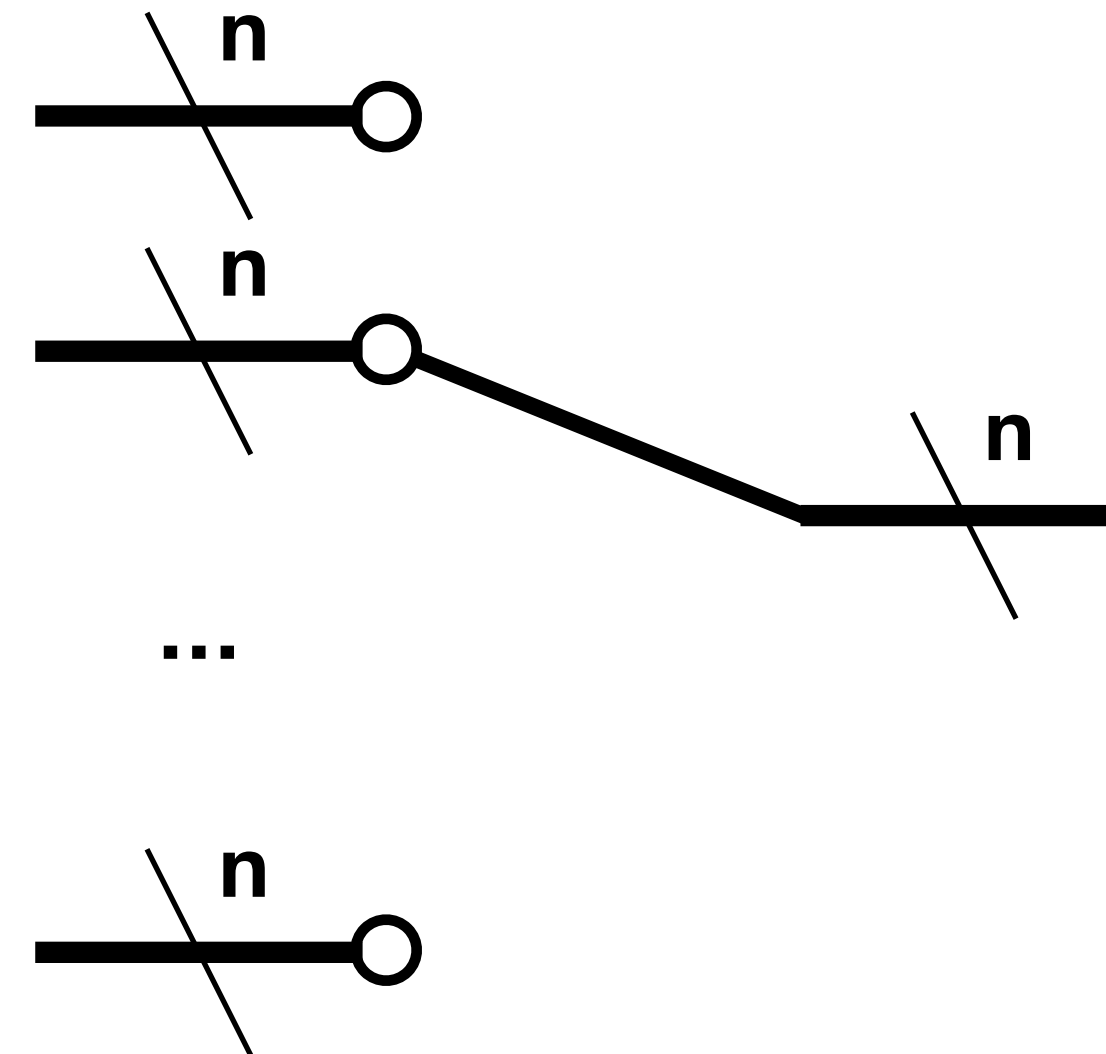
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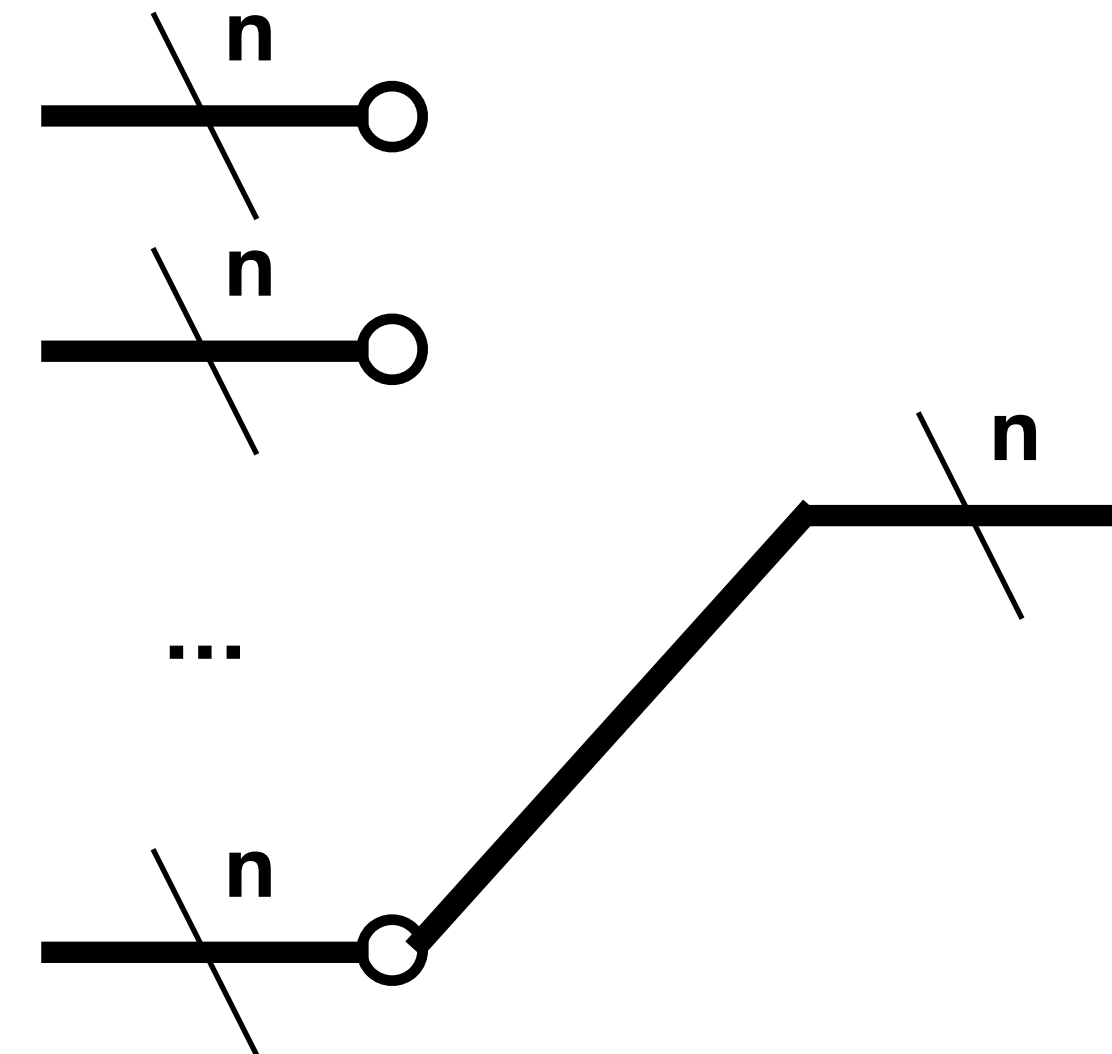
# Multiplexer

- Multiple  $n$ -variable input vectors
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# Multiplexer

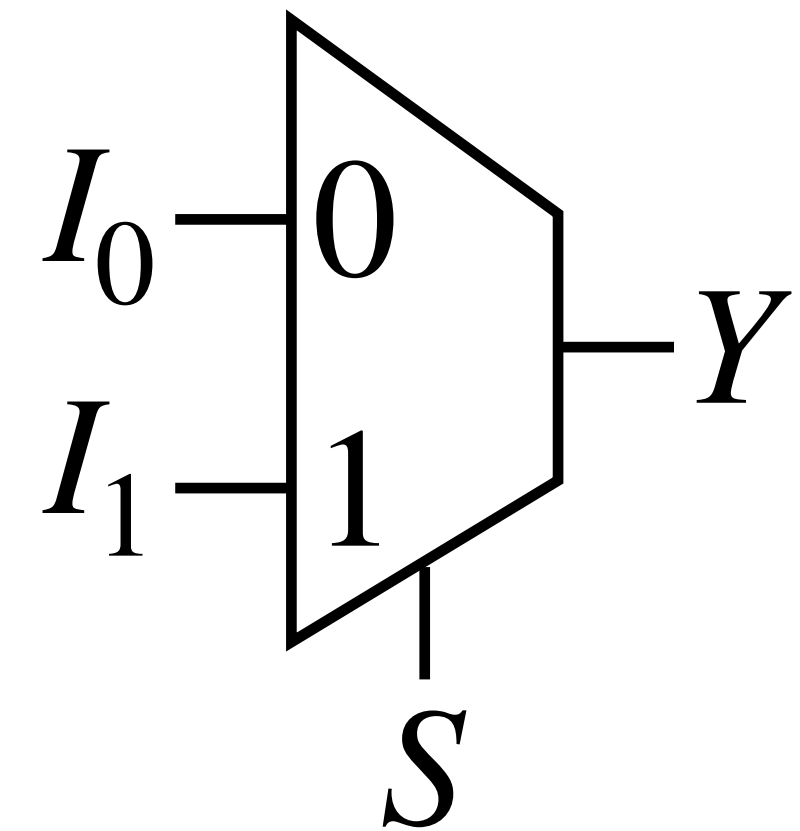
- Multiple  $n$ -variable input vectors
- Single  $n$ -variable output vector
- Switches: which input vectors to output





