



18.09.20 00:00

# CSCI 150

## Introduction to Digital and Computer System Design

### Lecture 1: Digital Information Representations II



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2020 Fall Semester (S3)

# Overview

- Focus: Number Systems
- Architecture: Digital Circuits
- Textbook v4: Ch1 1.3, 1.4; v5: Ch1 1.4, 1.5
- Core Ideas:
  1. Digital Number Systems
  2. Arithmetic Operations, Ranges
  3. Digital & Analog Conversion

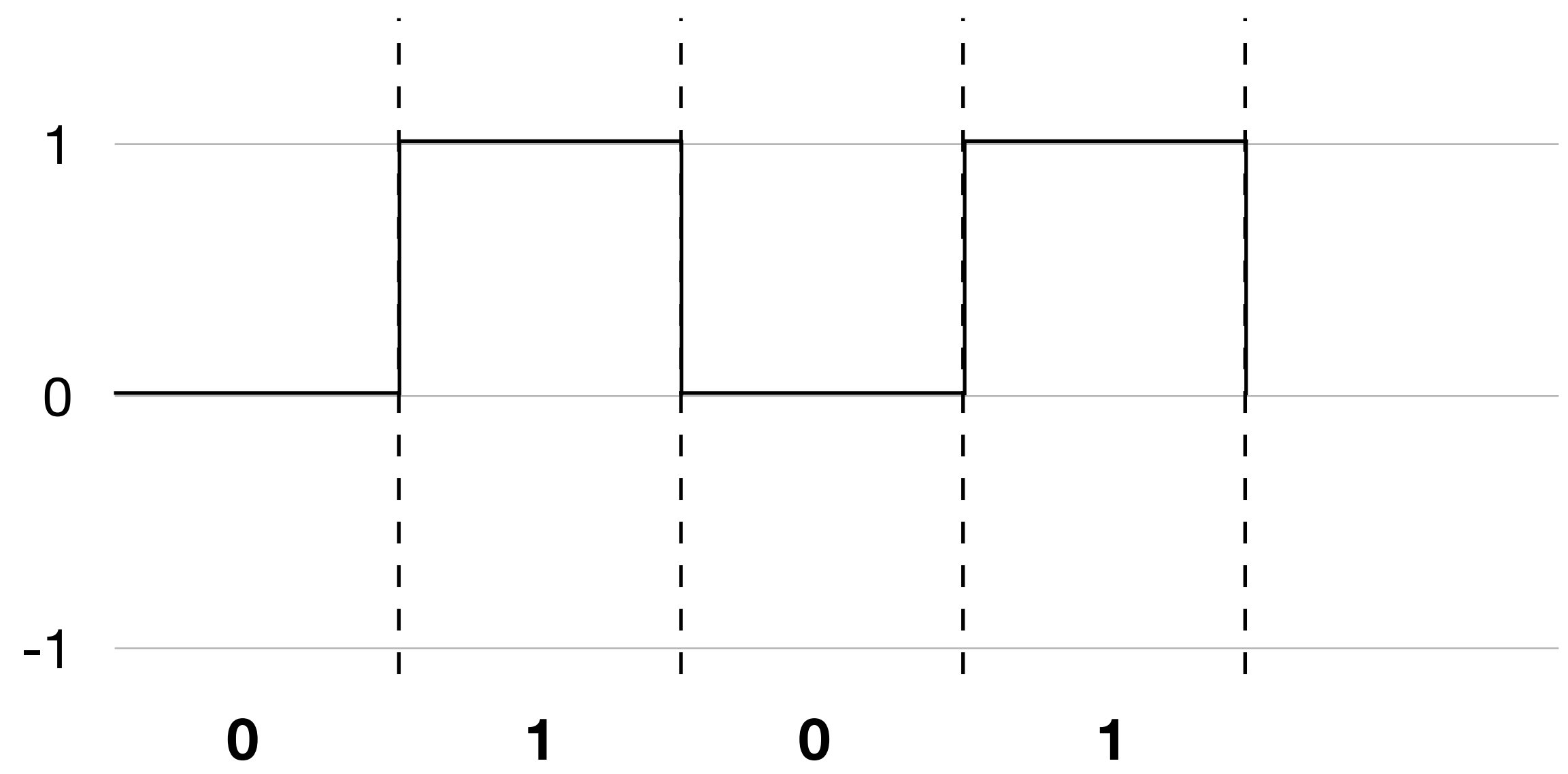
# Digital and Analog Circuits

- In Digital Circuits, information are represented by discrete values, usually 0/1s defined by HIGH/LOW voltages ranges
- In Analog Circuits, information representation is continuous
- Von Neumann computers: all-purpose design
  - CPU (control unit and functional/datapath unit), IO, memory
- Embedded systems: specific functions only, compact and robust

# Digital Number Systems

Binary, Octal and Hexadecimal Numbers;  
Number Ranges

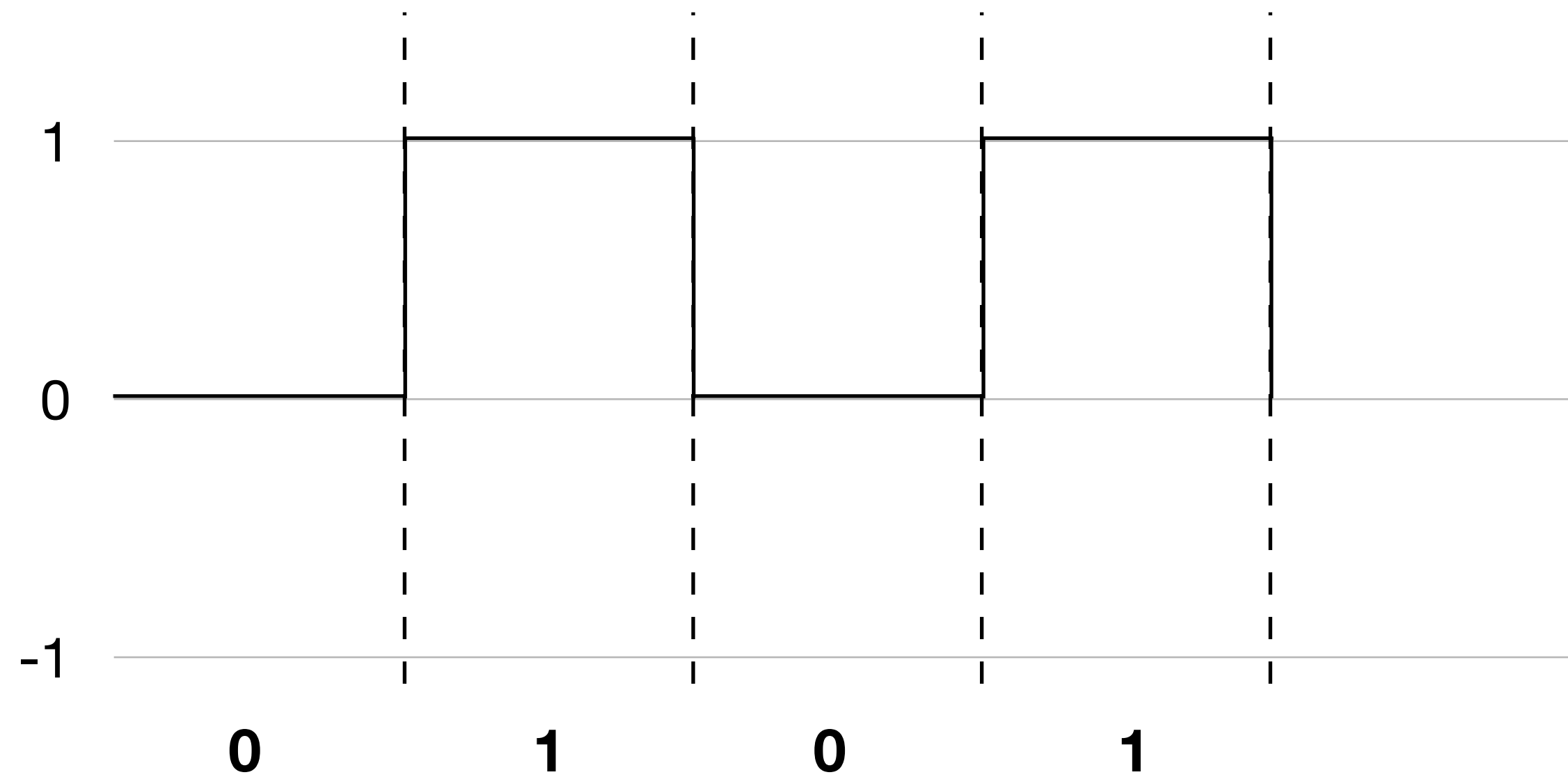
# Binary System



- Base 2 system
- A number is represented with a string of 1s and 0s, each called a *bit*
- $(0101)_2 = 5$

Concept

# Binary System



- Is it possible to use different bases in a digital circuit?
- If it is possible, why haven't we seen it very often?

