



CSCI 150

Introduction to Digital and Computer System Design

Lecture 2: Combinational Logical Circuits III



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Overview

- Focus: Boolean Algebra
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch2 2.3; v5: Ch2 2.3
- Core Ideas:
 1. Boolean Algebra II: Standard Forms

Boolean Algebra I

- Boolean Algebra vs Physical Implementation of Logic Circuits
- AND, OR, NOT Operators and Gates
 - Simple digital circuit implementation
 - Algebraic manipulation using Binary Identities

Boolean Algebra II: Standard Forms

Minterm/Maxterm;
Sum of Products; Product of Sums

Standard Forms

- Equivalent expressions can be written in a variety of ways
Standard forms: typical such ways that incorporates some **unique characteristics** -> **simplify the implementation** of these designs
- **Product terms** (AND terms): e.g. $\bar{X}YZ$
Literals with inverts connected through only AND operators
- **Sum terms** (OR terms): e.g. $X + \bar{Y} + Z$
Literals with inverts connected through only OR operators

Minterms and Maxterms

- Minterm**

Product term; Contains **all variables**; Has only **one Positive row** in the truth table

X	Y	$\bar{X}\bar{Y}$	$\bar{X}Y$	$X\bar{Y}$	XY
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

Minterms and Maxterms

- **Minterm**
Product term; Contains **all variables**; Has only **one Positive row** in the truth table

	X	Y	$\bar{X}\bar{Y}$	$\bar{X}Y$	$X\bar{Y}$	XY
$(00)_2=0$	0	0	1	0	0	0
$(01)_2=1$	0	1	0	1	0	0
$(10)_2=2$	1	0	0	0	1	0
$(11)_2=3$	1	1	0	0	0	1

Minterms and Maxterms

- Minterm**

Product term; Contains **all variables**; Has only **one Positive row** in the truth table

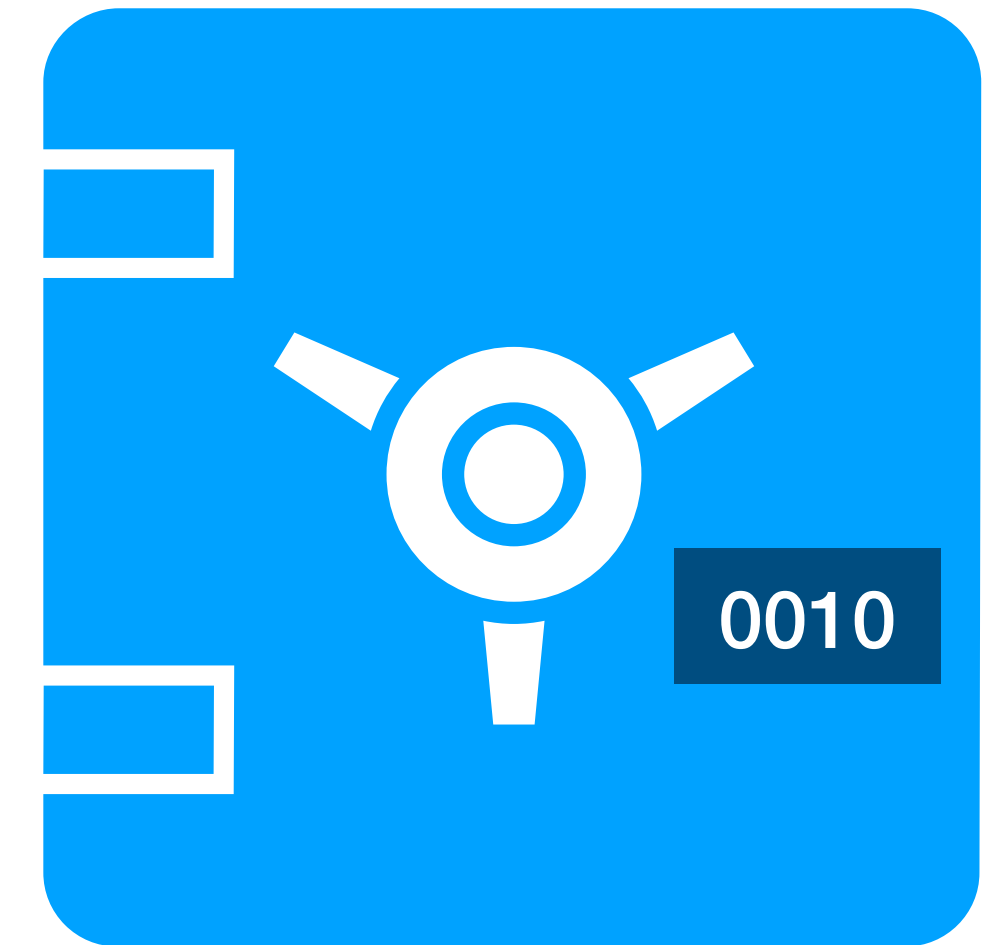
	X	Y	$m_0 = \bar{X}\bar{Y}$	$m_1 = \bar{X}Y$	$m_2 = X\bar{Y}$	$m_3 = XY$
$(00)_2=0$	0	0	1	0	0	0
$(01)_2=1$	0	1	0	1	0	0
$(10)_2=2$	1	0	0	0	1	0
$(11)_2=3$	1	1	0	0	0	1