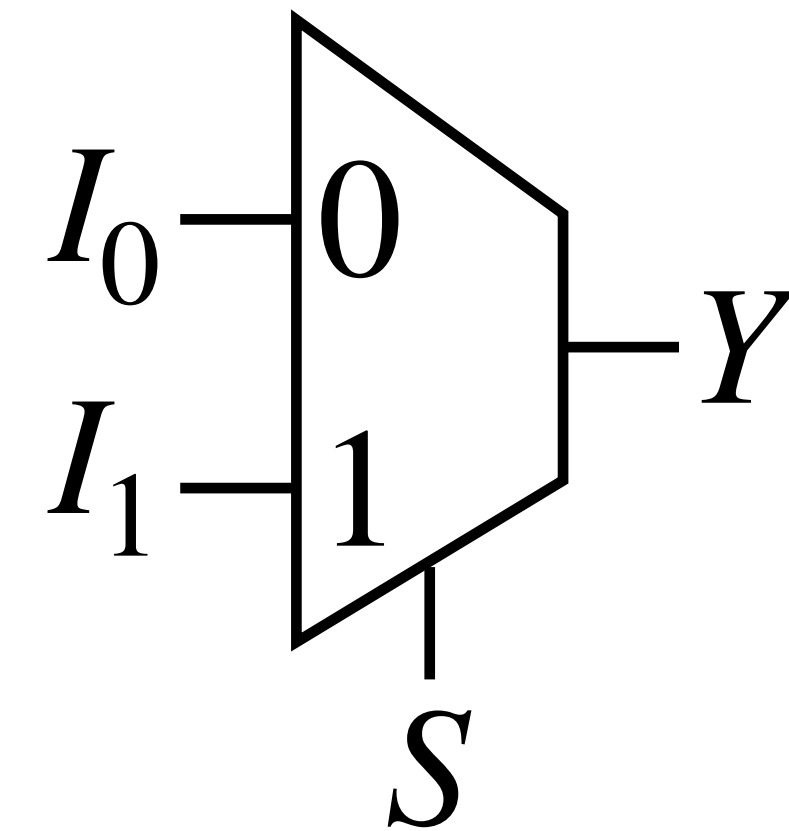
# Single-Bit 2-to-1 Multiplexer

- 2 single-bit inputs
- 1 single-bit output
- 1-bit switch



# Single-Bit 2-to-1 Multiplexer

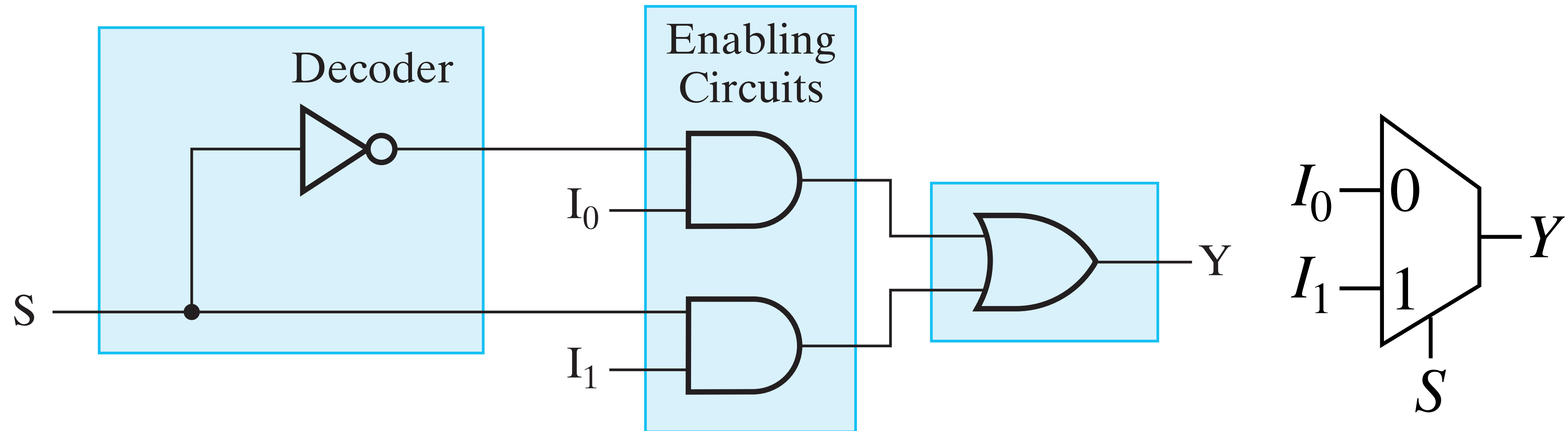
S	$I_0$	$I_1$	Y
0	0	X	0
0	1	X	1
1	X	0	0
1	X	1	1



# Single-Bit 2-to-1 Multiplexer

## Technology

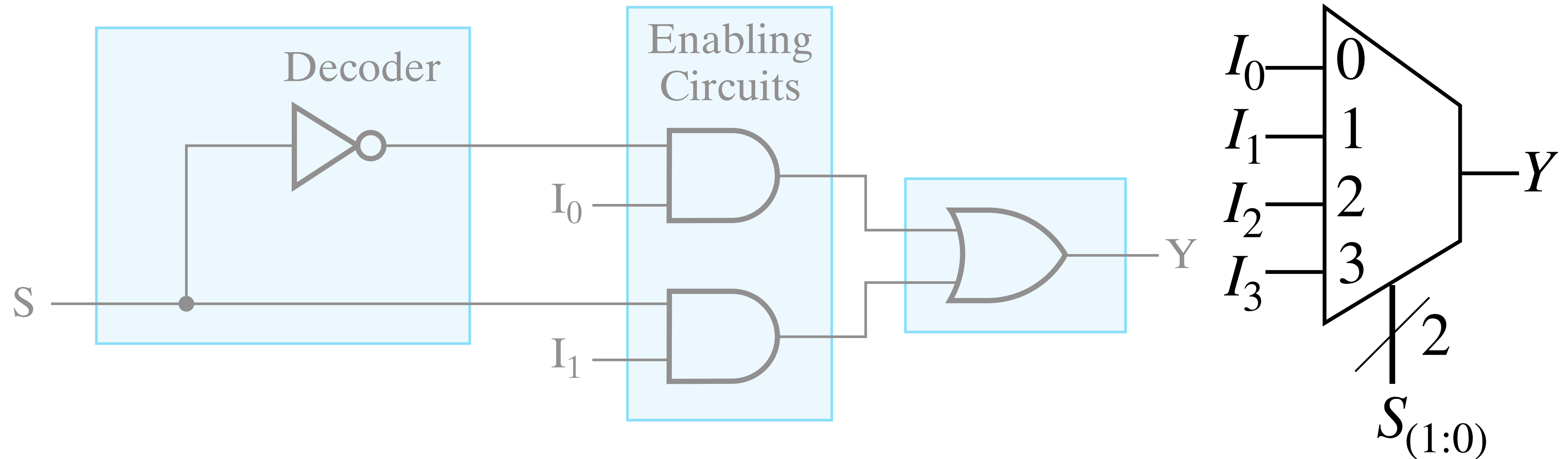
- 1 x 1-to-2 Decoder
- 2 x 1-bit Enabler
- 1 x 2-input OR Gate



