



24.09.20 13:35

CSCI 150

Introduction to Digital and Computer System Design

Lecture 2: Combinational Logical Circuits II



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2020 Fall Semester (S3)

Overview

- Focus: Boolean Algebra
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch2 2.2, 2.3; v5: Ch2 2.2, 2.3
- Core Ideas:
 1. Boolean Algebra I
 2. In-Class Exercises

Boolean Algebra

It's just math

Intro. to Boolean Algebra

$$L(X_1, X_2, \dots, X_n) = Y_1, Y_2, \dots, Y_m$$

- **Boolean Expression**

An algebraic expression formed by using binary variables, the constants 0 and 1, the logic operation symbols, and parentheses

- **Boolean Function**

A Boolean equation consisting of a binary variable identifying the function, followed by an equals sign and a **Boolean Expression**

- **Single-Output / Multi-Output Boolean Function**

Multiple Boolean function variables as input, **value 0/1 (single)** or **combinations of 0/1s (multi)** as output

Intro. to Boolean Algebra

$$L(X_1, X_2, \dots, X_n) = Y_1, Y_2, \dots, Y_m$$

- If a boolean function as n input variables and m output variables, how many rows will its Truth Table have?

Basic Identities

1. $X + 0 =$

2. $X \cdot 1 =$

3. $X + 1 =$

4. $X \cdot 0 =$

5. $X + X =$

6. $X \cdot X =$

7. $X + \bar{X} =$

8. $X \cdot \bar{X} =$

9. $\bar{\bar{X}} =$

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Basic Identities

- Communicative

$$10. X + Y = Y + X$$

$$11. XY = YX$$

- Associative

$$12. X + (Y + Z) = (X + Y) + Z$$

$$13. X(YZ) = (XY)Z$$

- Distributive

$$14. X(Y + Z) = XY + XZ$$

$$15. X + (YZ) = (X + Y)(X + Z)$$

- DeMorgan's

$$16. \overline{X + Y} = \bar{X} \cdot \bar{Y}$$

$$17. \overline{\bar{X} \cdot \bar{Y}} = X + Y$$

DeMorgan's

$$\overline{X + Y} = \bar{X} \cdot \bar{Y}$$

$$\overline{X \cdot Y} = \bar{X} + \bar{Y}$$

Truth Table

X	Y	$\overline{X + Y}$	$\bar{X} \cdot \bar{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Truth Table-1

X	Y	$\overline{X \cdot Y}$	$\bar{X} + \bar{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Basic Identities

A. $X + XY = X$

B. $XY + X\bar{Y} = X$

C. $X + \bar{X}Y = X + Y$

D. $X(X + Y) = X$

E. $(X + Y)(X + \bar{Y}) = X$

F. $X(\bar{X} + Y) = XY$

Basic Identities

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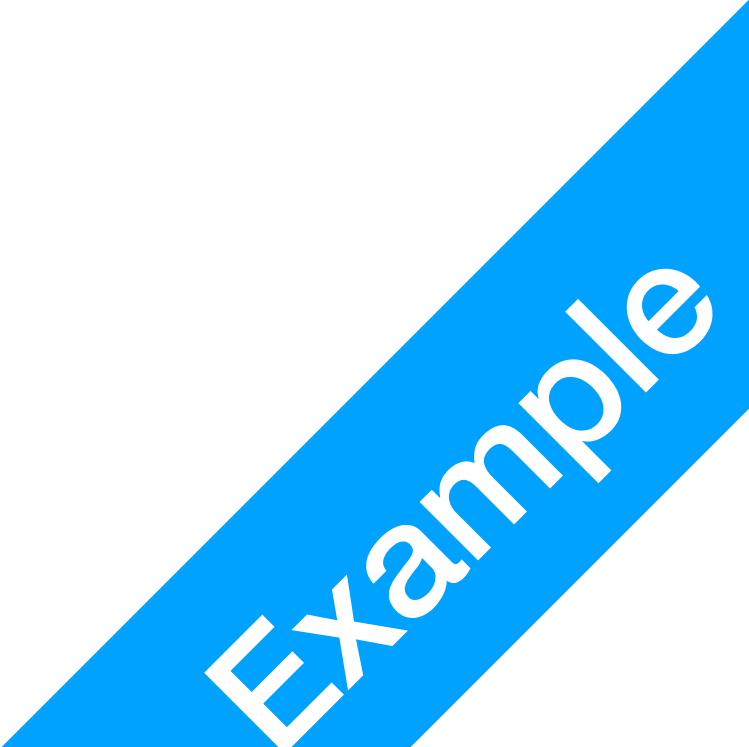
6. $X \cdot X = X$

Basic Identities

Example

Basic Identities

- Prove



Basic Identities

- Prove

- $\bar{X} \cdot \bar{Y} + \bar{X} \cdot Y + X \cdot Y = \bar{X} + Y$

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 - $\bar{X} \cdot \bar{Y} + \bar{X} \cdot Y + X \cdot Y = \bar{X} + Y$
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 - $\bar{X} \cdot \bar{Y} + \bar{X} \cdot Y + X \cdot Y = \bar{X} + Y$
 - $\bar{A} \cdot B + \bar{B} \cdot \bar{C} + A \cdot B + \bar{B} \cdot C = 1$
 - $Y + \bar{X} \cdot Z + X \cdot \bar{Y} = X + Y + Z$