Minterms

X	Y	Z	m_0 $\overline{X}\overline{Y}\overline{Z}$	m_1 $\overline{X}\overline{Y}Z$	m_2 $\overline{X}Y\overline{Z}$	m_3 $\overline{X}YZ$	m_4 $X\overline{Y}\overline{Z}$	m_5 $X\overline{Y}Z$	m_6 $XY\overline{Z}$	m_7 XYZ
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

Example

Minterms



- Minterm
 - The output is 1 when a unique combination of input (condition) is met (like a combination lock)

Minterms

- Minterms to Boolean conversions m_i
 - Write down i in binary $(\alpha_1\alpha_2 \dots \alpha_n)_2$
 - List all the variables, connect them with AND, if $\alpha_i = 0$ then invert the i th variable

Minterms

- With variables X, Y, Z, A , write down
 - m_4
 - m_{10}

Minterms

- With variables X, Y, Z, A , write down

- m_4 $m_4 = \bar{X}Y\bar{Z}\bar{A}$

- m_{10}

Minterms

- With variables X, Y, Z, A , write down

- m_4 $m_4 = \bar{X}Y\bar{Z}\bar{A}$

- m_{10} $m_{10} = X\bar{Y}Z\bar{A}$

Minterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

- Write down the Sum of Minterm of the truth table on the left

Minterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

- Write down the Sum of Minterm of the truth table on the left

$$m_0 + m_2 + m_5$$

Minterms

- With variables X, Y, Z , write down
 - $m_2 + m_3$
 - $m_1 + m_3 + m_5 + m_7$

Minterms

- With variables X, Y, Z , write down

- $m_2 + m_3$

$$m_2 + m_3 = \bar{X}Y\bar{Z} + \bar{X}YZ$$

- $m_1 + m_3 + m_5 + m_7$

Minterms

- With variables X, Y, Z , write down

- $m_2 + m_3$

$$m_2 + m_3 = \bar{X}Y\bar{Z} + \bar{X}YZ$$

- $m_1 + m_3 + m_5 + m_7$

$$m_1 + m_3 + m_5 + m_7 = \bar{X}\bar{Y}Z + \bar{X}YZ + X\bar{Y}Z + XYZ$$

Minterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

- Write down the Sum of Minterm of the truth table on the left

Minterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

- Write down the Sum of Minterm of the truth table on the left

$$m_0 + m_3 + m_6$$

Minterms and Maxterms

- Maxterm**

Sum term; Contains **all variables**; Has only **one Negative row** in the truth table

	X	Y	$M_0 = X + Y$	$M_1 = X + \bar{Y}$	$M_2 = \bar{X} + Y$	$M_3 = \bar{X} + \bar{Y}$
$(00)_2=0$	0	0	0	1	1	1
$(01)_2=1$	0	1	1	0	1	1
$(10)_2=2$	1	0	1	1	0	1
$(11)_2=3$	1	1	1	1	1	0