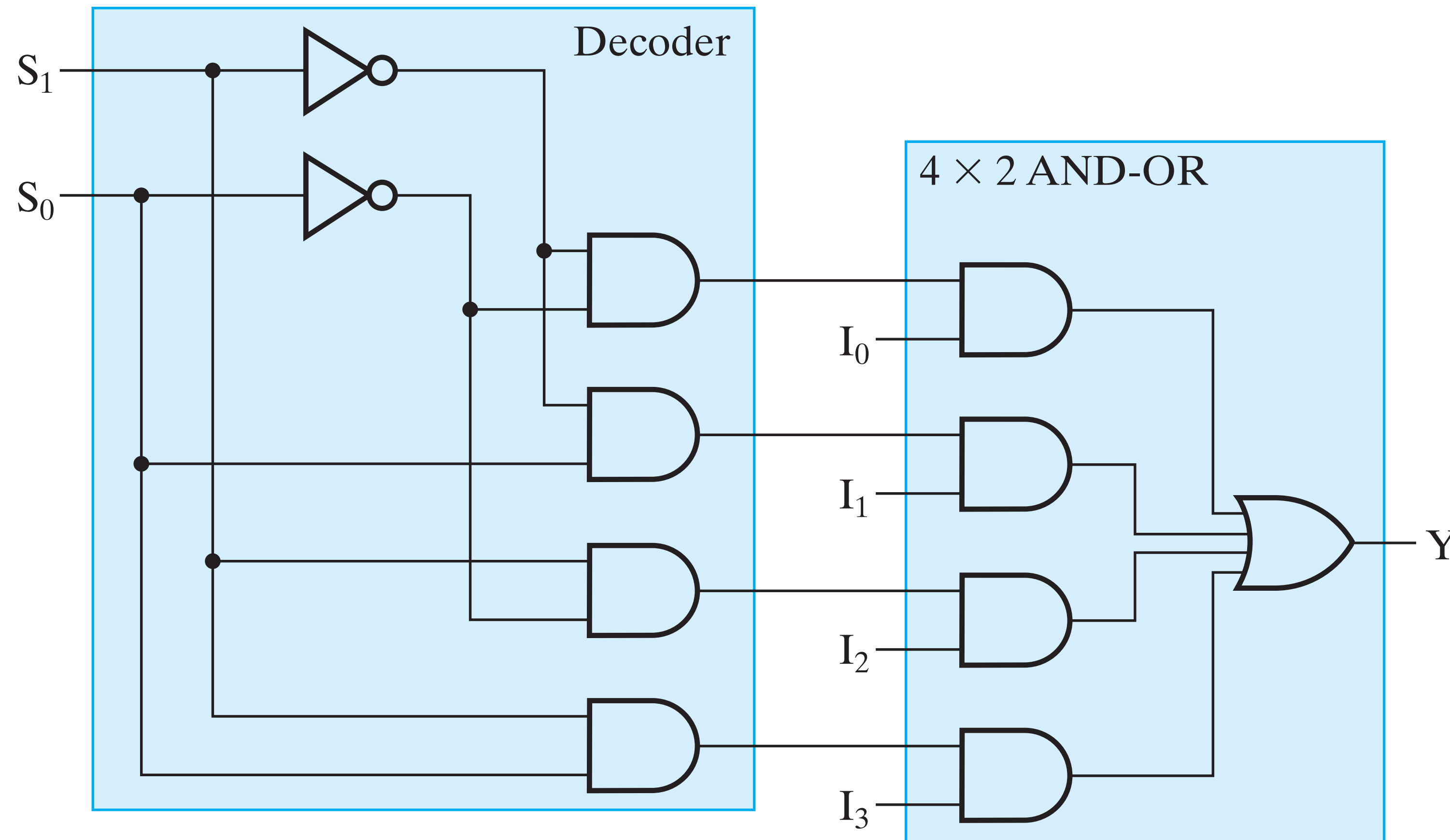
# Single-Bit 4-to-1 Multiplexer

## Technology

- 1 x 2-to-4 Decoder
- 4 x 1-bit Enabler
- 1 x 4-input OR Gate

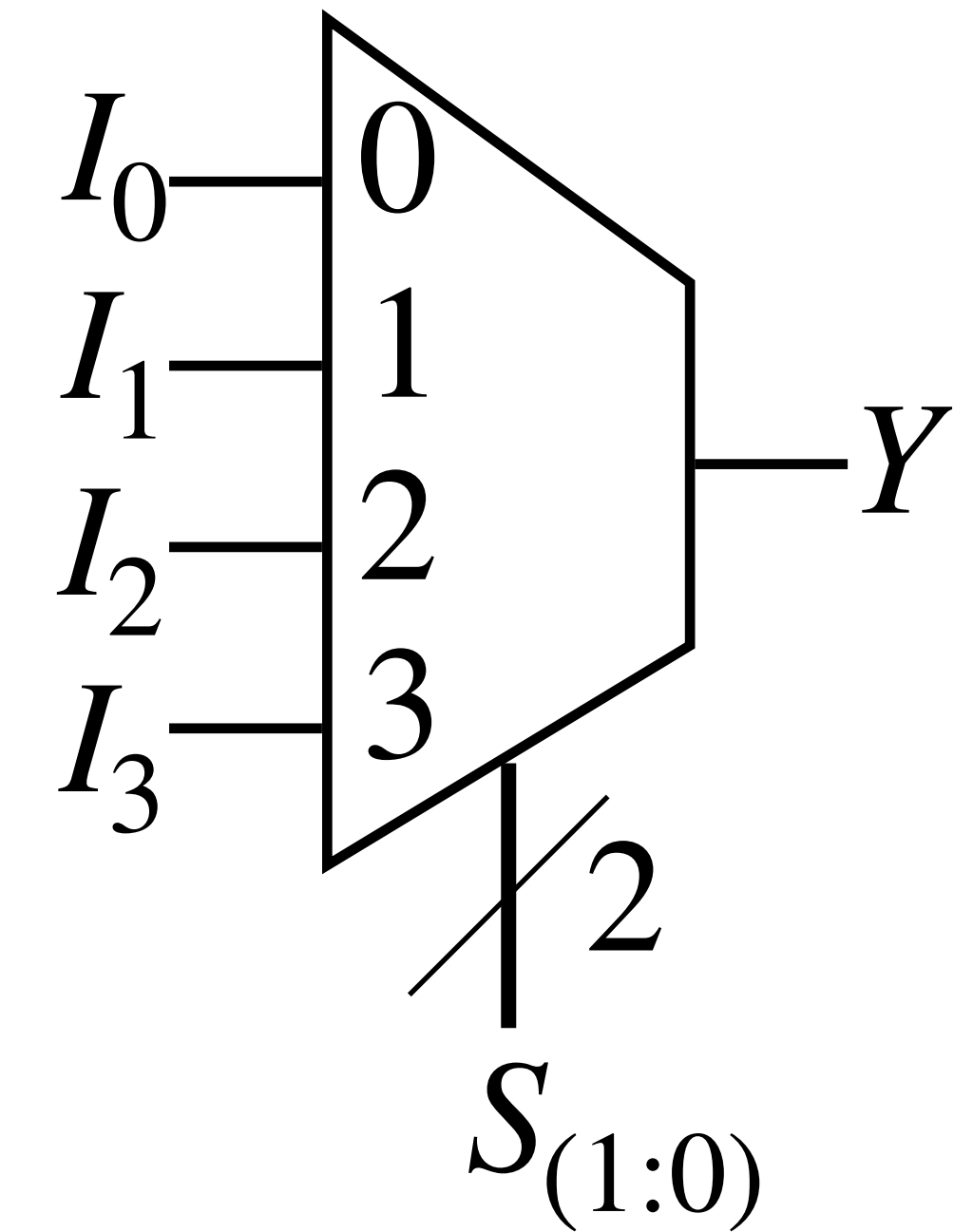


# Single-Bit 4-to-1 Multiplexer



## Technology

- 1 x 2-to-4 Decoder
- 4 x 1-bit Enabler
- 1 x 4-input OR Gate



# Circuit Drawing Time!

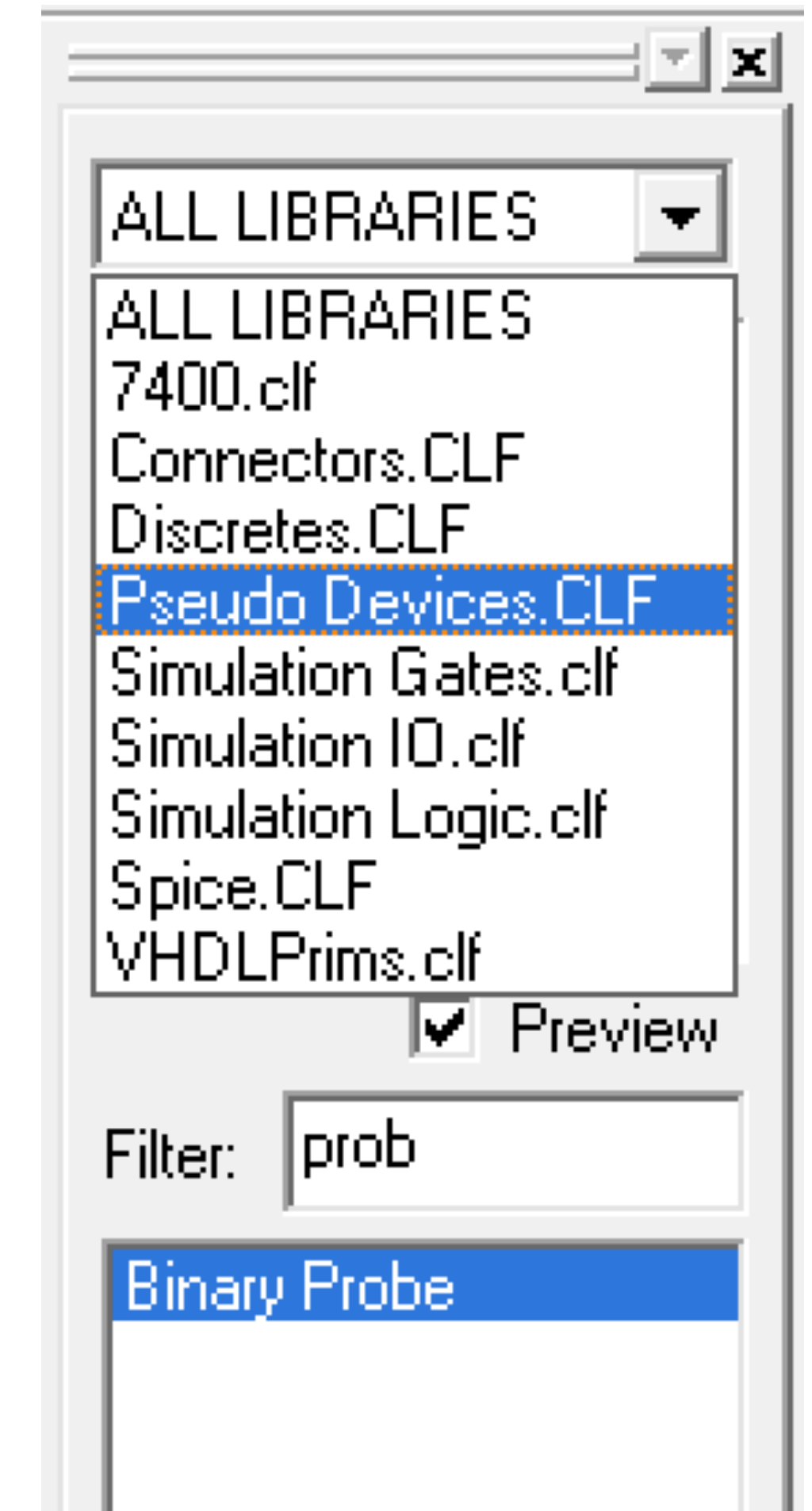
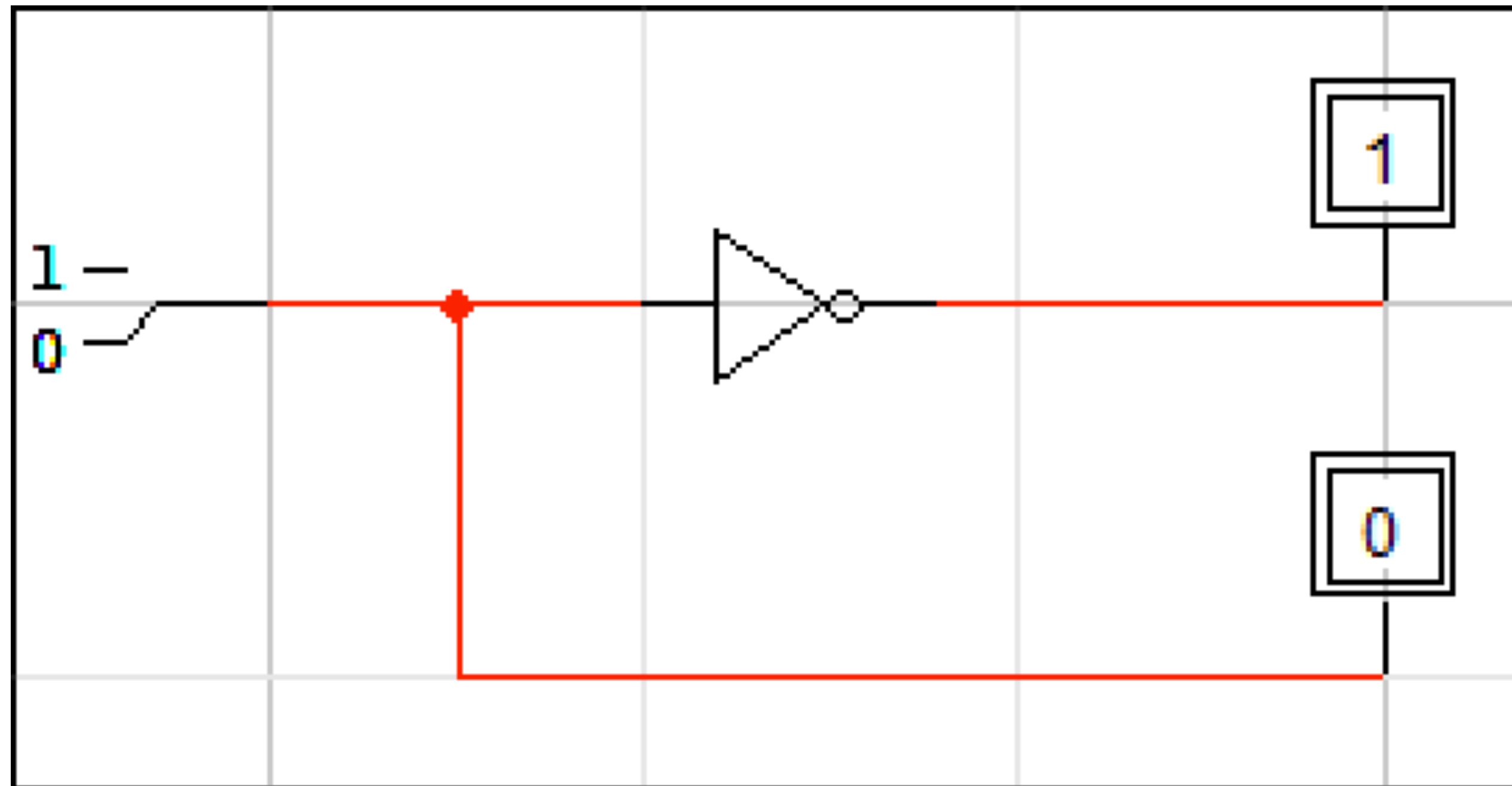
It's OK, this is the last time...  
Sorta...

