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CSCI 150
Introduction to Digital and Computer System Design
Lecture 2: Combinational Logical Circuits IV

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2020 Summer Semester (S2)

Overview

- Focus: Boolean Algebra
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch2 2.4, 2.5; v5: Ch2 2.4, 2.5
- Core Ideas:

    1. Boolean Algebra III: K-Map

P0
Review

Boolean Algebra I&II

- AND, OR, NOT Operators and Gates
  - Simple digital circuit implementation
  - Algebraic manipulation using Binary Identities
- Standard Forms
  - Minterm & Maxterm
  - Sum of Products & Product of Sums

Review

P1
Optimisation

Boolean Algebra III: K-Map

Cost Criteria;
Map and Map Manipulation

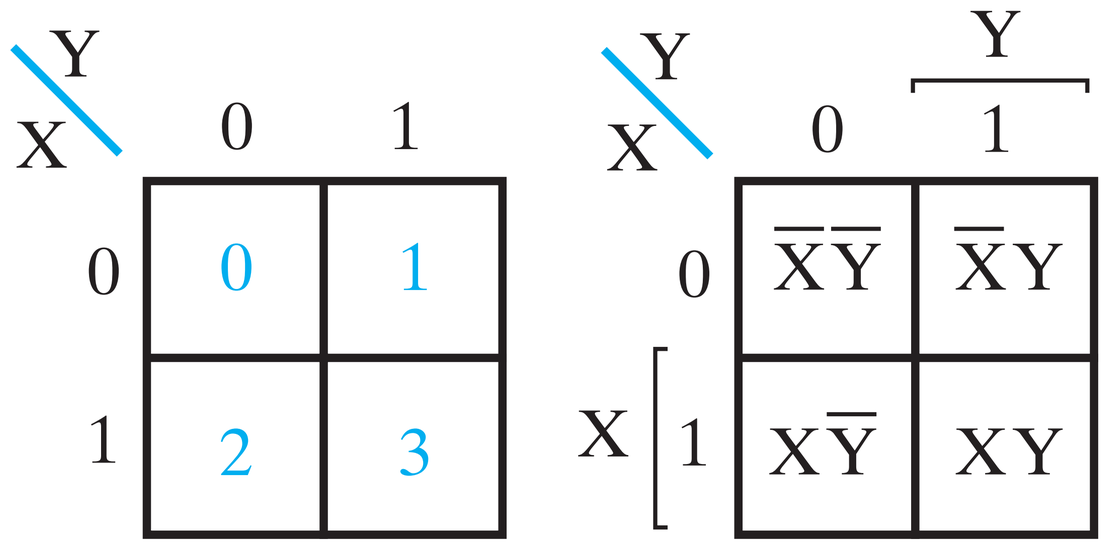
Summary

K-Map

- Karnaugh Map, or just K-Map
  - For optimising 2-4 variable boolean expressions
    - Skip: 5,6 variable K-Maps can also be drawn but are not very intuitive to use