Minterms and Maxterms

- Maxterm**

Sum term; Contains **all variables**; Has only **one Negative row** in the truth table

$$M_i = \overline{m_i}$$

	X	Y	$M_0 = X + Y$	$M_1 = X + \overline{Y}$	$M_2 = \overline{X} + Y$	$M_3 = \overline{X} + \overline{Y}$
$(00)_2=0$	0	0	0	1	1	1
$(01)_2=1$	0	1	1	0	1	1
$(10)_2=2$	1	0	1	1	0	1
$(11)_2=3$	1	1	1	1	1	0

Maxterms

- Maxterms to Boolean conversion M_i
 - Write down i in binary $(\alpha_1\alpha_2 \dots \alpha_n)_2$
 - List all the variables, connect them with OR, if $\alpha_i = 1$ then invert the i th variable

Maxterms

- With variables X, Y, Z, A , write down
 - M_4
 - M_{10}

Maxterms

- With variables X, Y, Z, A , write down

- M_4

$$M_4 = (X + \bar{Y} + Z + A)$$

- M_{10}

Maxterms

- With variables X, Y, Z, A , write down

- M_4

$$M_4 = (X + \bar{Y} + Z + A)$$

- M_{10}

$$M_{10} = (\bar{X} + Y + \bar{Z} + A)$$

Maxterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

- Write down the Product of Maxterm of the truth table on the left

Maxterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

- Write down the Product of Maxterm of the truth table on the left

$$M_1 M_3 M_4 M_6$$

Maxterms

- With variables X, Y, Z , write down
 - $M_2 \cdot M_3$
 - $M_1 \cdot M_3 \cdot M_5 \cdot M_7$

Maxterms

- With variables X, Y, Z , write down

- $M_2 \cdot M_3$ $(X + \bar{Y} + Z)(X + \bar{Y} + \bar{Z})$

- $M_1 \cdot M_3 \cdot M_5 \cdot M_7$

Maxterms

- With variables X, Y, Z , write down

- $M_2 \cdot M_3$ $(X + \bar{Y} + Z)(X + \bar{Y} + \bar{Z})$

- $M_1 \cdot M_3 \cdot M_5 \cdot M_7$
 $(X + Y + \bar{Z})(X + \bar{Y} + \bar{Z})(\bar{X} + Y + \bar{Z})(\bar{X} + \bar{Y} + \bar{Z})$

Maxterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

- Write down the Product of Maxterm of the truth table on the left

Maxterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

- Write down the Product of Maxterm of the truth table on the left

$$M_1 M_2 M_4 M_7$$

Minterms and Maxterms

- e.g. $M_3 = X + \bar{Y} + \bar{Z} = \overline{\bar{X}Y\bar{Z}} = \bar{m}_3$
- Sum of Minterms
 - e.g. $F = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XYZ = m_0 + m_2 + m_5 + m_7$
 $= \Sigma m(0,2,5,7)$
- Product of Maxterm
 - e.g. $F = (X + Y + Z)(X + \bar{Y} + Z)(\bar{X} + Y + \bar{Z})(\bar{X} + \bar{Y} + \bar{Z})$
 $= M_0M_2M_5M_7$
 $= \Pi M(0,2,5,7)$

Minterms and Maxterms

- $F(X, Y, Z) = \Sigma m(1,4,5)$
- Write down F in boolean expression
- Write down \overline{F} in Product of Maxterm form

Minterms and Maxterms

- $F(X, Y, Z) = \Sigma m(1,4,5)$
- Write down F in boolean expression $F = \bar{X}\bar{Y}Z + X\bar{Y}\bar{Z} + X\bar{Y}Z$
- Write down \bar{F} in Product of Maxterm form