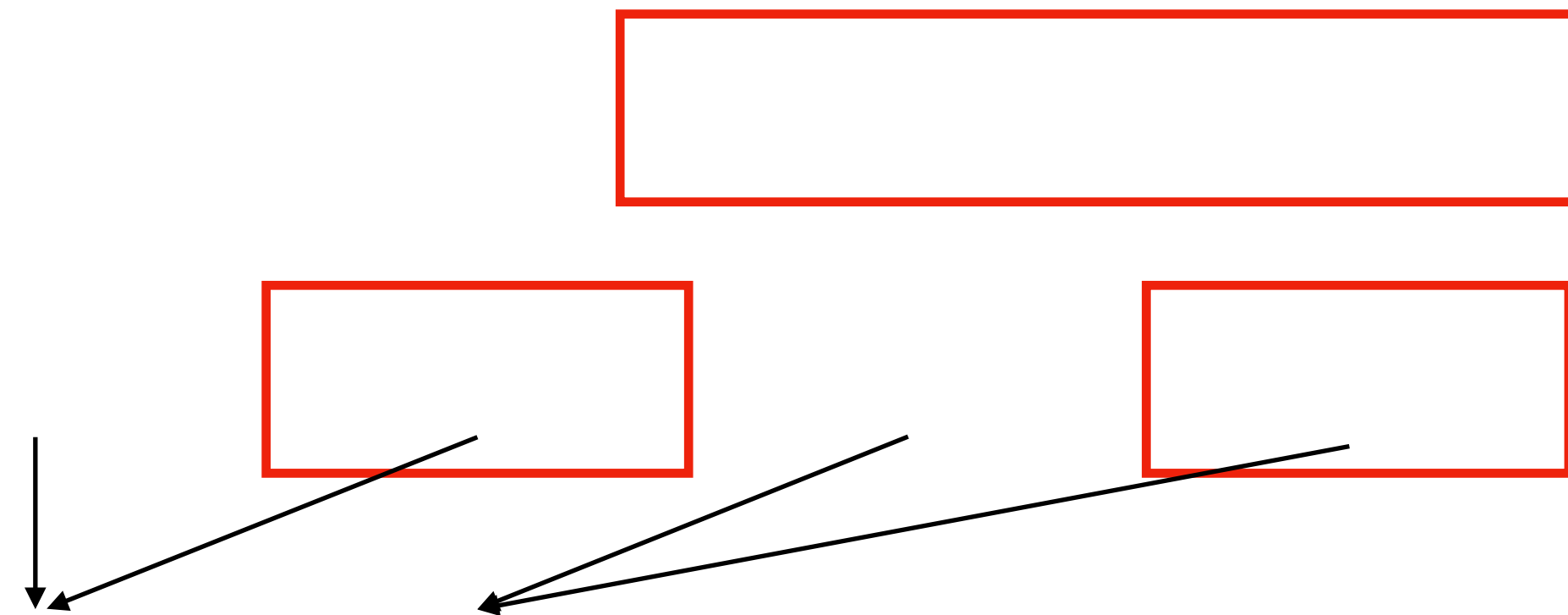
Basic Identities

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 - $\bar{X} \cdot \bar{Y} + \bar{Y} \cdot Z + X \cdot Z = \bar{X} \cdot \bar{Y} + X \cdot Z$
 - $\bar{X}\bar{Y} + \bar{Y}Z + XZ + XY + Y\bar{Z} = \bar{X} \cdot \bar{Y} + X \cdot Z + Y \cdot \bar{Z}$

Basic Identities

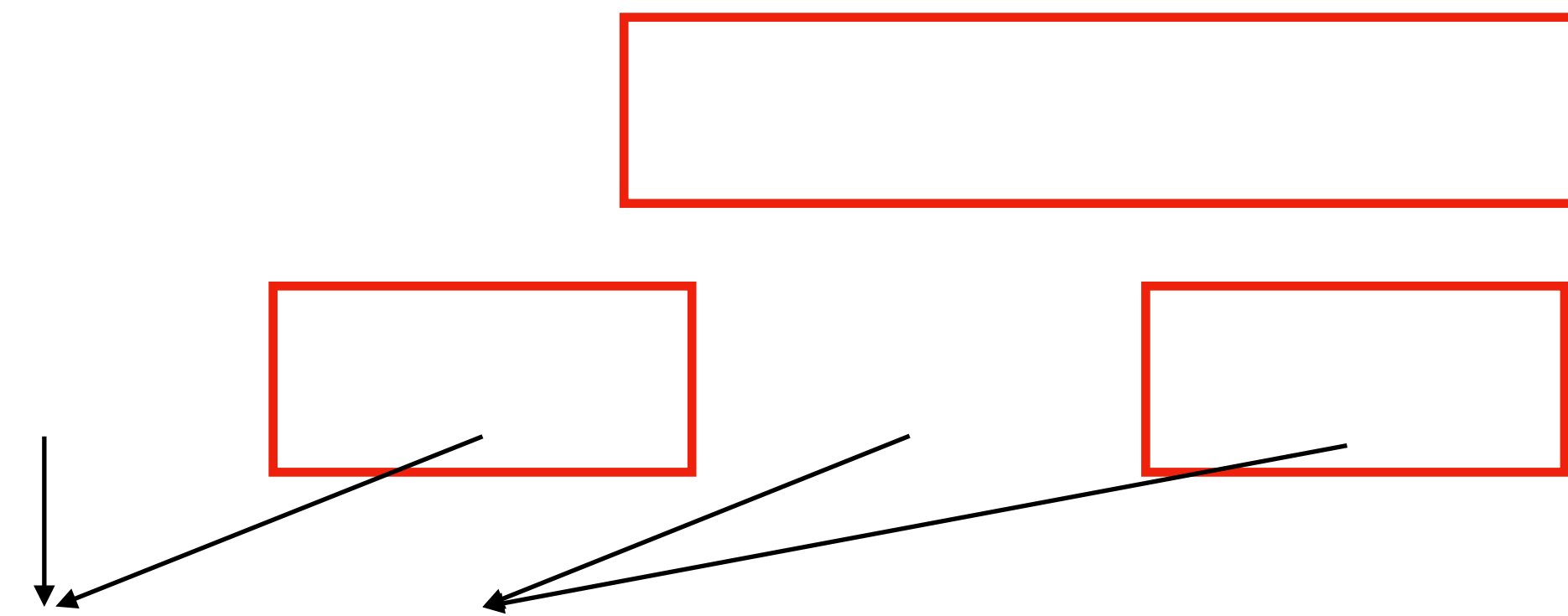


Rule B
Rule A x 2

Example

Basic Identities

- $\bar{X} \cdot \bar{Y} + \bar{Y} \cdot Z + X \cdot Z = \bar{X} \cdot \bar{Y} + X \cdot Z$