# Binary Systems in Computers

- Every 8bit is called a Byte
- 32bit OS
- A single number is represented by 32bits
- Range (int): 0 - 4,294,967,295
- OS vs Processor?
- Compatibility mode

Windows 8 Pro with Media Center  
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System

Rating: 3.1 Windows Experience Index

Processor: Intel(R) CPU 2.20GHz 2.19 G

Installed memory (RAM):

System type: 32-bit Operating System, 64-based processor

Pen and Touch: No Pen or Touch input is available for this Display

Computer name, domain, and workgroup settings

Computer name:

Full computer name:

Computer description:

Workgroup: WORKGROUP

Rating: 6.9 Windows Experience Index

Processor: Intel(R) Core(TM)2 Quad CPU Q6600 @ 2.40GHz

Installed memory (RAM): 4.00 GB

System type: 64-bit Operating System

Pen and Touch: No Pen or Touch Input is available for this Display



Concept

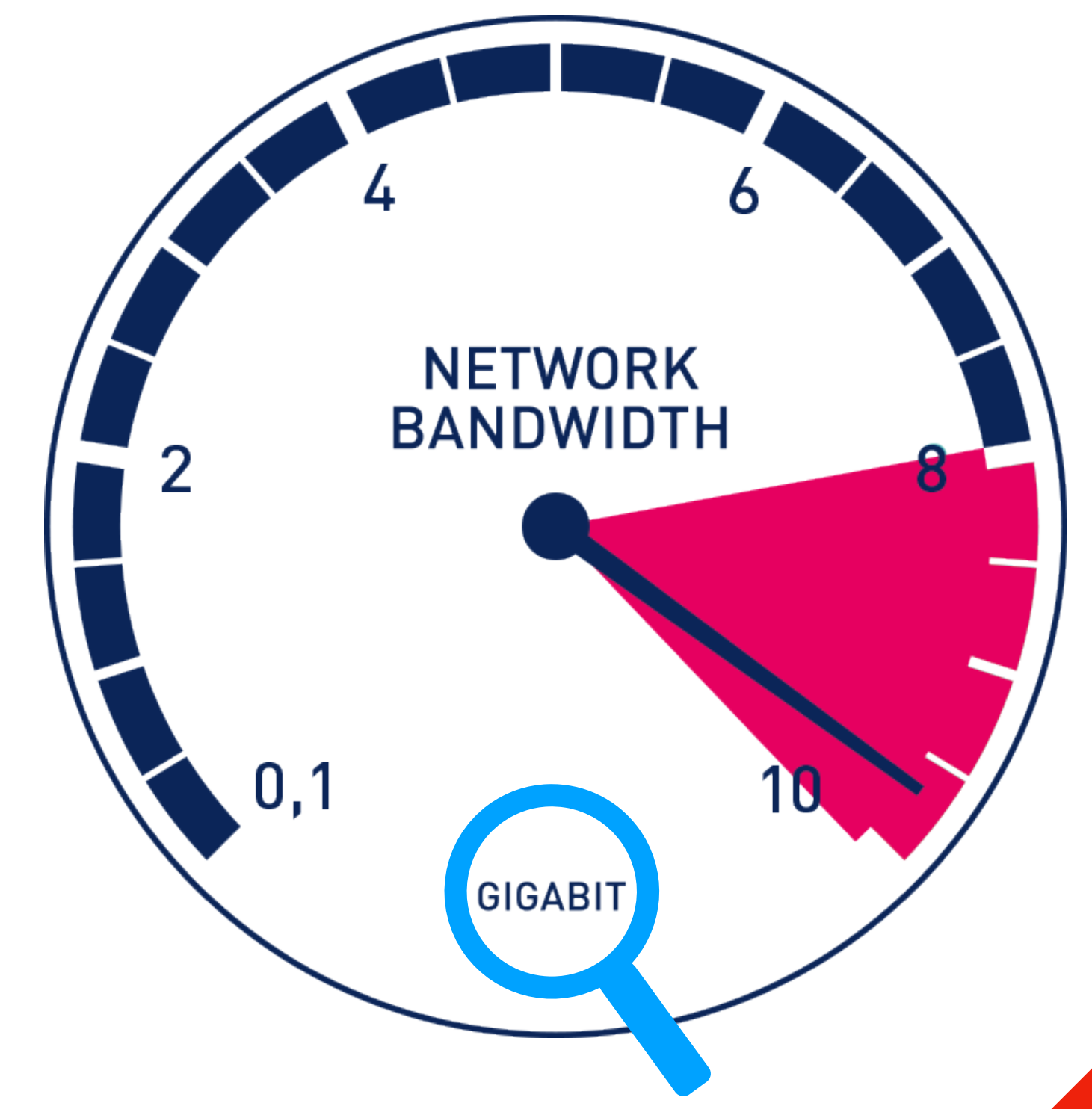
# Binary Systems in Computers

- Every 8bit is called a Byte
- $1,024 = 2^{10}$  is called K (Kilo)
- $1,024 \times 1,024 = 2^{20}$  is called M (Mega)
- $1,024 \times 1,024 \times 1,024 = 2^{40}$  is called G (Giga)
- Tera, Peta, Exa, Zetta, Yotta



# Binary Systems in Computers

- What is the difference between MBps and Mbps?
- MegaBytes per second vs MegaBits per second
- 8x difference!



Concept

# Octal and Hexadecimal Systems

- Octal: base 8
  - digits: 0-7
- Hexadecimal: base 16
  - digits: 0-9, A-F (10-15)

# Octal and Hexadecimal Systems

Decimal (Base 10)	Binary (Base 2)	Octal (Base 8)	Hexadecimal (Base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Concept

# Summary

- Digital binary systems
- Octal and Hexadecimal systems

# Arithmetic Operations

Additions, Subtractions, Conversions

# Arithmetics

- The same as decimal (mostly)
- 

$$\begin{array}{r} 0010 \\ +0011 \\ \hline 0101 \end{array} \qquad \begin{array}{r} 0101 \\ -0011 \\ \hline 0010 \end{array}$$

Example (binary)

# Arithmetics

Binary additions and subtractions

Carries

Augend

01101

Addend

+00101

---

Sum

# Arithmetics

## Binary additions and subtractions

<b>Carries</b>	11010
<b>Augend</b>	01101
<b>Addend</b>	+00101
	<hr/>
<b>Sum</b>	10010

<b>Borrows</b>	
<b>Minuend</b>	10110
<b>Subtrahend</b>	-10011
	<hr/>
<b>Difference</b>	



# Arithmetics

## OCTAL Multiplication

$$\begin{array}{r} 762 \\ \times 54 \\ \hline 4672 \\ 3710 \\ \hline 43772 \end{array}$$

$$5 \times 2 = 12$$

$$5 \times 6 + 1 = 37$$

$$5 \times 7 + 3 = 46$$

...

# Arithmetics

## OCTAL Multiplication

Octal

$$\begin{array}{r} \text{Octal} \\ 762 \\ \times 54 \\ \hline 4672 \\ 3710 \\ \hline 43772 \end{array}$$

$$5 \times 2 = 12$$

$$5 \times 6 + 1 = 37$$

$$5 \times 7 + 3 = 46$$

...

Decimal

$$10 = (12)_8$$

$$31 = (37)_8$$

$$38 = (46)_8$$

...