# Last Circuit Drawing Practice

1. Sub-circuit
2. Implementing 2-to-4 Decoder using drawing tools
3. Implementing 3-to-8 Decoder using 2-to-4 Decoders
4. Implementing Octal-to-Binary Priority Encoder using drawing tools<sup>1</sup>
5. Implementing Multiplexer using drawing tools<sup>1</sup>

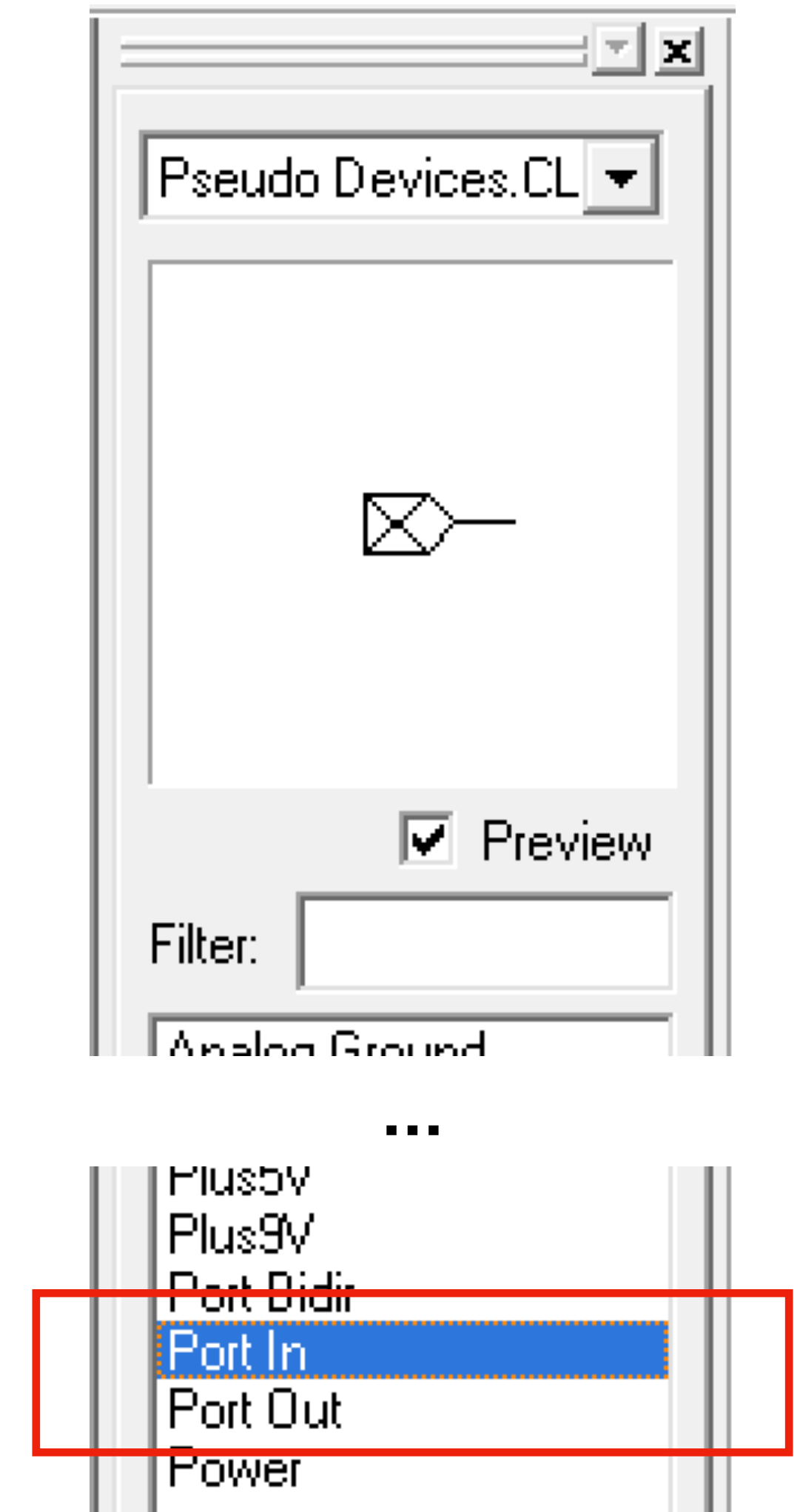
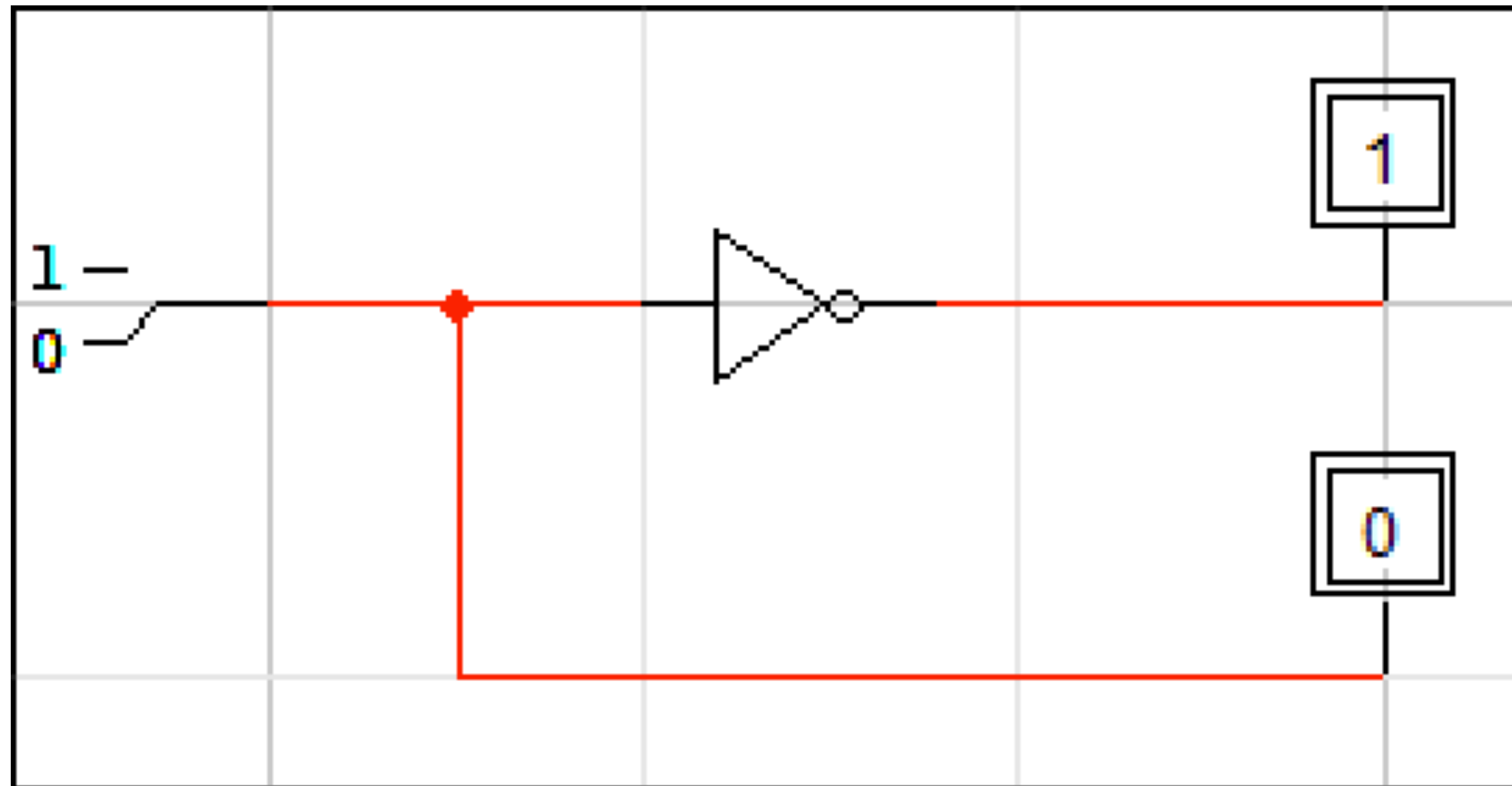
1. You will be reusing these designs in later lectures and assignments