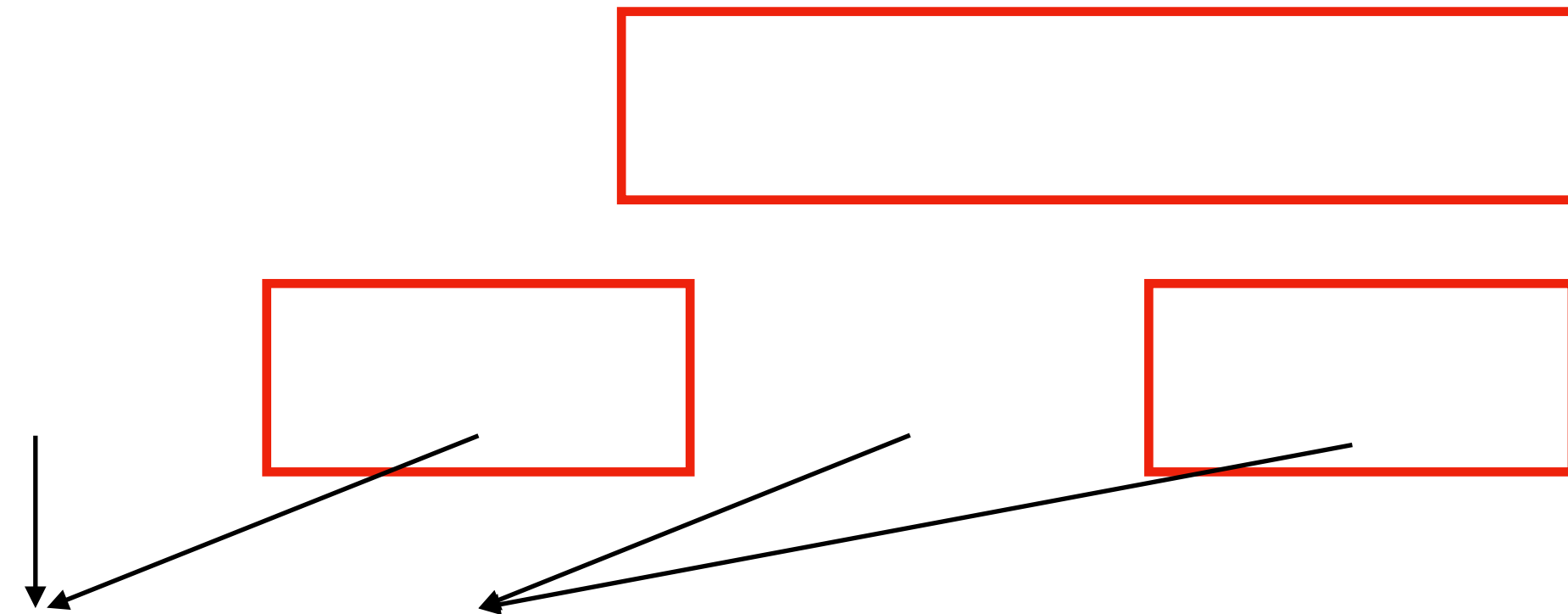


Rule B
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Example

Basic Identities

- $\bar{X} \cdot \bar{Y} + \bar{Y} \cdot Z + X \cdot Z = \bar{X} \cdot \bar{Y} + X \cdot Z$
- Since $\bar{Y} \cdot Z = X \cdot \bar{Y} \cdot Z + \bar{X} \cdot \bar{Y} \cdot Z$



Rule B

Rule A x 2

Example

Basic Identities

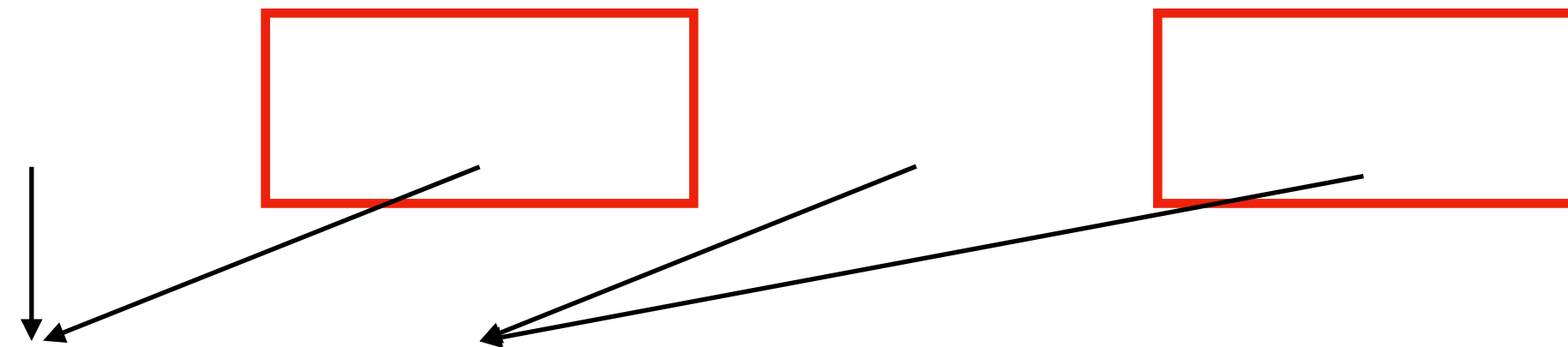
- $\bar{X} \cdot \bar{Y} + \bar{Y} \cdot Z + X \cdot Z = \bar{X} \cdot \bar{Y} + X \cdot Z$