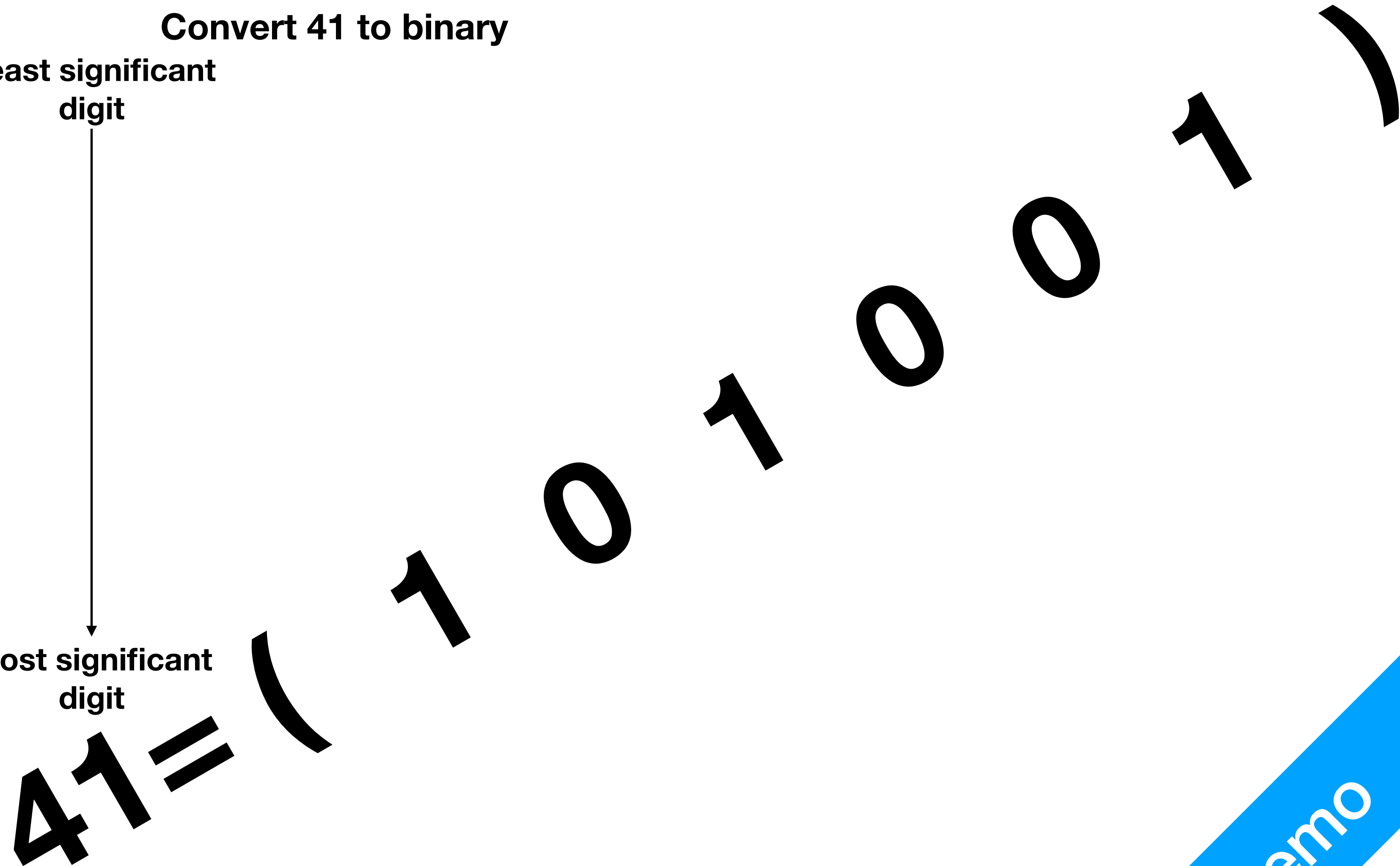
# Arithmetics

Convert 41 to binary

$$\begin{aligned} 41 \div 2 &= 20 \quad \text{mod } 1 \\ 20 \div 2 &= 10 \quad \text{mod } 0 \\ 10 \div 2 &= 5 \quad \text{mod } 0 \\ 5 \div 2 &= 2 \quad \text{mod } 1 \\ 2 \div 2 &= 1 \quad \text{mod } 0 \\ 1 \div 2 &= 0 \quad \text{mod } 1 \end{aligned}$$

Least significant  
digit

Most significant  
digit



# Arithmetics

Convert 0.6875 to binary

$$0.6875 \times 2 = 1 + 0.3750$$

$$0.3750 \times 2 = 0 + 0.7500$$

$$0.7500 \times 2 = 1 + 0.5000$$

$$0.5000 \times 2 = 1 + 0.0000$$

Most significant  
digit



Least significant  
digit

$$0.6875 = (1011)$$

# Signed Integers

- Integers in digital circuits have limited bits
- e.g. 8bit for every number
- How do we represent negative values in digital circuits?

$$\begin{array}{r} 00000101 \\ -00001011 \\ \hline 0000???? \end{array}$$

(binary, 8bit, unsigned)

# Signed & Unsigned Integers

- Unsigned 8bit:
  - $(11111111)_2 = 255$
- Signed 8bit (only in digital circuits):
  - $127 \rightarrow '01111111'$
  - $-127 \rightarrow '11111111'$

First digit:

- 0 for positive
- 1 for negative

**1**0001111

(binary, 8bit, signed)

# Signed & Unsigned Integers

- Unsigned 8bit integer: 0 - 255
  - Signed 8bit integer: -128 - 127
- Unsigned 32bit integer: 0 - 4,294,967,295
  - Signed 32bit integer: -2,147,483,648 - 2,147,483,647
- Unless otherwise specified, treat as unsigned