Minterms and Maxterms

- $F(X, Y, Z) = \Sigma m(1,4,5)$
- Write down F in boolean expression $F = \bar{X}\bar{Y}Z + X\bar{Y}\bar{Z} + X\bar{Y}Z$
- Write down \bar{F} in Product of Maxterm form

$$\bar{F} = \Pi_M(1,4,5)$$

Minterms and Maxterms

- $F(X, Y, Z) = \Pi M(2,3,7)$
- Write down F in boolean expression
- Write down \bar{F} in Sum of Minterm form

Minterms and Maxterms

- $F(X, Y, Z) = \Pi M(2,3,7)$

- Write down F in boolean expression

$$F = (X + \bar{Y} + Z)(X + \bar{Y} + \bar{Z})(\bar{X} + \bar{Y} + \bar{Z})$$

- Write down \bar{F} in Sum of Minterm form

Minterms and Maxterms

- $F(X, Y, Z) = \Pi M(2,3,7)$

- Write down F in boolean expression

$$F = (X + \bar{Y} + Z)(X + \bar{Y} + \bar{Z})(\bar{X} + \bar{Y} + \bar{Z})$$

- Write down \bar{F} in Sum of Minterm form

$$\bar{F} = \Sigma_m(2,3,7)$$

Sum of Products

- Sum of Minterm can come directly from the truth table
- Sum of Product: simplified version of Sum of Minterm
- $F(X, Y, Z) = \Sigma m(0,1,2,6)$

Sum of Products

- Sum of Minterm can come directly from the truth table
- Sum of Product: simplified version of Sum of Minterm
- $F(X, Y, Z) = \Sigma m(0,1,2,6)$
 $= \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + XY\bar{Z}$

Sum of Products

- Sum of Minterm can come directly from the truth table
- Sum of Product: simplified version of Sum of Minterm
- $$\begin{aligned} F(X, Y, Z) &= \Sigma m(0,1,2,6) \\ &= \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + XY\bar{Z} \\ &= \bar{X}\bar{Y} + Y\bar{Z} \end{aligned}$$

Product of Sums

- Product of Maxterms can also come directly from the truth table
- Product of Sums: simplified version of Product of Maxterms
- $F(X, Y, Z) = \Pi M(0,1,2,6)$

Product of Sums

- Product of Maxterms can also come directly from the truth table
- Product of Sums: simplified version of Product of Maxterms
- $F(X, Y, Z) = \Pi M(0,1,2,6)$
 $= (X + Y + Z)(X + Y + \bar{Z})(X + \bar{Y} + Z)(\bar{X} + \bar{Y} + Z)$

Product of Sums

- Product of Maxterms can also come directly from the truth table
- Product of Sums: simplified version of Product of Maxterms
- $$\begin{aligned} F(X, Y, Z) &= \Pi M(0, 1, 2, 6) \\ &= (X + Y + Z)(X + Y + \bar{Z})(X + \bar{Y} + Z)(\bar{X} + \bar{Y} + Z) \\ &= (X + Y)(\bar{Y} + Z) \end{aligned}$$

Summary

Summary

- Minterm/Maxterm

Summary

- Minterm/Maxterm
- Sum of Products

Summary

- Minterm/Maxterm
- Sum of Products
- Product of Sums

Boolean Algebra II

Exercises!

Boolean Algebra

Difficulty: Simple

Obtain the truth table of the following function, and express each function in sum-of-minterms and product-of-maxterms form:

- $(XY + Z)(Y + XZ)$

Boolean Algebra

Difficulty: Simple

For the Boolean functions E and F , as given in the following truth table:

- List the minterms and maxterms of each function
- List the minterms of \bar{E} and \bar{F}
- List the minterms of $E + F$ and EF

X	Y	Z	E	F
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	0
1	1	1	0	1

Boolean Algebra

Difficulty: Simple

For the Boolean functions E and F , as given in the following truth table:

- Express E and F in sum-of-minterms algebraic form
- Simplify E and F to expressions with a minimum of literals

X	Y	Z	E	F
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	0
1	1	1	0	1

Boolean Algebra

Difficulty: Mid

Convert the following expressions into sum-of-products and product-of-sums form

- $(AB + C)(B + \overline{C}D)$
- $\overline{X} + X(X + \overline{Y})(Y + \overline{Z})$