- Since $\bar{Y} \cdot Z = X \cdot \bar{Y} \cdot Z + \bar{X} \cdot \bar{Y} \cdot Z$

- $\bar{X} \cdot \bar{Y} + \bar{Y} \cdot Z + X \cdot Z = \bar{X} \cdot \bar{Y} + X \cdot Z + X \cdot \bar{Y} \cdot Z + \bar{X} \cdot \bar{Y} \cdot Z$

Rule B

Rule A x 2



Basic Identities

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Rule B

$$= \bar{X} \cdot \bar{Y} + \bar{X} \cdot \bar{Y} \cdot Z + X \cdot Z + X \cdot \bar{Y} \cdot Z$$

Rule A x 2

Basic Identities

- $\bar{X} \cdot \bar{Y} + \bar{Y} \cdot Z + X \cdot Z = \bar{X} \cdot \bar{Y} + X \cdot Z$

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Rule B

$$= \bar{X} \cdot \bar{Y} + \bar{X} \cdot \bar{Y} \cdot Z + X \cdot Z + X \cdot \bar{Y} \cdot Z$$

Rule A x 2

$$= \bar{X} \cdot \bar{Y} + X \cdot Z$$

Basic Identities

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- **Identify** rules **applicable** to the expression

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 - **Simplification**: reducing the number of variables and operators in an expression without changing its truth table values

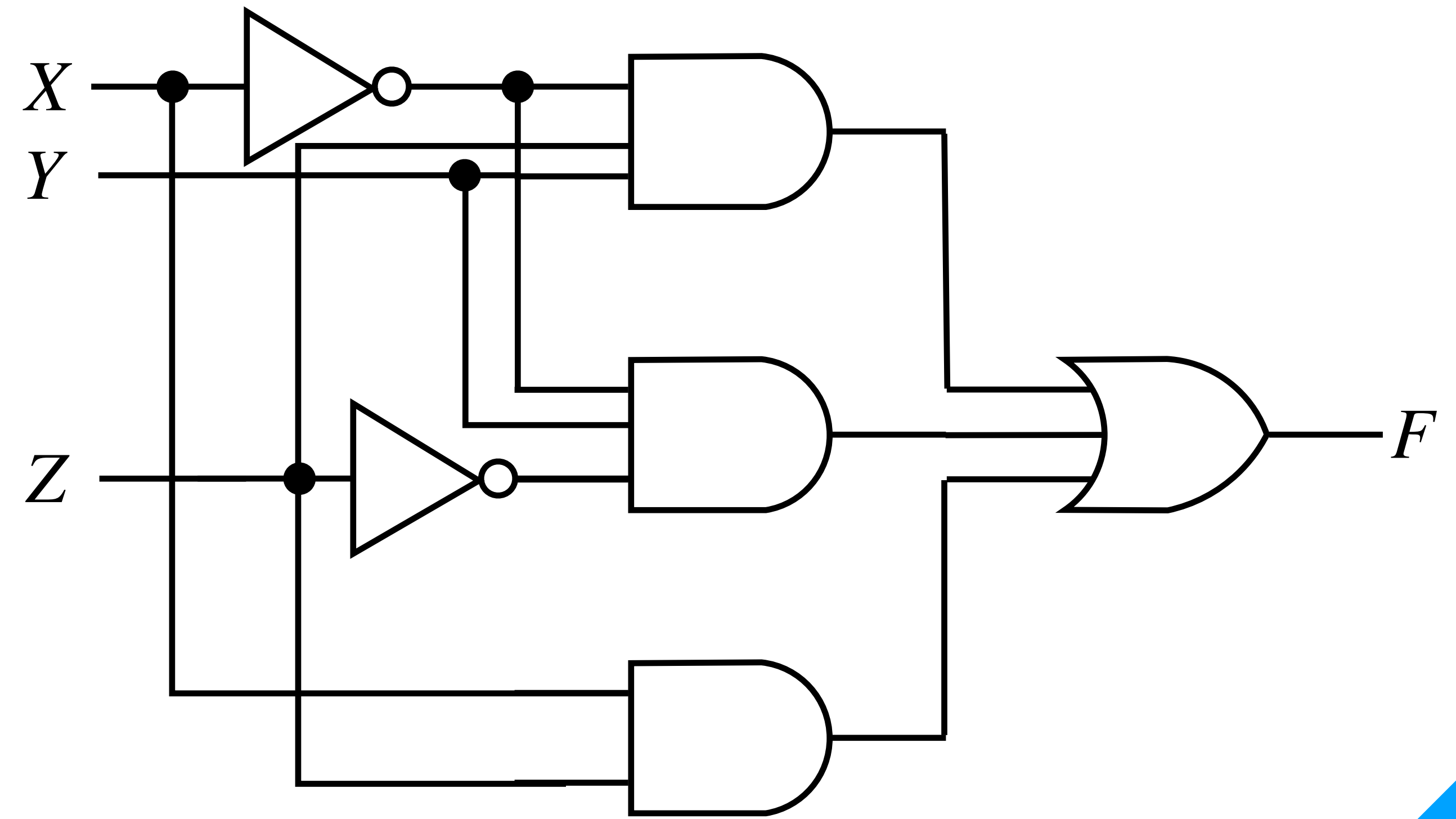
Basic Identities

- Boolean Algebra solving
 - **Identify** rules **applicable** to the expression
 - **Apply** rules that can help you **simplify** the expression
 - **Simplification:** reducing the number of variables and operators in an expression without changing its truth table values
 - **Atomic element:** an element that can't have the number of its variables and operators reduced any further