# Summary

- Additions, Subtractions, Multiplications in Binary, Octal, Hexadecimal systems

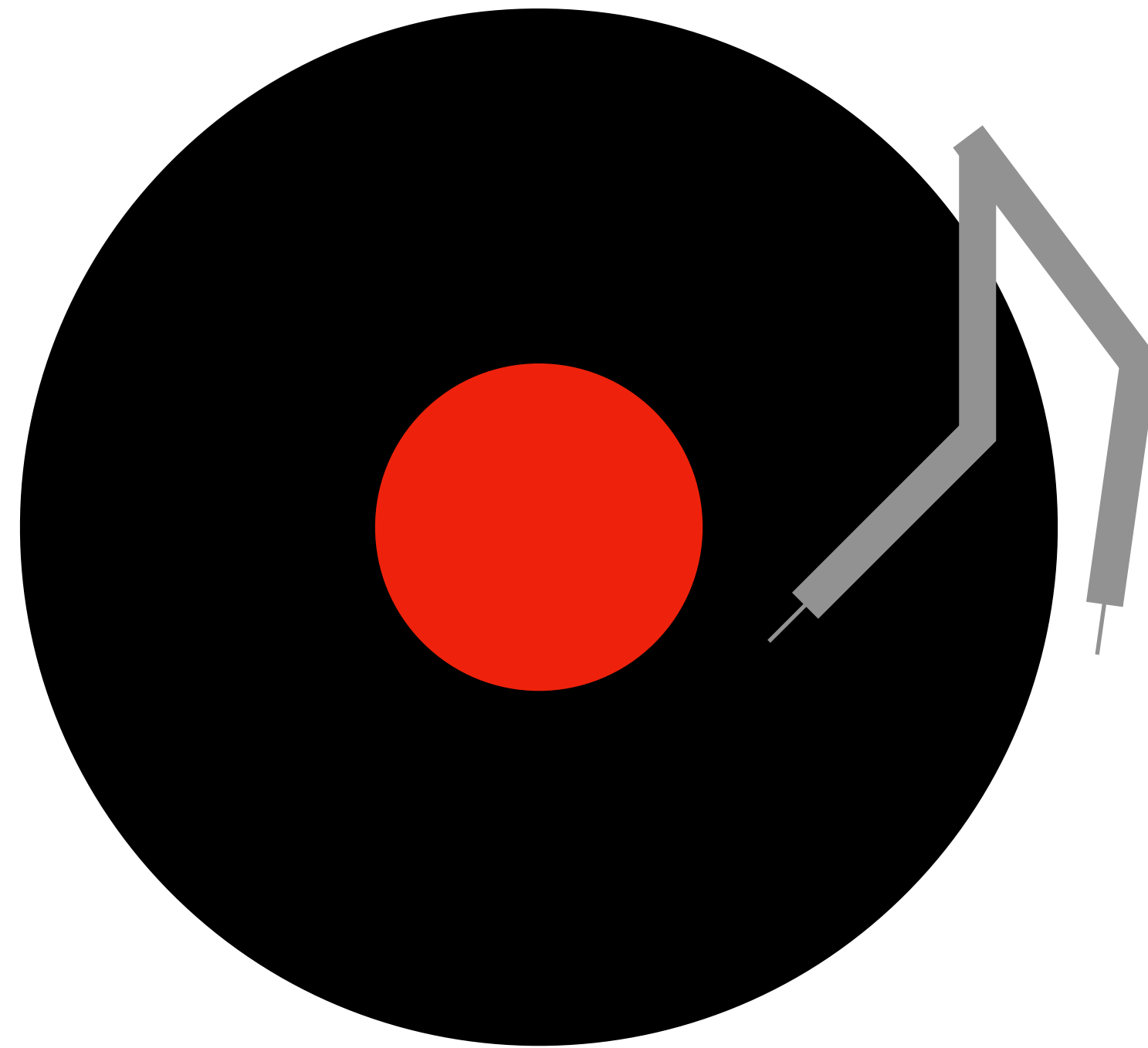


# Digital & Analog Conversions

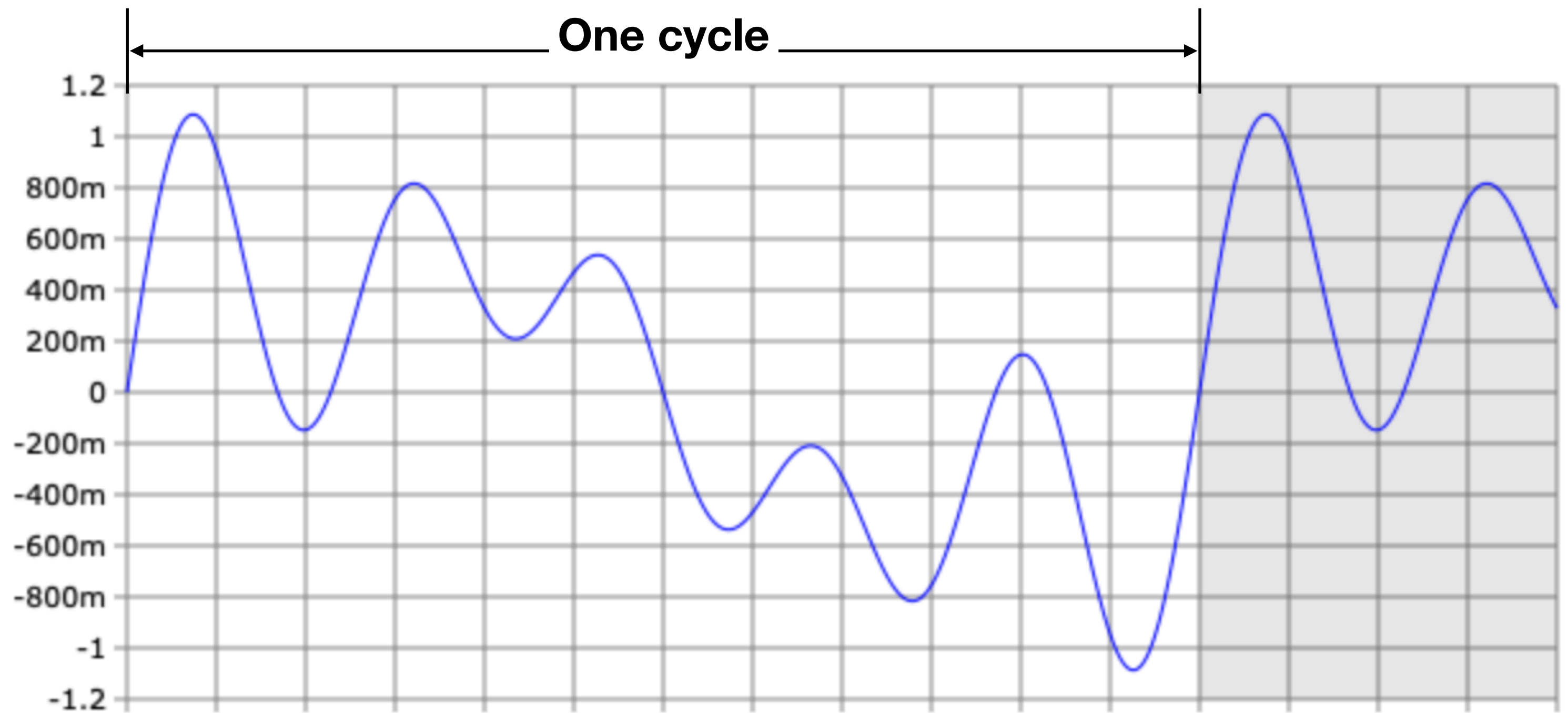
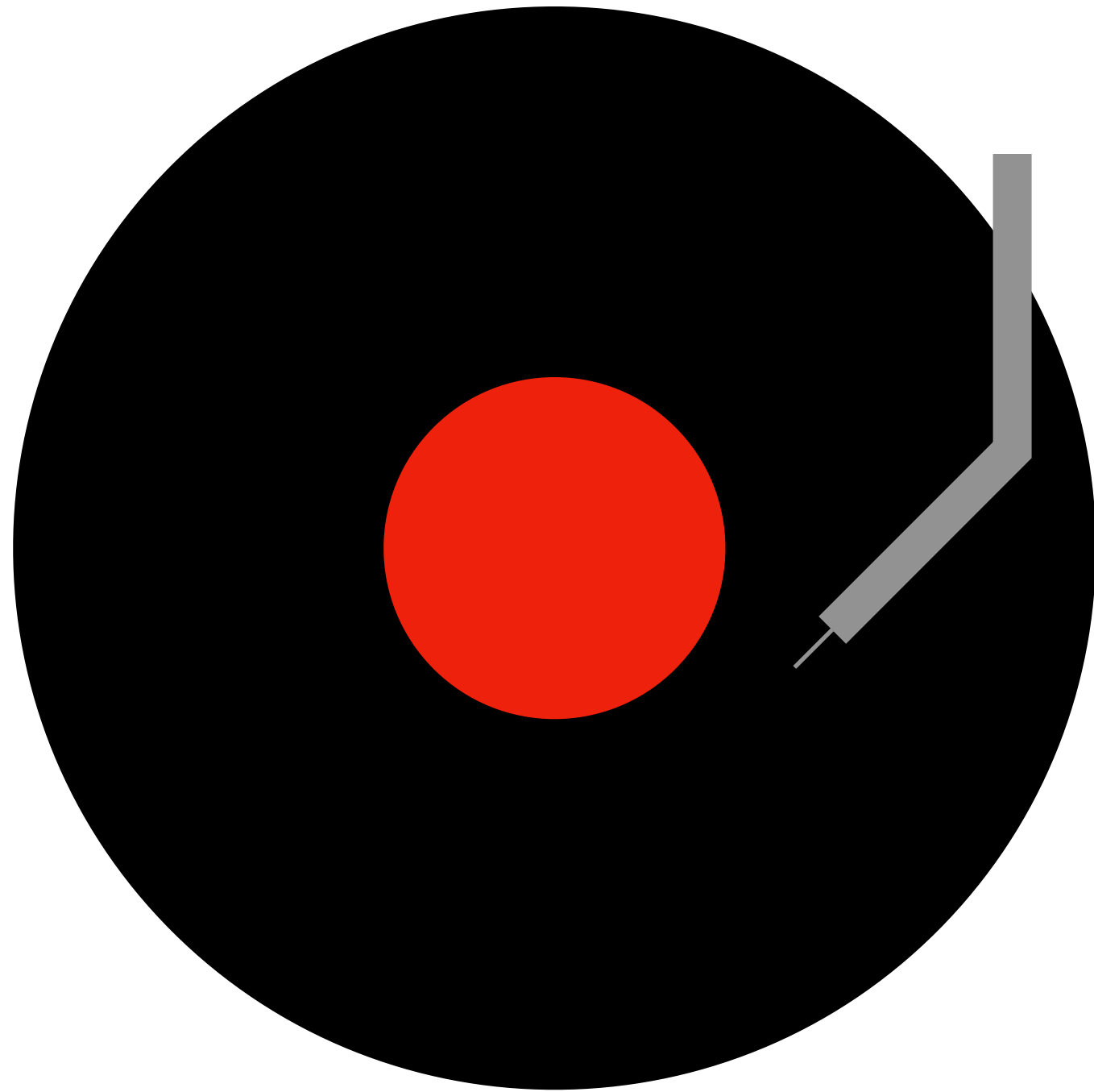
Demonstration