# Sub-circuit

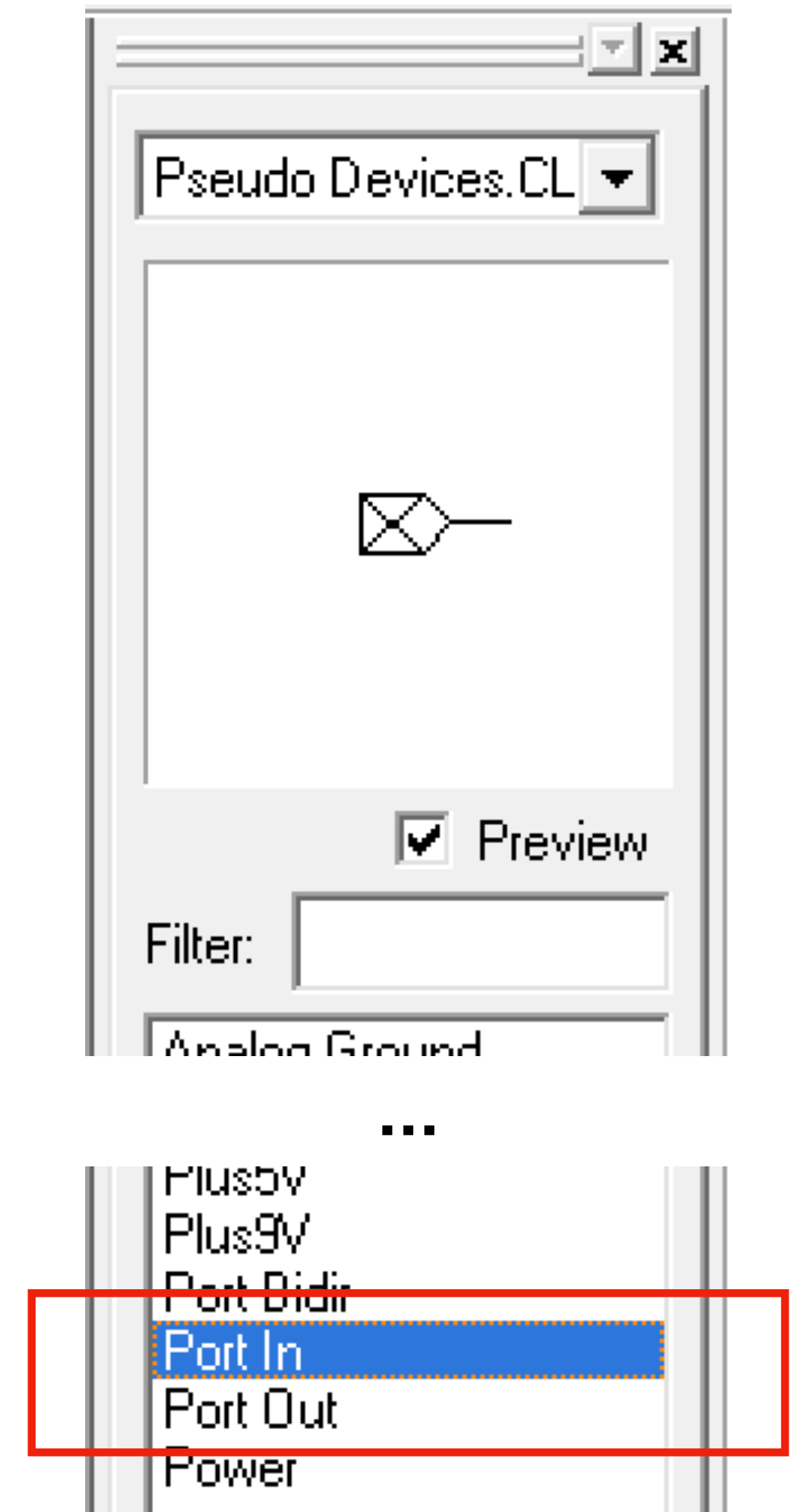
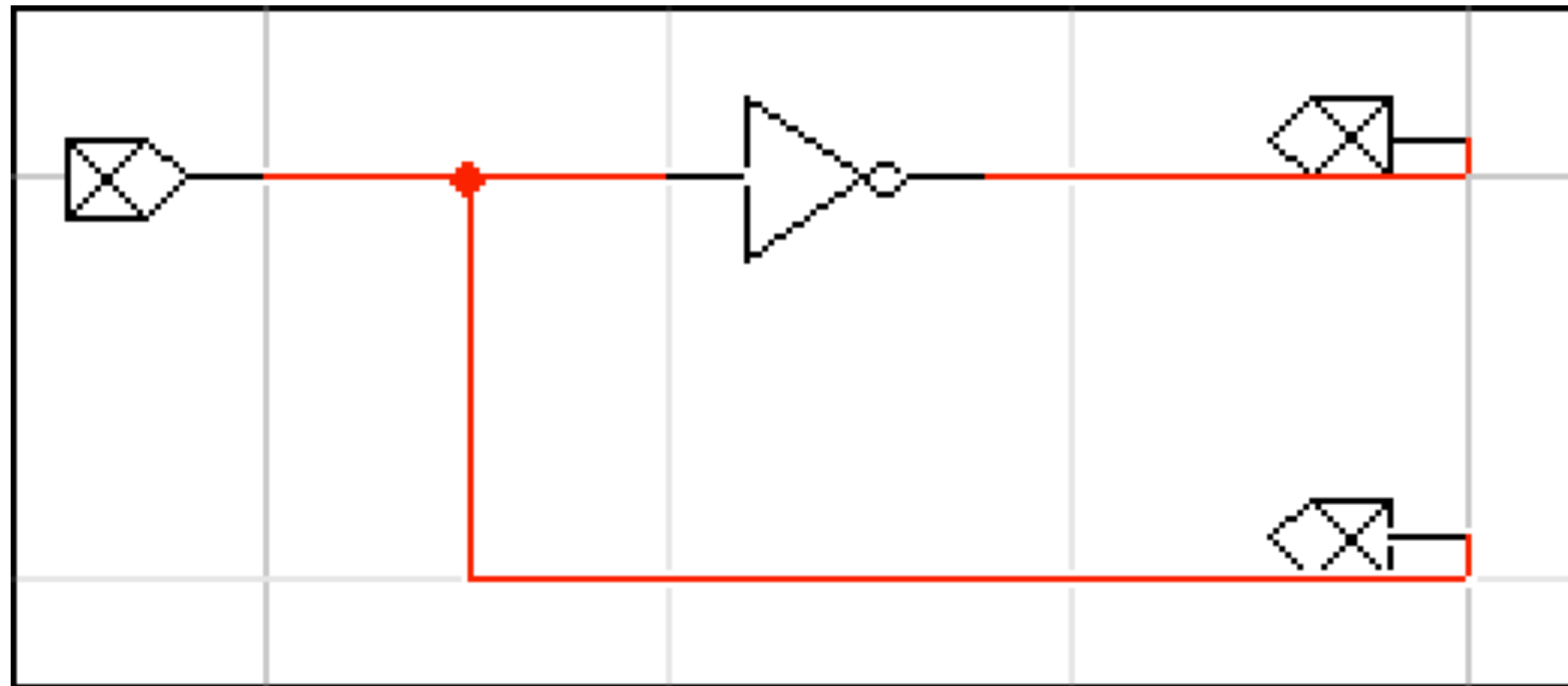