Algebraic Manipulation

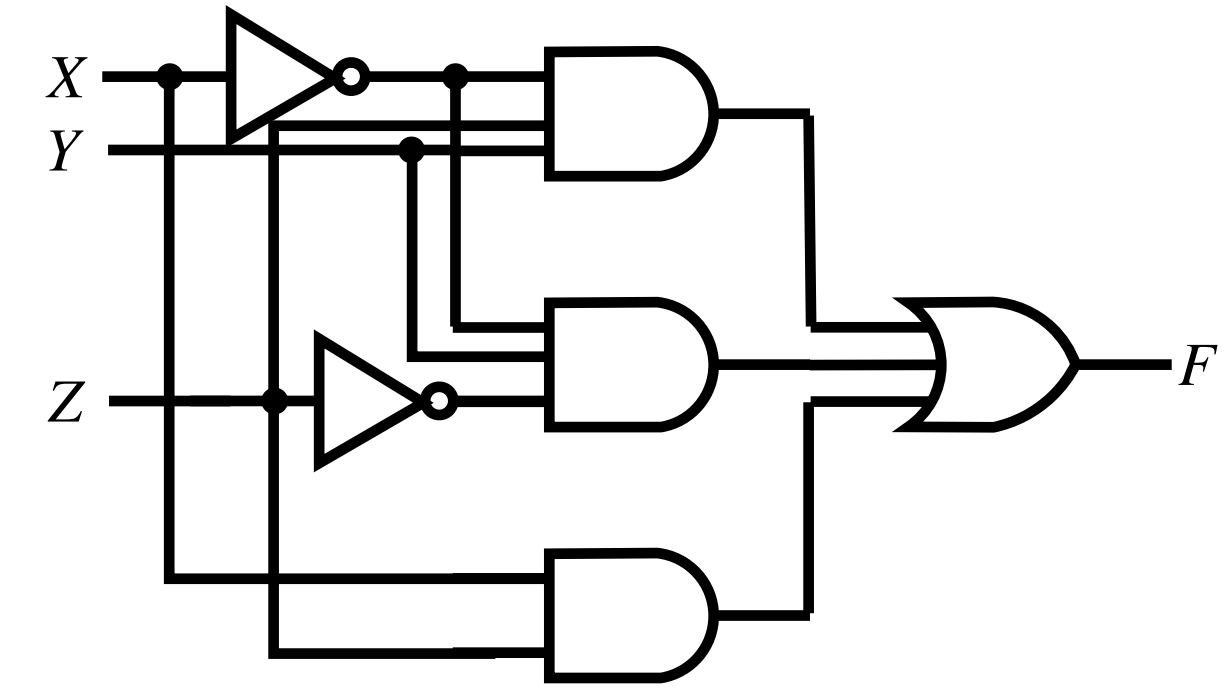
Truth Table

X	Y	Z	$F = \bar{X}YZ + \bar{X}Y\bar{Z} + XZ$
0	0	0	0
0	1	0	1
1	0	0	0
1	1	0	0
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1



Example

Algebraic Manipulation



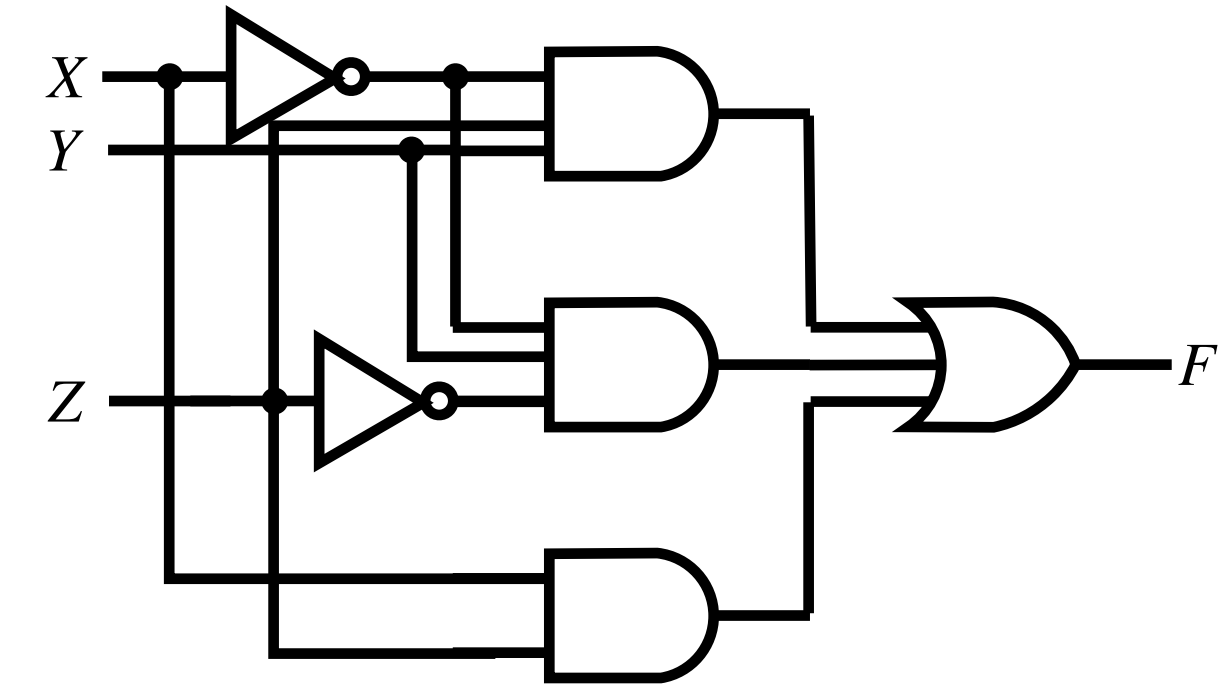
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Example