Concept

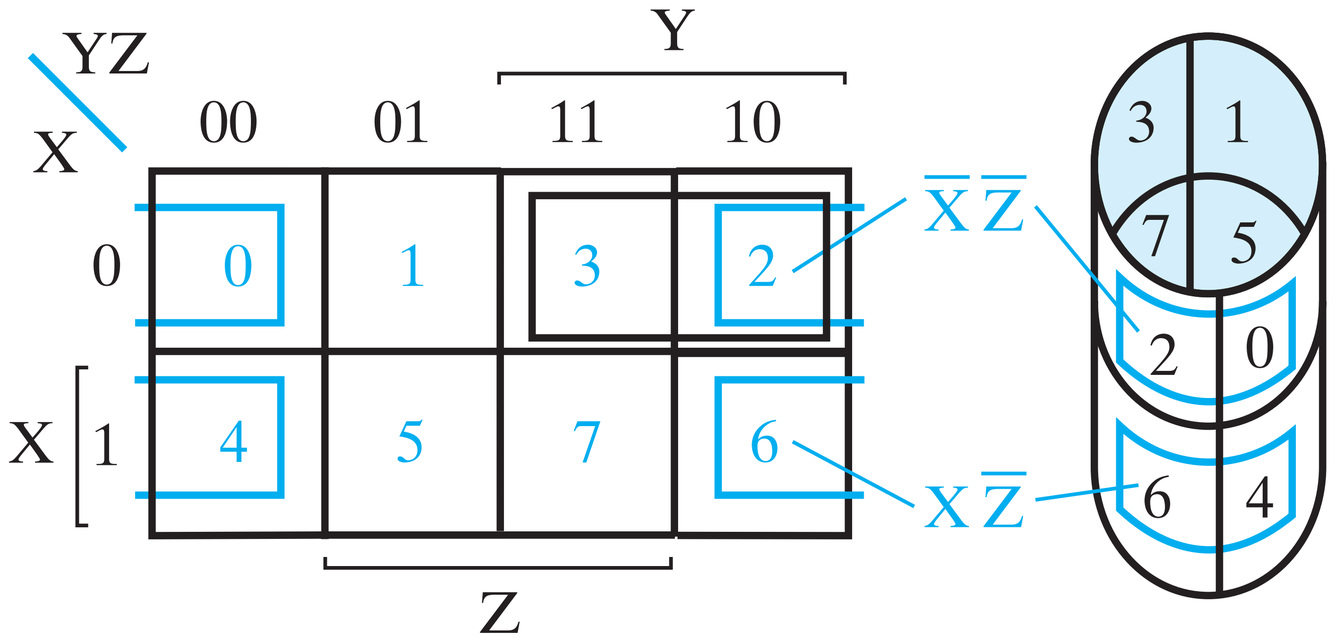
Two Variable Maps



- Number of squares in each map is equal to the number of minterms for the same number of variables, light blue digit above is the index (of minterm)
- Two squares are adjacent if they only differ in one variable
- Binary value inside at each position indicates the truth table value for that term

Concept

Three Variable Maps



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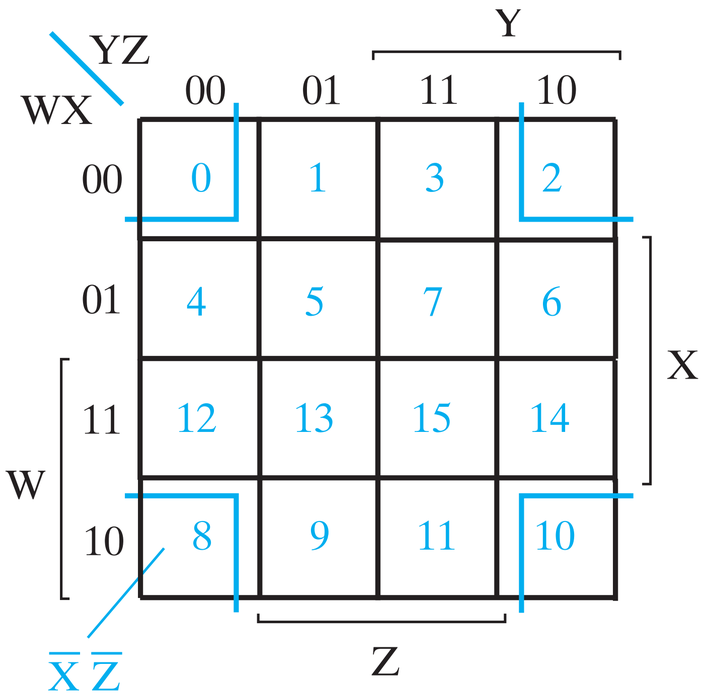
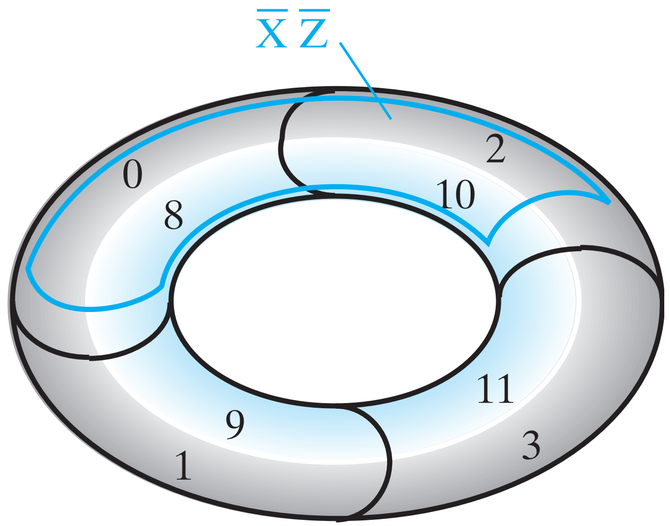
Concept