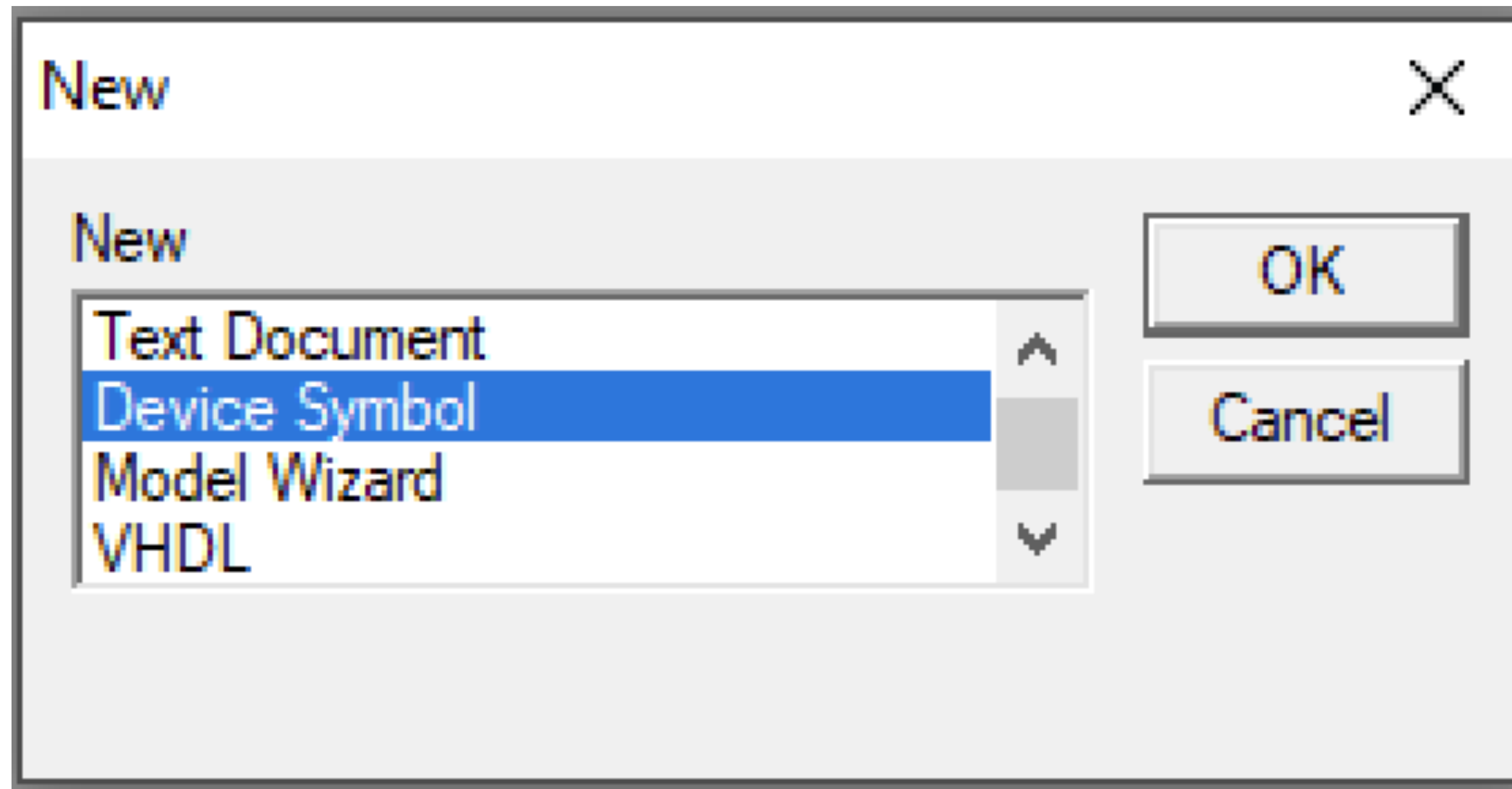
# Sub-circuit



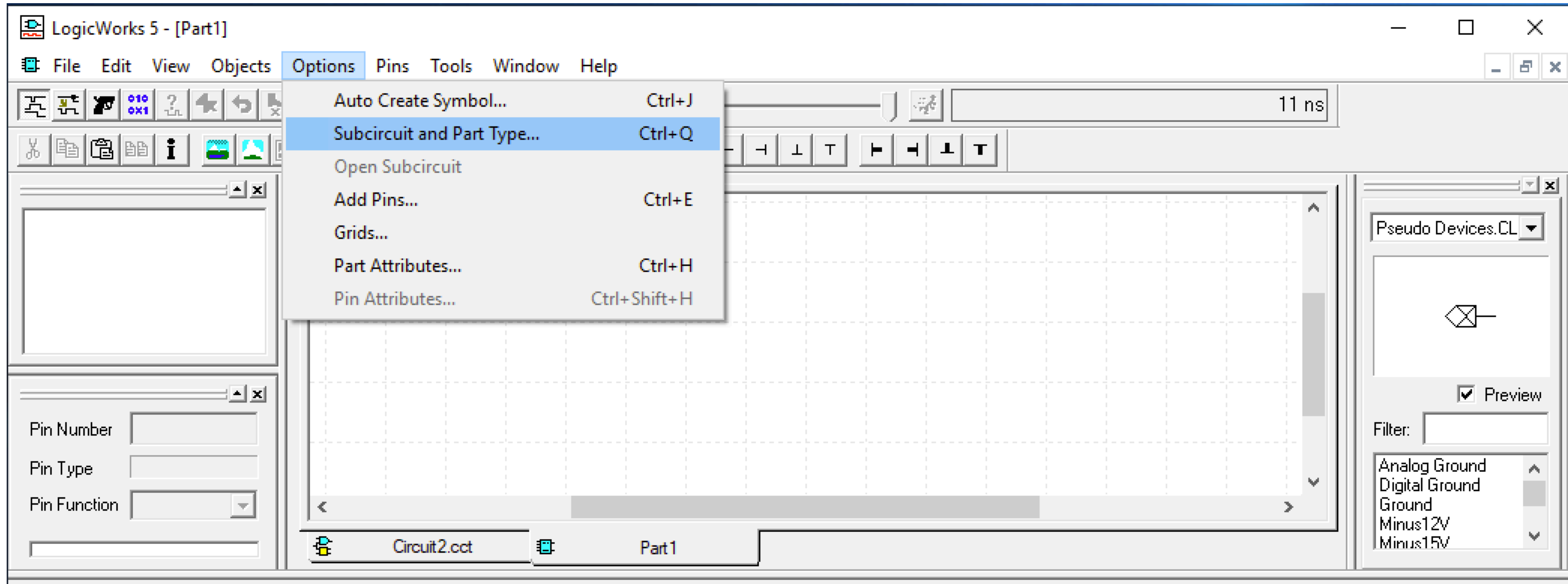
# Sub-circuit



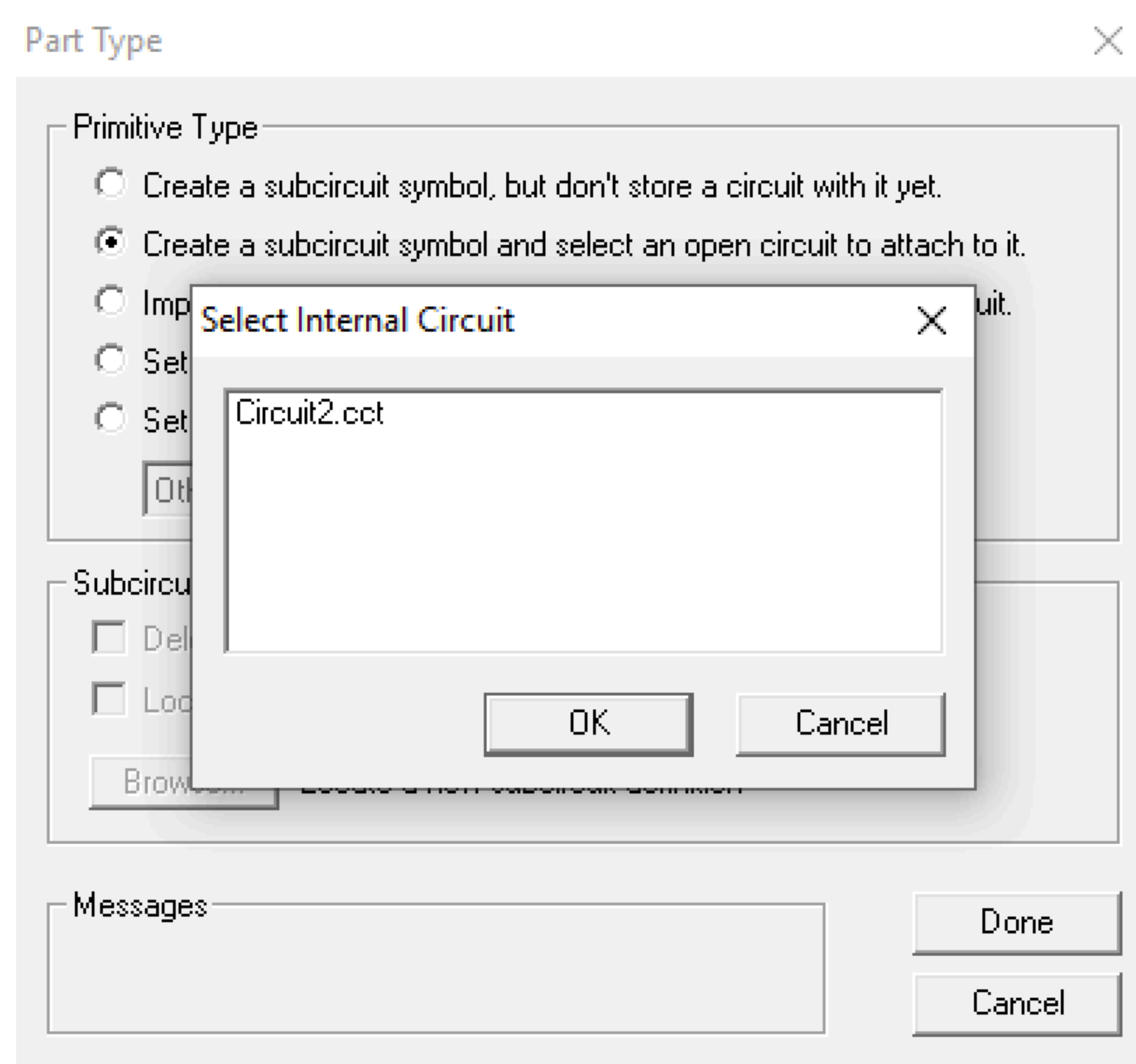
# Sub-circuit



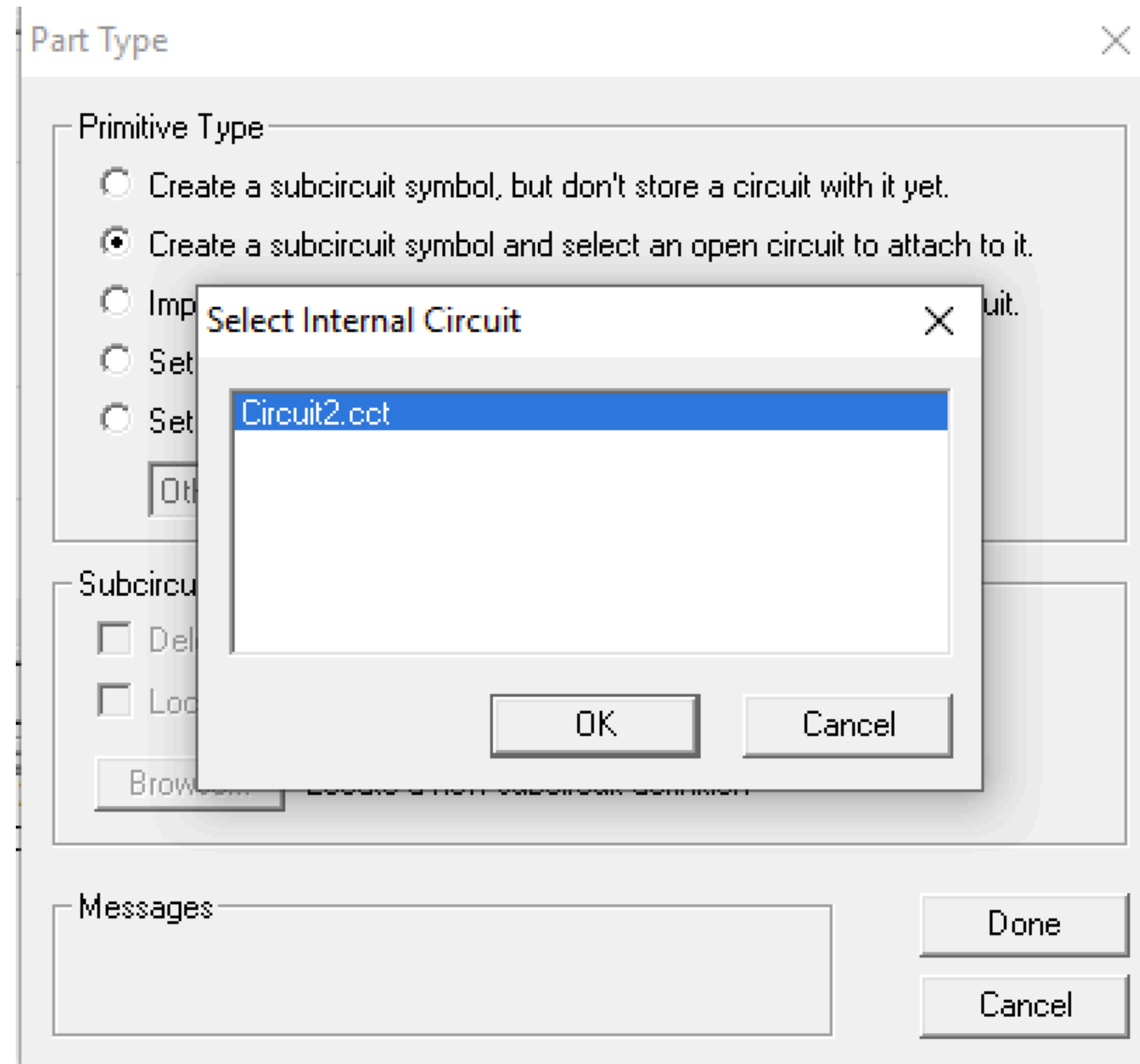
# Sub-circuit



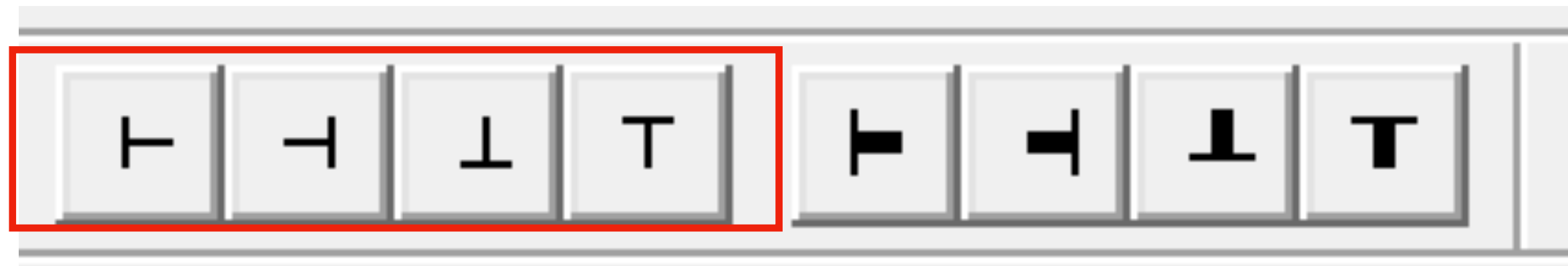
# Sub-circuit



# Sub-circuit



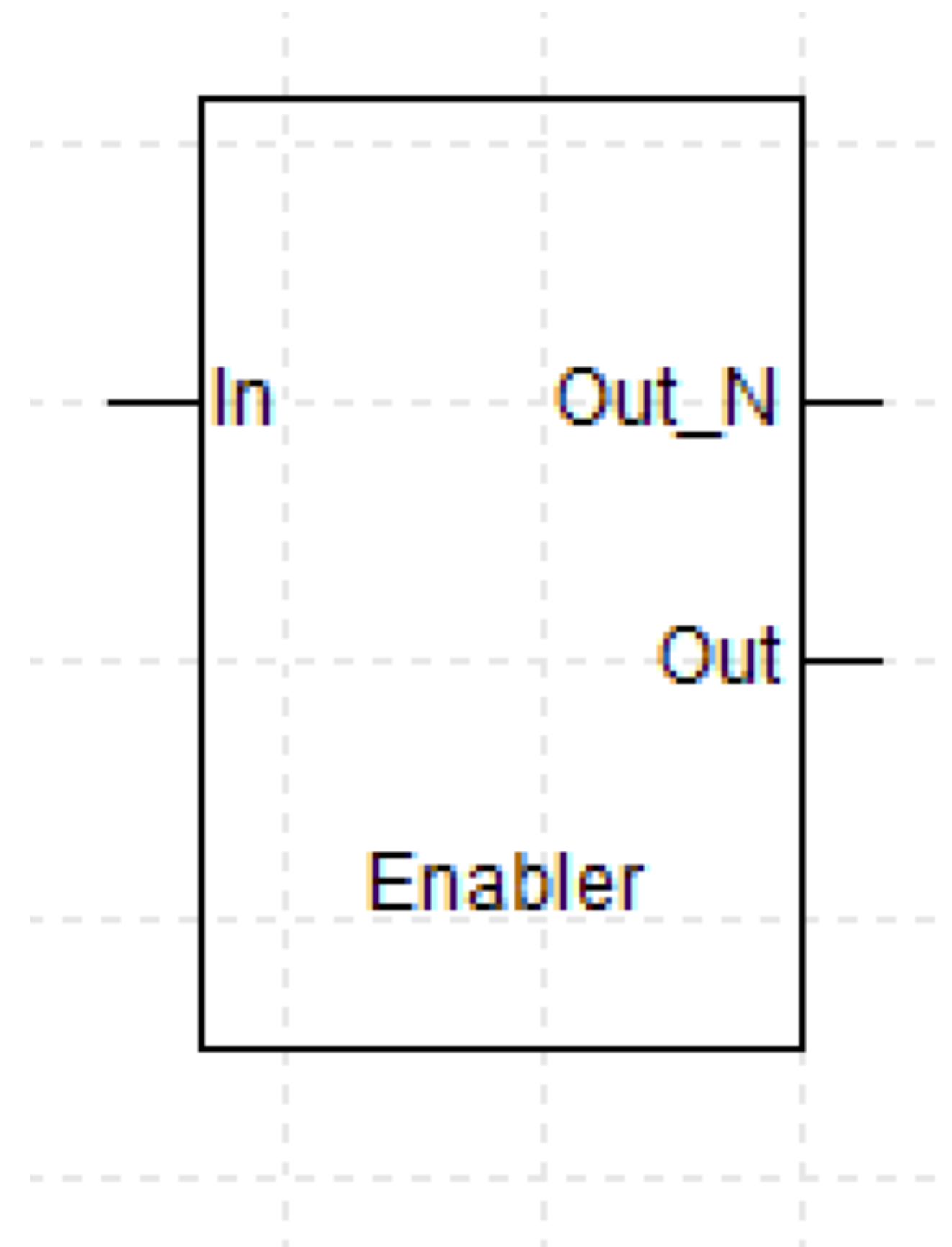