Algebraic Manipulation



Truth Table

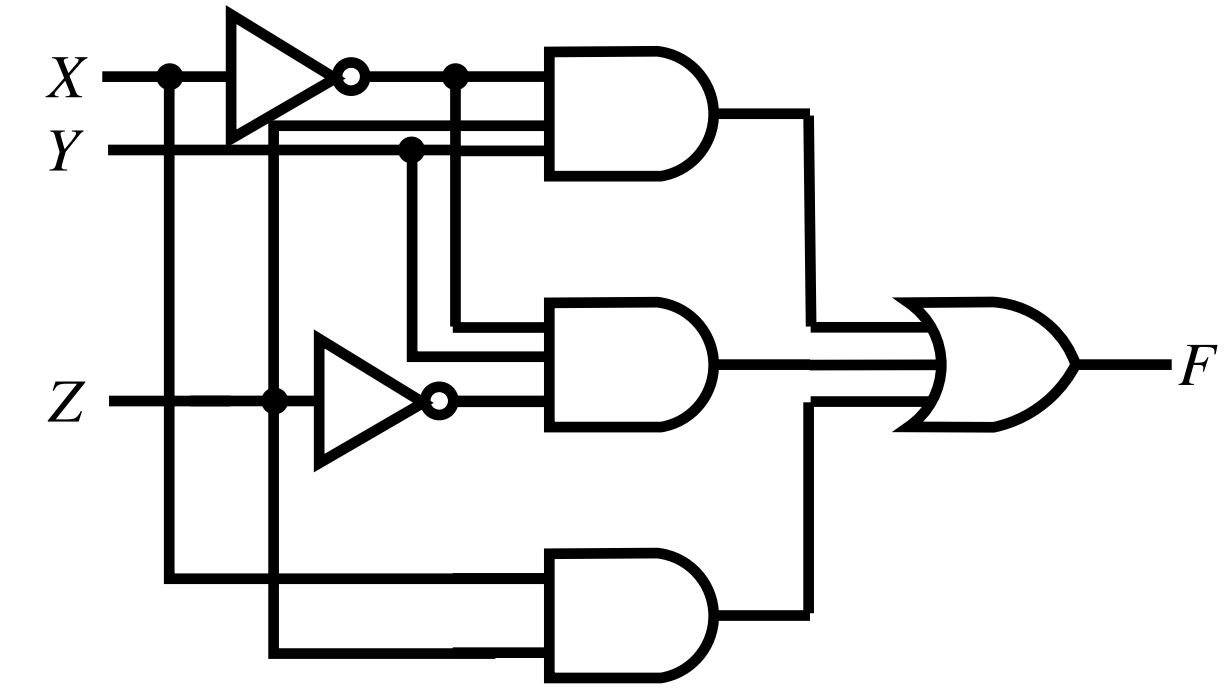
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$$\begin{aligned}
 F &= \bar{X}YZ + \bar{X}Y\bar{Z} + XZ \\
 &= \bar{X}Y(Z + \bar{Z}) + XZ
 \end{aligned}$$

Rule 14

Example

Algebraic Manipulation



Truth Table

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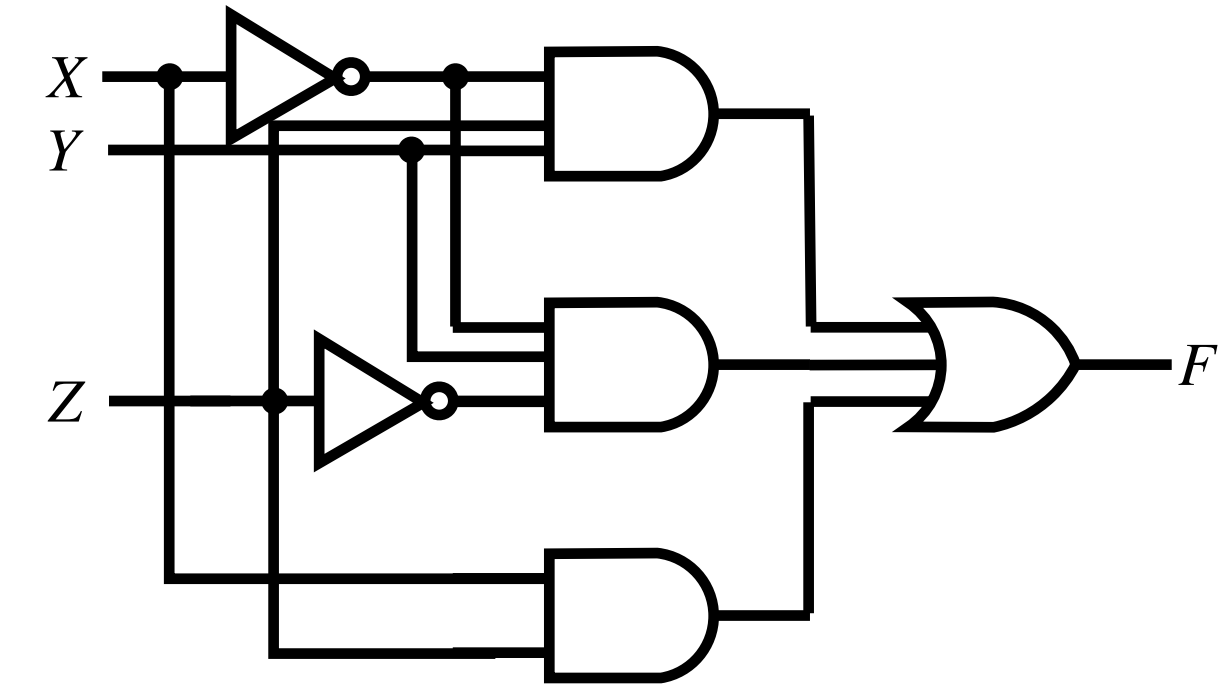
$$\begin{aligned}
 F &= \bar{X}YZ + \bar{X}Y\bar{Z} + XZ \\
 &= \bar{X}Y(Z + \bar{Z}) + XZ \\
 &= \bar{X}Y \cdot 1 + XZ
 \end{aligned}$$