Four Variable Maps





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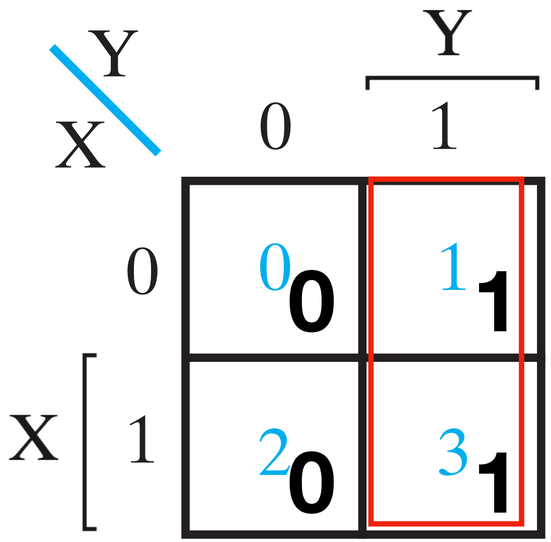
Concept

P1
Optimisation

Two Variable Maps Optimisation



Truth Table

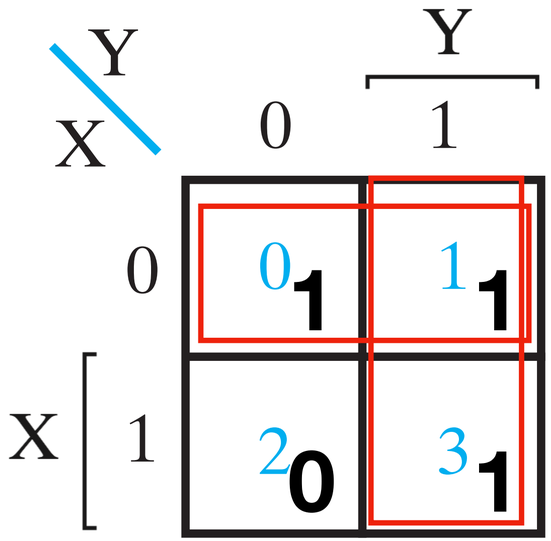
X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Concept

P1
Optimisation

Two Variable Maps Optimisation



Truth Table

X	Y	F
0	0	1
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1	0	0
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$$\overline{X} + Y$$

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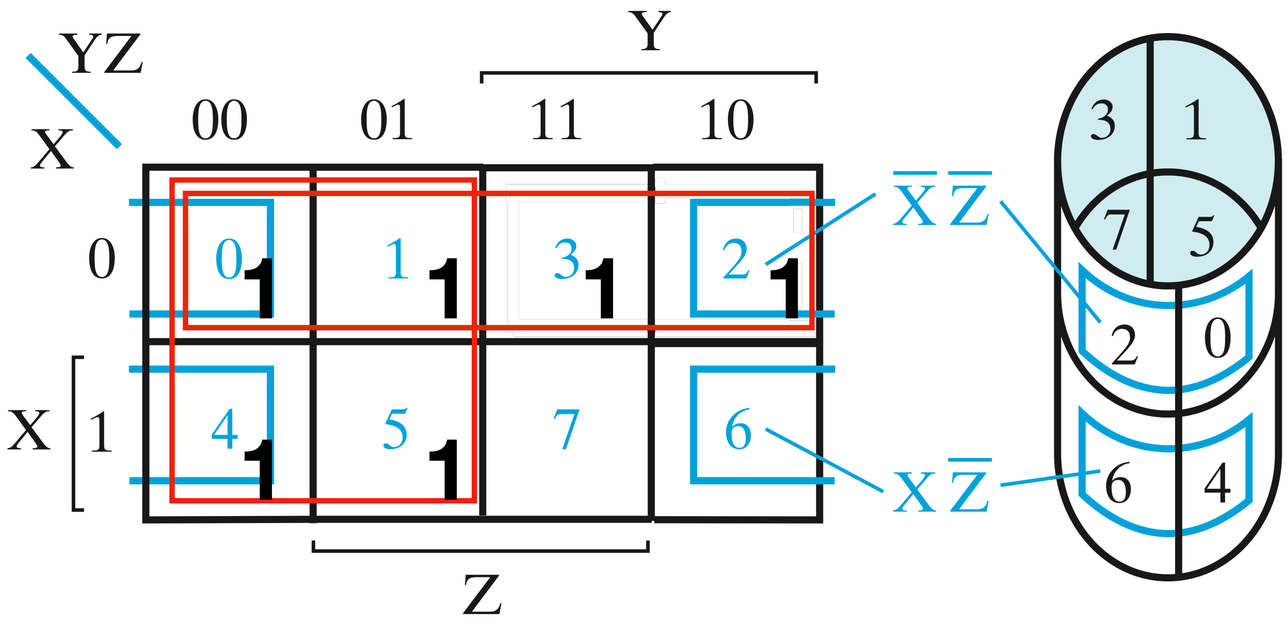
Demo