WARNING: inbound signed Binary/Hexadecimal operations

# Case Study: Audio Signal Representation

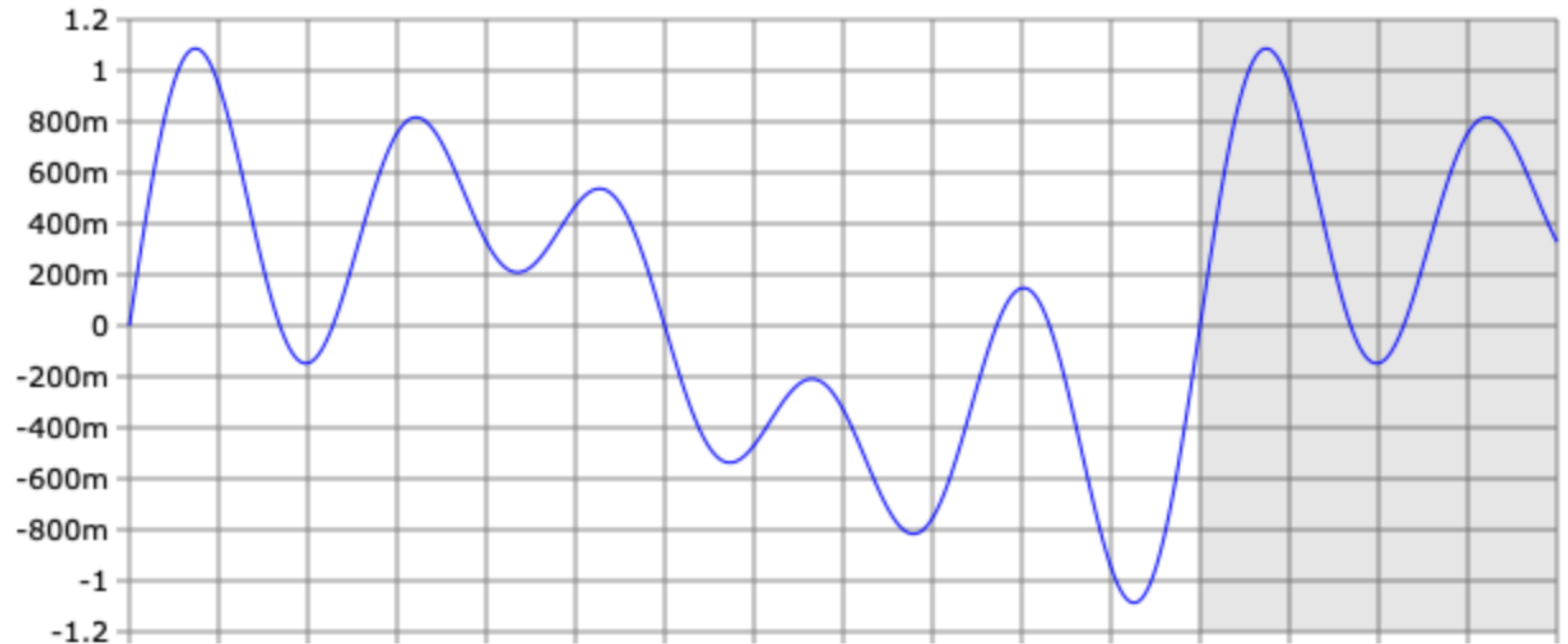
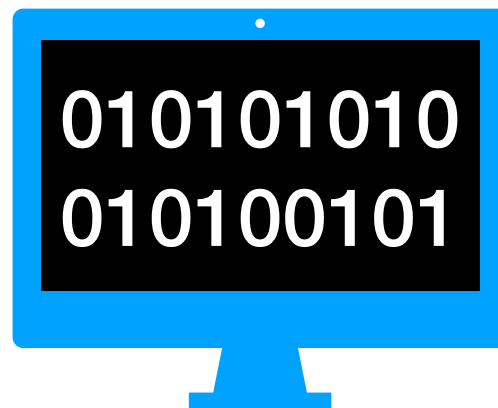