Rule 14

Rule 7

Example

Algebraic Manipulation



Truth Table

X	Y	Z	$F = \bar{X}YZ + \bar{X}Y\bar{Z} + XZ$
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 \end{aligned}$$

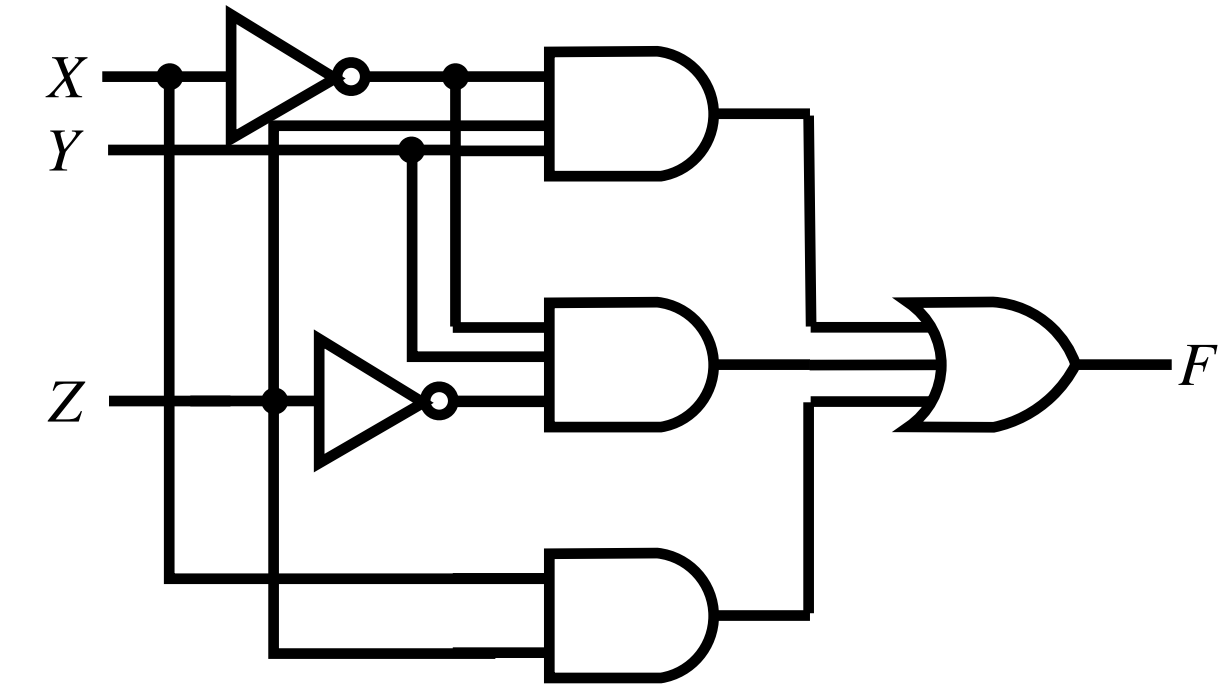
Rule 14

Rule 7

Rule 2

Example

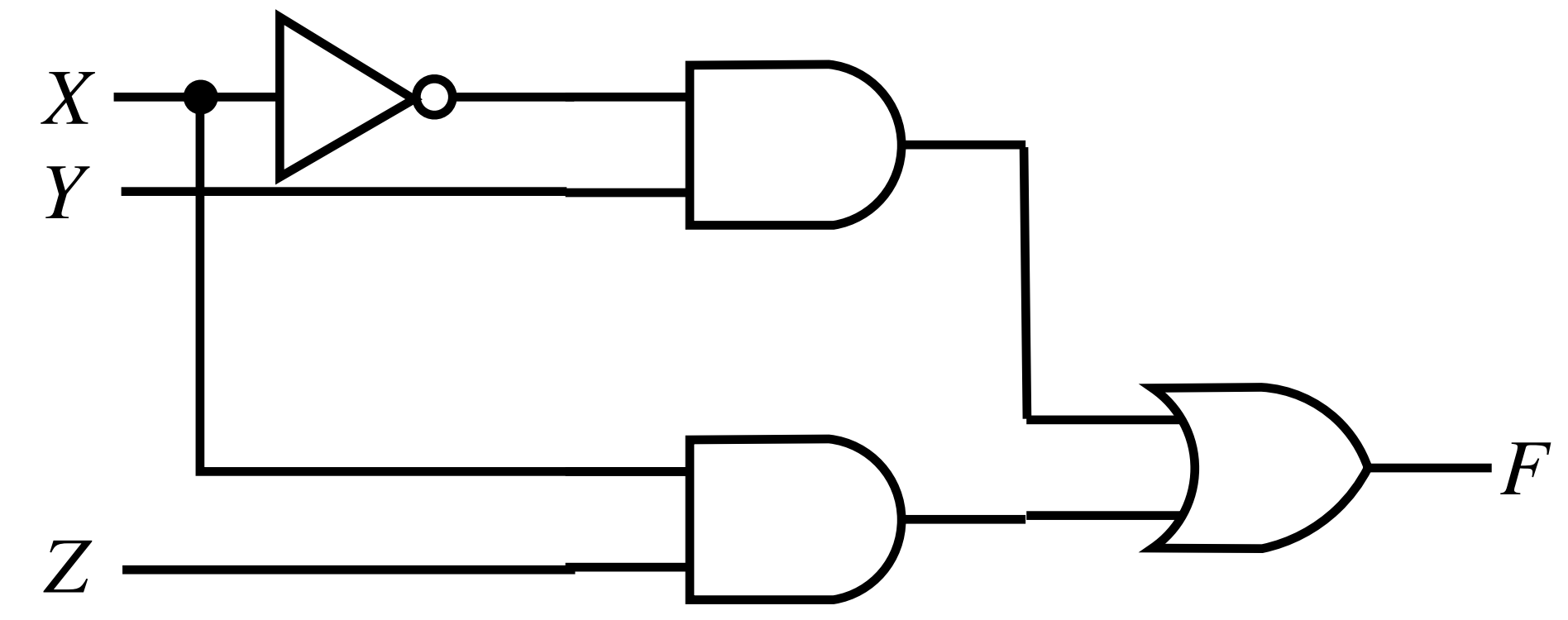
Algebraic Manipulation



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$$\begin{aligned}
 F &= \bar{X}YZ + \bar{X}Y\bar{Z} + XZ \\
 &= \bar{X}Y(Z + \bar{Z}) + XZ && \text{Rule 14} \\
 &= \bar{X}Y \cdot 1 + XZ && \text{Rule 7} \\
 &= \bar{X}Y + XZ && \text{Rule 2}
 \end{aligned}$$



Example

Algebraic Manipulation

- Algebraic Manipulation can help reduce the number of gates in a circuit
 - easier to implement and debug
 - more efficient

Complementation

- \bar{F} : complement (invert) representation for a function F , obtained from an interchange of 1s to 0s and 0s to 1s for the values of F in the truth table
- Apply DeMorgan's Rule

$$16. \overline{X_1 + X_2 + \dots + X_n} = \bar{X}_1 \cdot \bar{X}_2 \cdot \dots \cdot \bar{X}_n$$

$$17. \overline{X_1 \cdot X_2 \cdot \dots \cdot X_n} = \bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_n$$

Complementation

- $F_1 = \bar{X}Y\bar{Z} + \bar{X}\bar{Y}Z$
- $F_2 = X(\bar{Y}\bar{Z} + YZ)$

Algebra Solving 1

- Known $BC + D = 1; \bar{C} = 1$
- Calculate: $\bar{D}(\bar{B} + \bar{C})$
- Calculate: $(D + B)(D + C)$

Algebra Solving 2

- Known $AB + C = 1; A + \bar{D} = 0$
- Calculate: $C + AB$
- Calculate: $\bar{C}\bar{A} + \bar{C}\bar{B}$

Algebra Solving 3

- Known $A + B + \bar{C} = 1$; $\bar{A}\bar{C} + \bar{A}D = 1$
- Calculate: $(\bar{C} + B)(\bar{C} + D)$
- Calculate: $\bar{C} + BD$

Boolean Algebra

Exercises! Use HS401 to help you!

Boolean Algebra

Difficulty: Simple

Prove by truth table that