P1
Optimisation

Three Variable Maps Optimisation



$$F(X, Y, Z) = \Sigma m(0,1,2,3,4,5)$$
$$= \overline{X} + \overline{Y}$$

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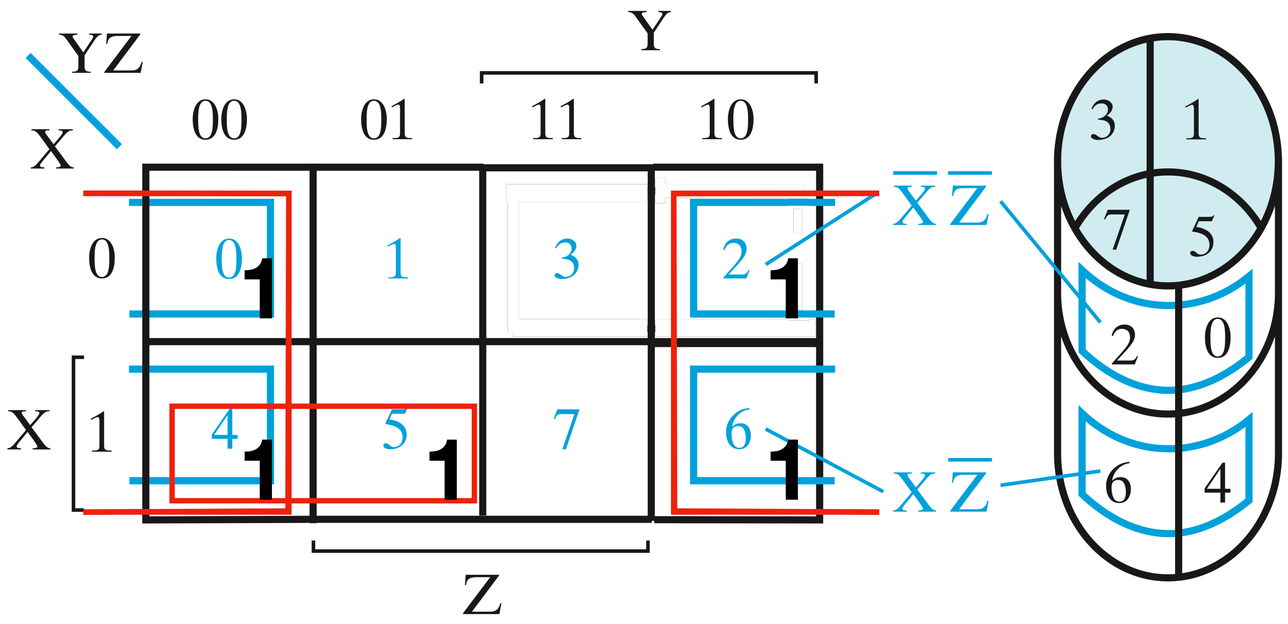
Concept