# Case Study: Audio Signal Representation



440 Hz = 440 cycles per sec

# Case Study: Audio Signal Representation

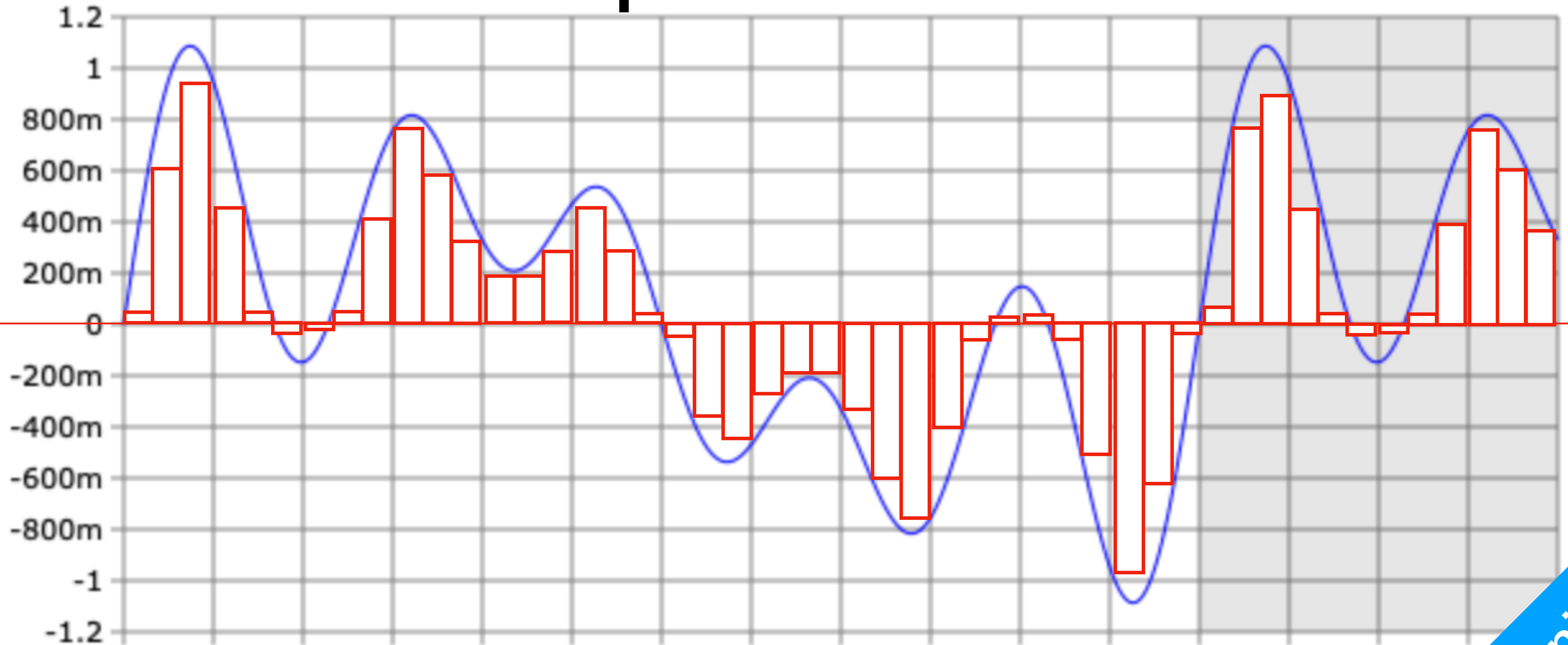


440 Hz = 440 cycles per sec

# Case Study: Audio Signal Representation

P3

DA Conversion



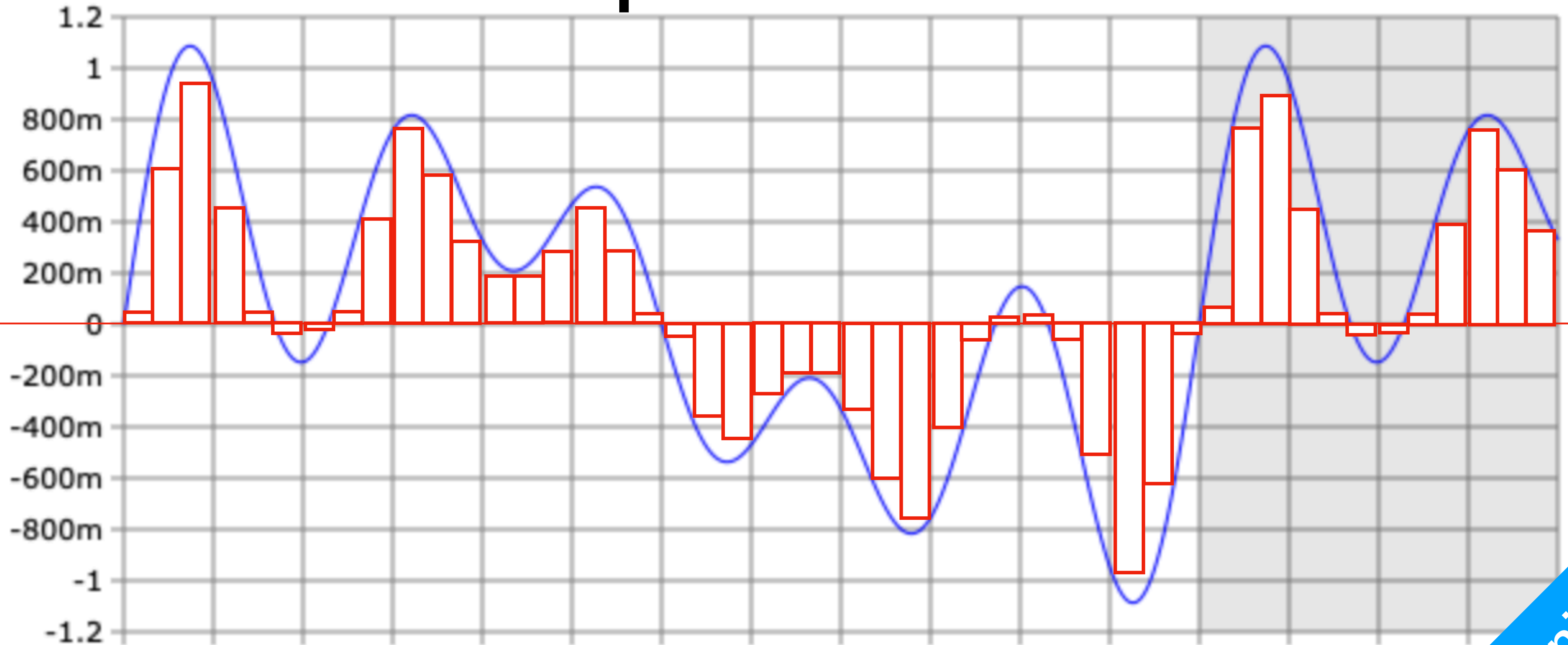
440 Hz = 440 cycles per sec, Sample rate  $36 \times 440 = 15,840 = 16\text{kps}$

Technical

# Case Study: Audio Signal Representation

P3

DA Conversion



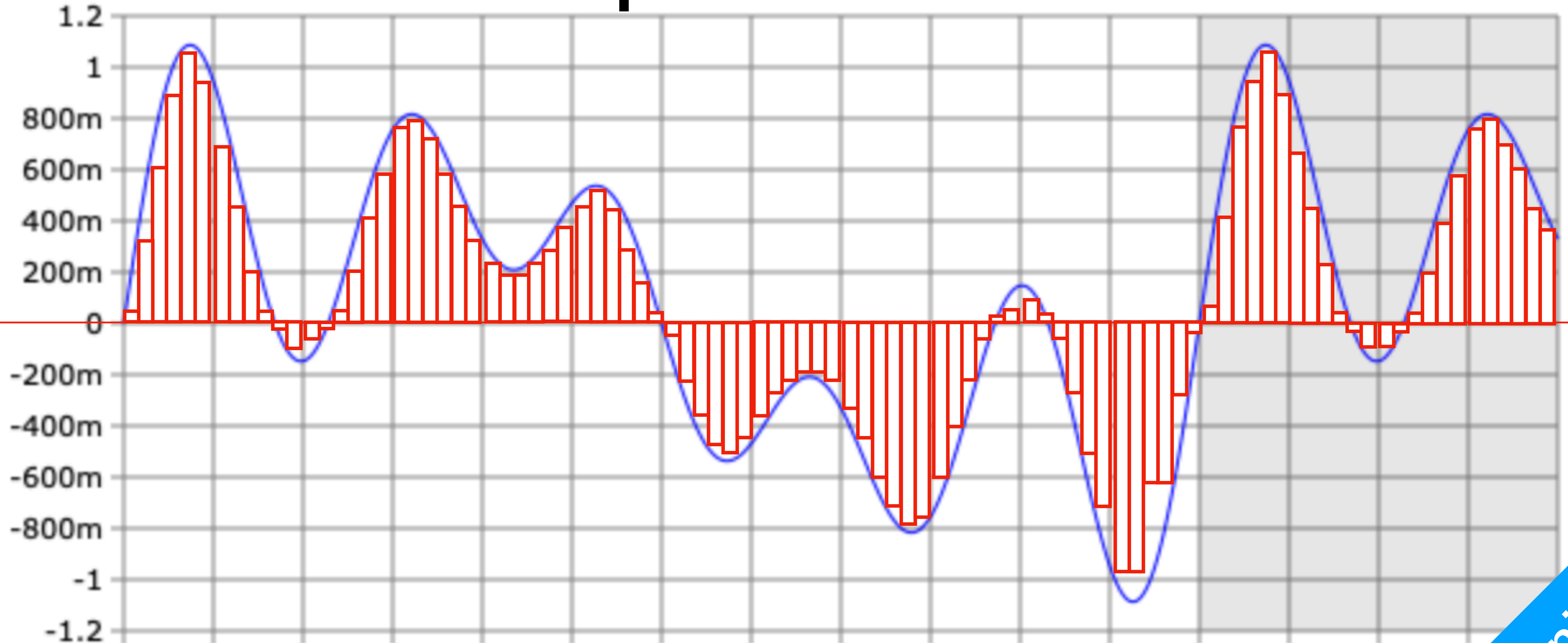
440 Hz = 440 cycles per sec, Sample rate  $36 \times 440 = 15,840 = 16\text{kps}$