# Sub-circuit



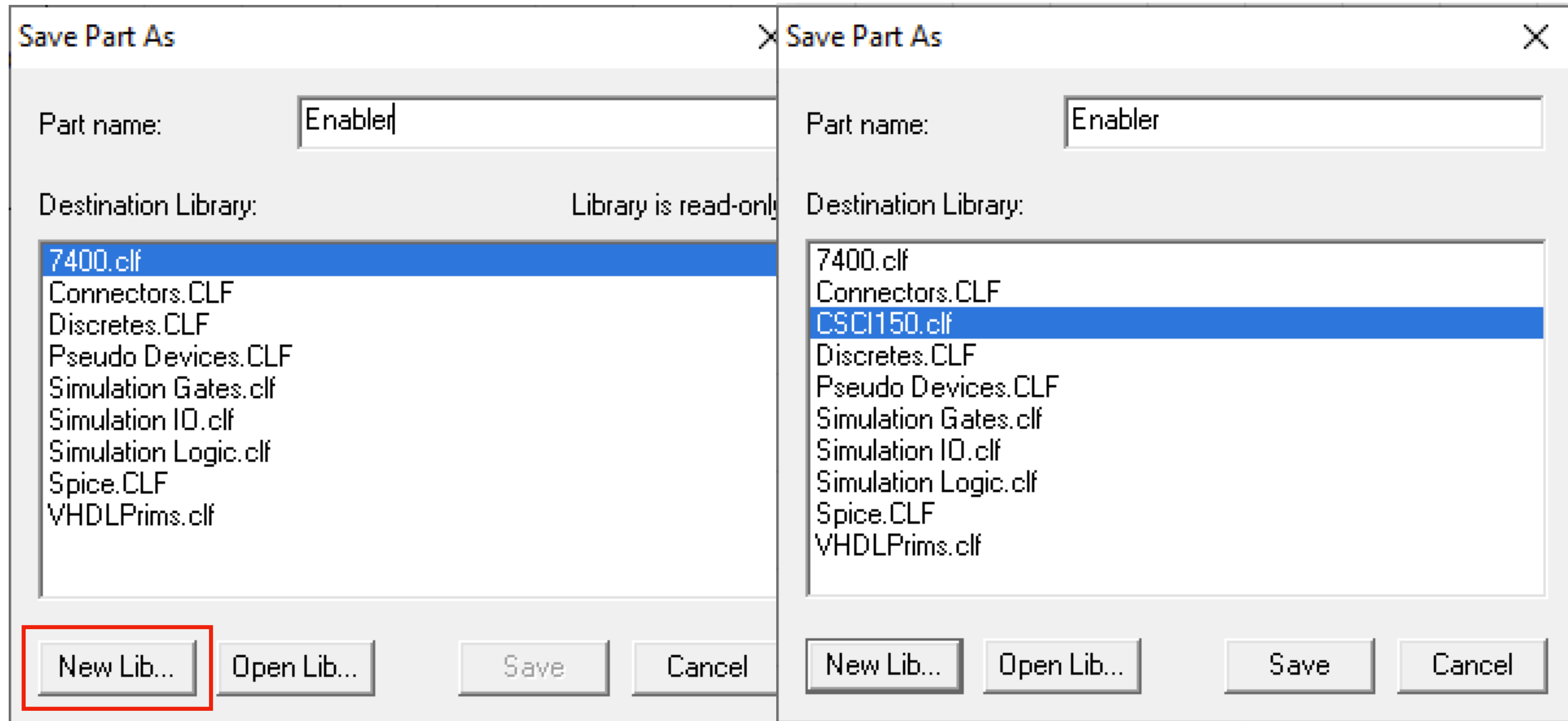
Use these pins only!



Add a text description

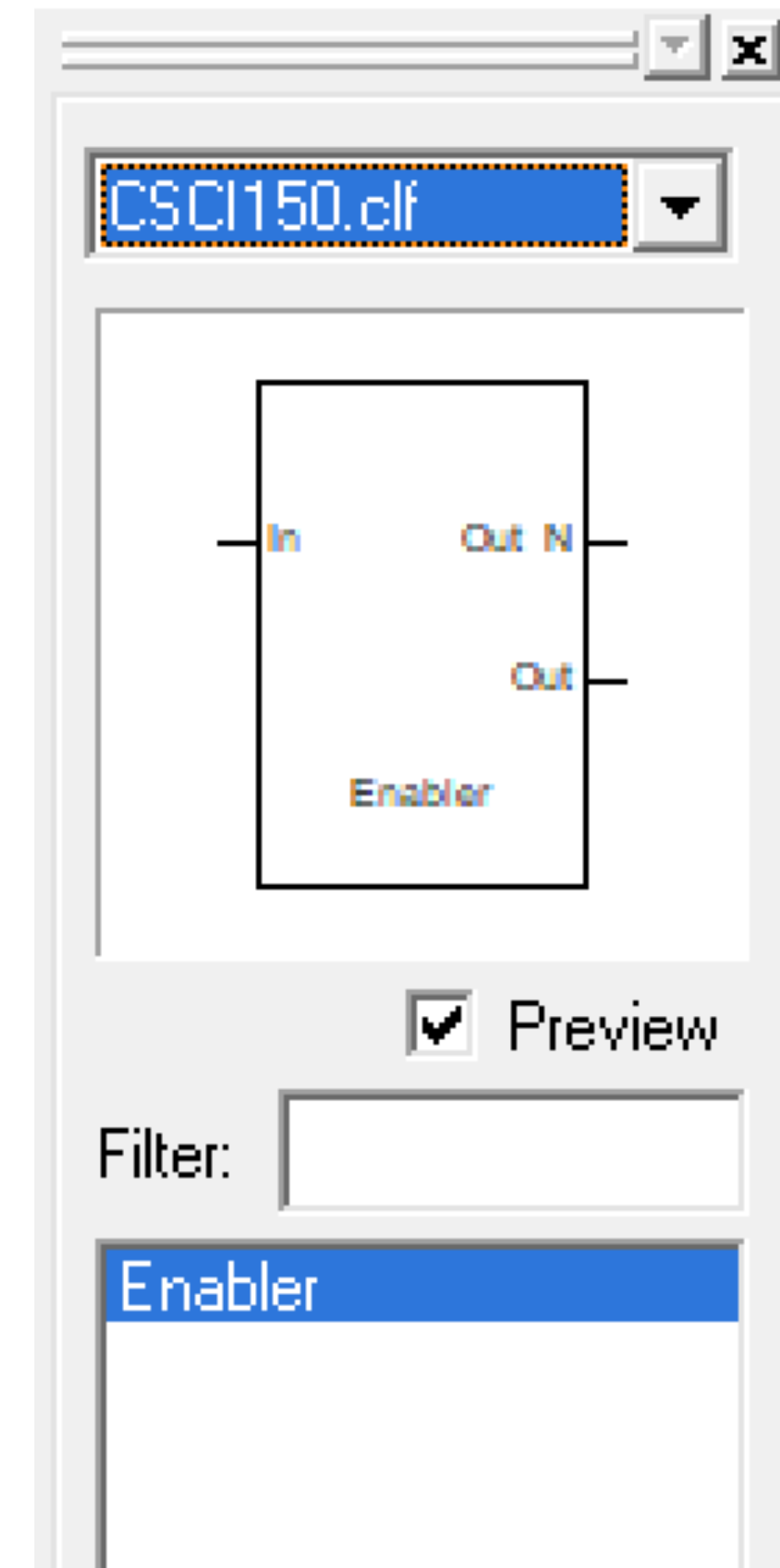
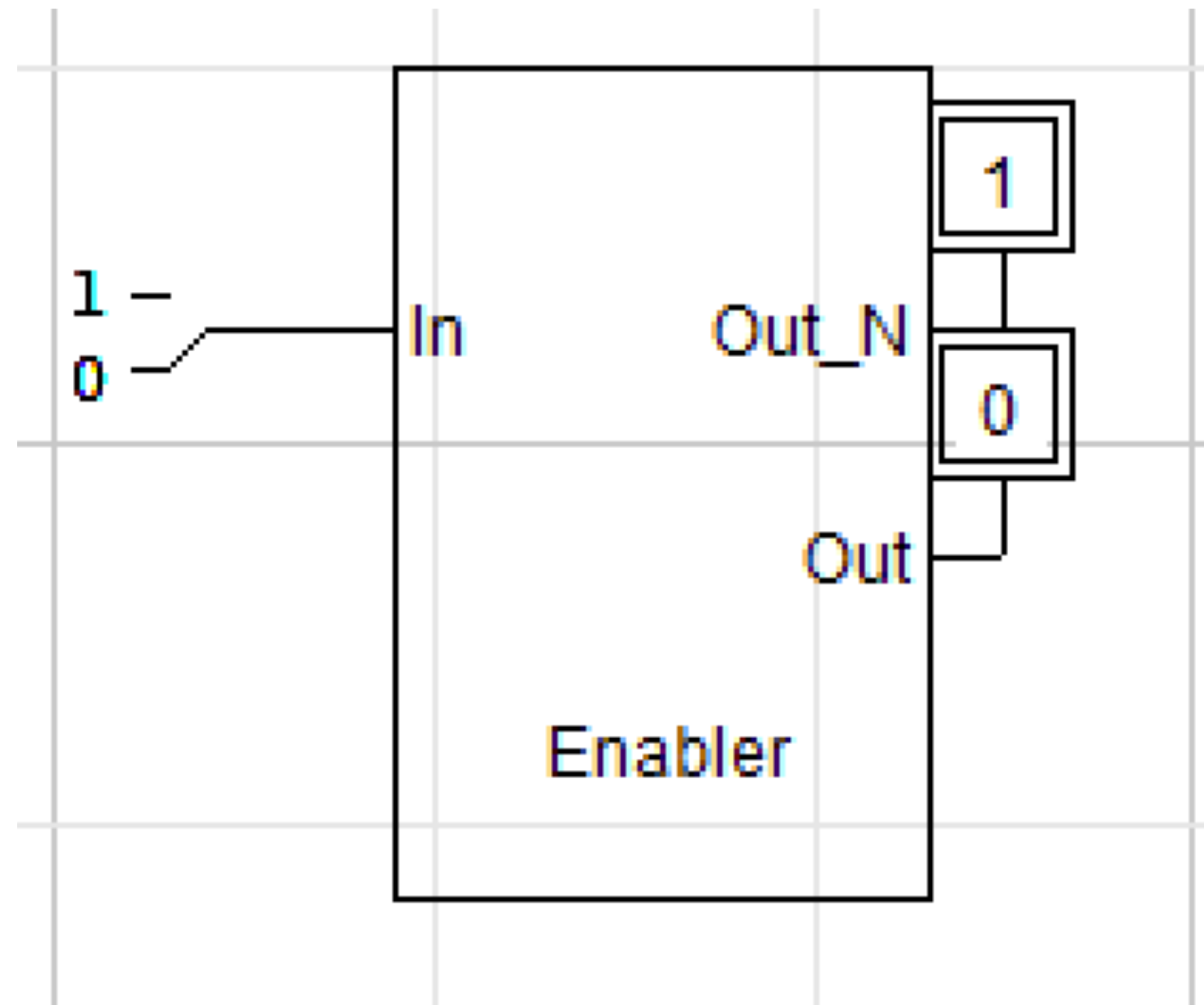


# Sub-circuit





# Sub-circuit



# Last Circuit Drawing Practice

1. Sub-circuit
2. Implementing 2-to-4 Decoder using drawing tools
3. Implementing 3-to-8 Decoder using 2-to-4 Decoders
4. Implementing Octal-to-Binary Priority Encoder using drawing tools<sup>1</sup>
5. Implementing Multiplexer using drawing tools<sup>1</sup>

1. You will be reusing these designs in later lectures and assignments