- $\bar{X}Y + \bar{Y}Z + X\bar{Z} = X\bar{Y} + Y\bar{Z} + \bar{X}Z$

Algebraic Manipulation

Difficulty: Simple

Use DeMorgan's Rules to transform the following expression to one WITHOUT AND operator

- $\overline{ABC} + CD$
- $A\overline{B}C + \overline{A} \cdot \overline{C} + AB$

Logic Diagram

Difficulty: Simple

Draw the logic diagram for the following expression (you don't have to perform any manipulation/transformation)

- $XYZ + \overline{X}\overline{Y} + \overline{X}\overline{Z}$
- $B(\overline{A} \cdot \overline{C} + AC) + \overline{B}(A + \overline{B}C)$

Algebraic Manipulation

Difficulty: Simple

Simplify the following expressions

- $\bar{X} \cdot \bar{Y} + XYZ + \bar{X}Y$
- $X + Y(Z + \overline{X + Z})$

Algebraic Manipulation

Difficulty: Mid

Simplify the following expressions

- $\bar{W}X(\bar{Z} + \bar{Y}Z) + X(W + \bar{W}YZ)$
- $(AB + \bar{A}\bar{B})(\bar{C}\bar{D} + CD) + AC$

Algebraic Manipulation

Difficulty: Mid

Simplify the following expressions

- $\bar{A} \cdot \bar{C} + \bar{A}BC + \bar{B}C$

- $\overline{A + B + C} \cdot \overline{ABC}$

Algebraic Manipulation

Difficulty: Mid

Simplify the following expressions

- $ABC\bar{C} + AC$
- $\bar{A} \cdot \bar{B}D + \bar{A} \cdot \bar{C}D + BD$

Algebraic Manipulation

Difficulty: HARDCORE

Given that $AB = 0$ and $A + B = 1$, prove that

- $(A + C)(\bar{A} + B)(B + C) = BC$

Algebraic Manipulation

Difficulty: HARDCORE

Prove the identity of each of the following Boolean equations

- $ABC\bar{C} + BC\bar{D} + BC + \bar{C}D = B + \bar{C}D$
- $WY + \bar{W}Y\bar{Z} + WXZ + \bar{W}X\bar{Y} = WY + \bar{W}X\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}Z$
- $A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C = (\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + B + C + D)$