Technical

# Case Study: Audio Signal Representation

P3

DA Conversion



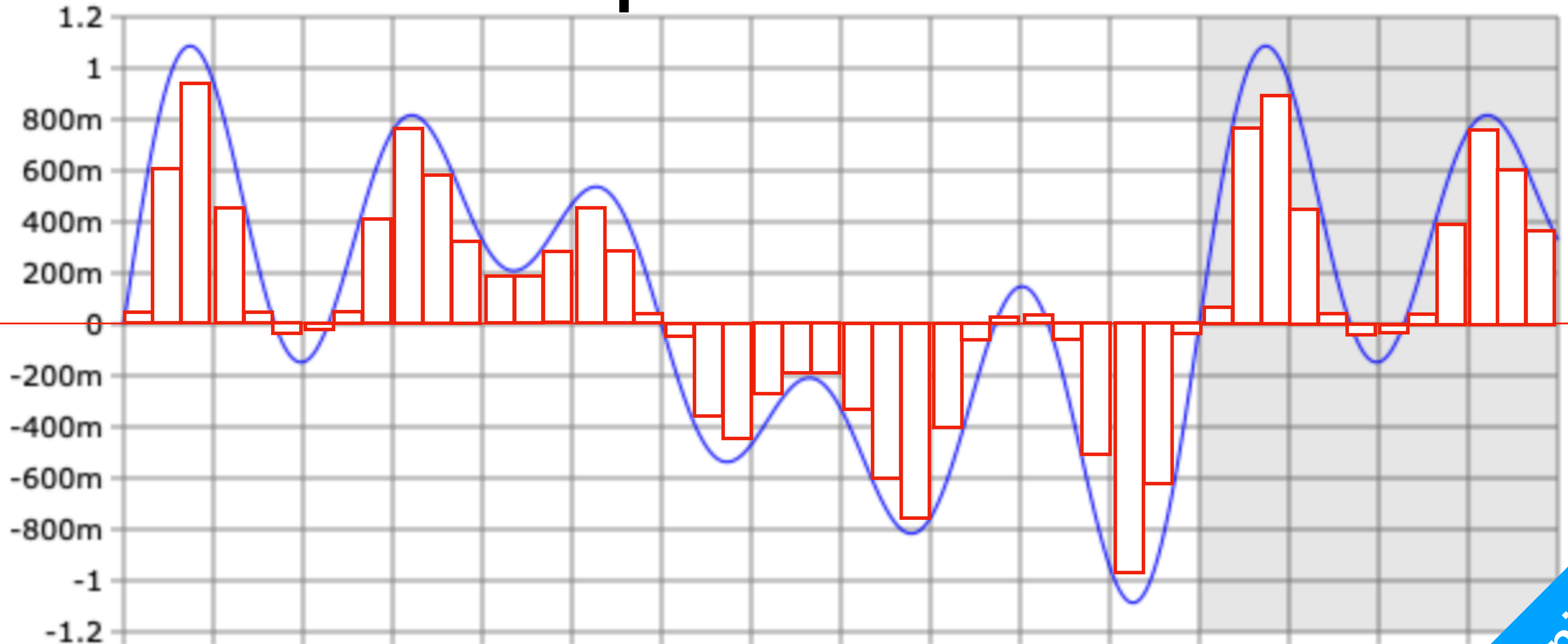
440 Hz = 440 cycles per sec, Sample rate  $64 \times 440 = 31,680 = 32\text{kps}$

Technical

# Case Study: Audio Signal Representation

P3

DA Conversion



440 Hz = 440 cycles per sec, Sample rate  $36 \times 440 = 15,840 = 16\text{kps}$

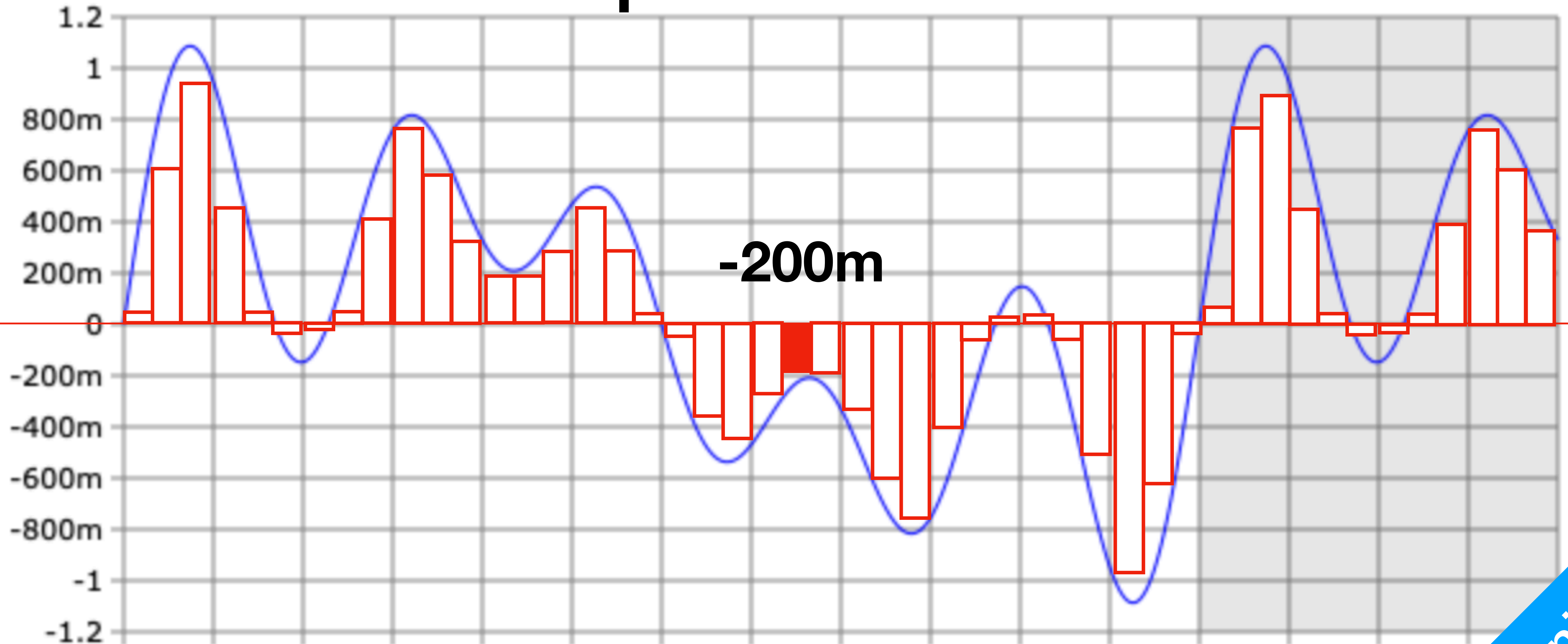
Technical



# Case Study: Audio Signal Representation

P3

DA Conversion



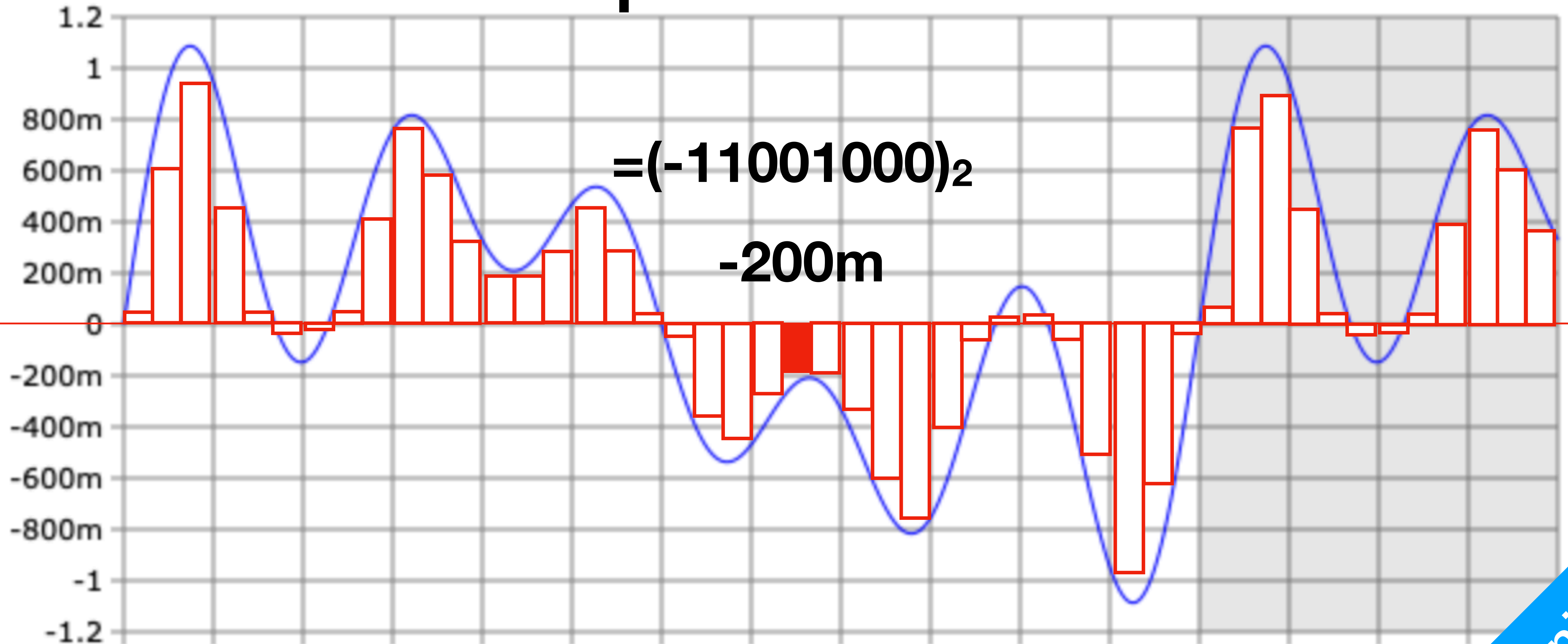
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Technical

# Case Study: Audio Signal Representation

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DA Conversion



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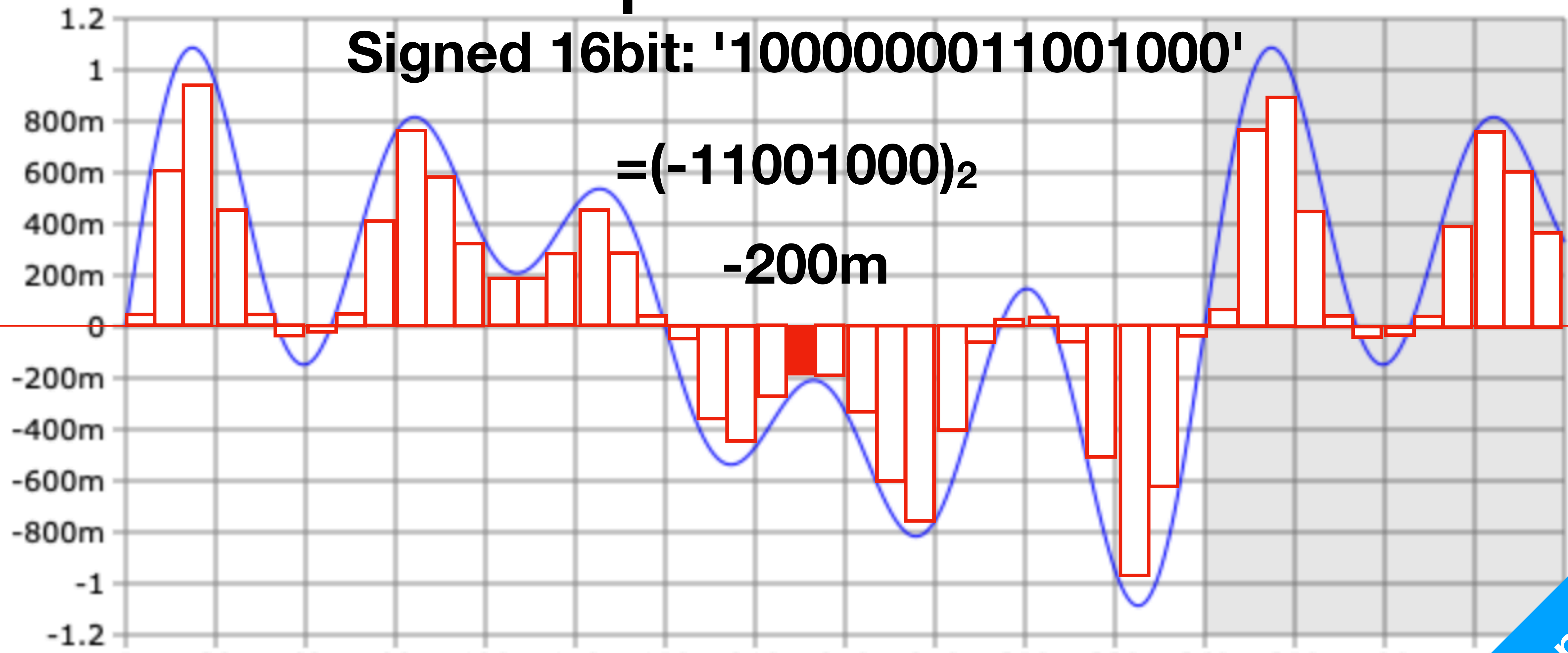
Technical

# Case Study: Audio Signal Representation

Signed 16bit: '1000000011001000'

$$= (-11001000)_2$$

-200m



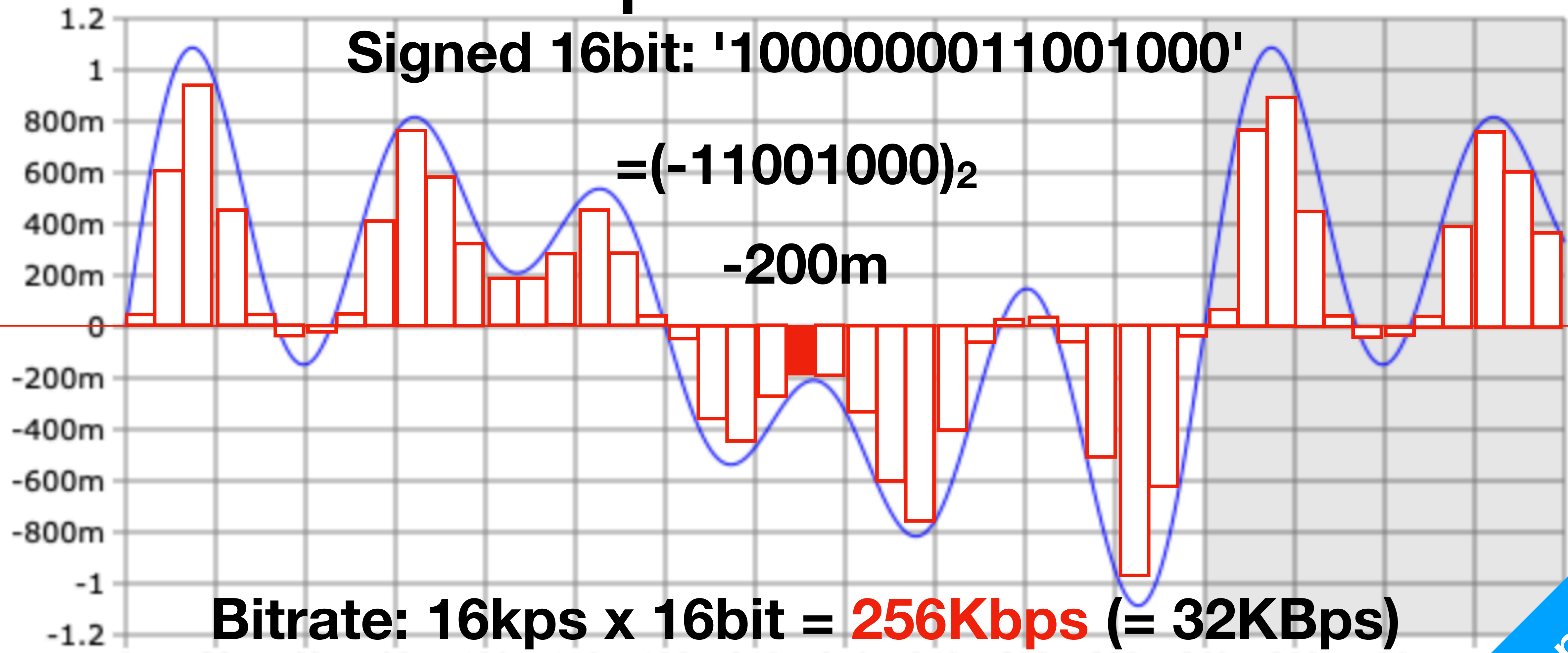
440 Hz = 440 cycles per sec, Sample rate 36 x 440 = 15,840 = 16kps

# Case Study: Audio Signal Representation

Signed 16bit: '1000000011001000'

$$=(-11001000)_2$$

-200m



Bitrate: 16kps x 16bit = **256Kbps** (= 32KBps)

440 Hz = 440 cycles per sec, Sample rate 36 x 440 = 15,840 = 16kps

Technical

# Case Study: Audio Signal Representation

- Standard Mp3 audio file
- Sampling rate: 44,100
- Bitrate: 256 kBit/s (Kbps)
- 4 min audio file size?  $> 4 \times 60 \times 256 \div 8 = 7680\text{KB}$

# Summary

- Digital to Analog Conversion
  - Frequency: number of cycles per second
  - Sample rate: number of samples per unit time
  - Bitrate: number of bits per second