P1
Optimisation

Three Variable Maps Optimisation



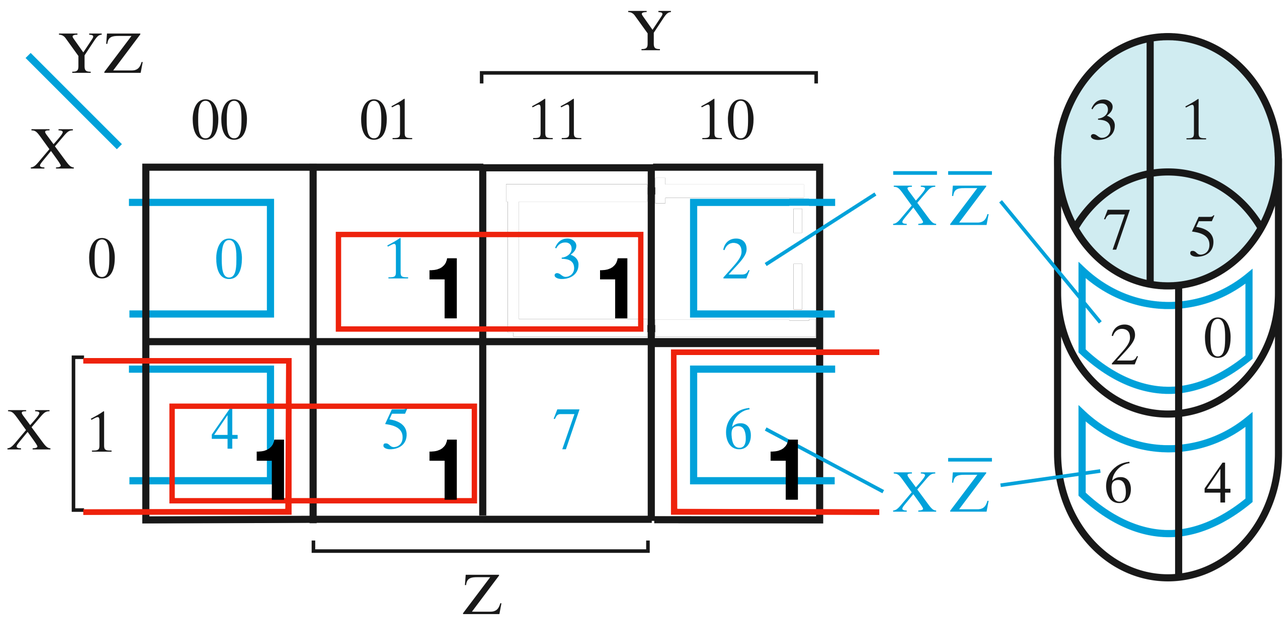
$$F(X, Y, Z) = \Sigma m(0,2,4,5,6)$$
$$= X\overline{Y} + \overline{Z}$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Demo

P1
Optimisation

Three Variable Maps Optimisation



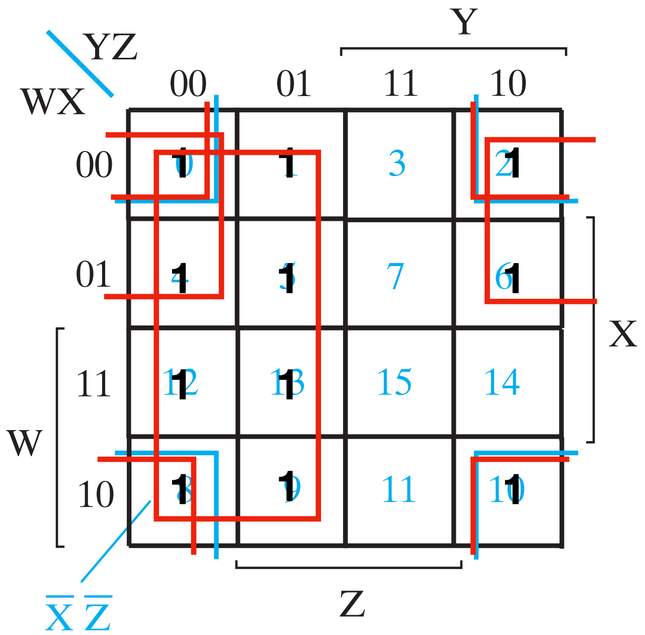
$$F(X, Y, Z) = \Sigma m(1,3,4,5,6)$$
$$= \overline{X}Z + X\overline{Y} + X\overline{Z}$$

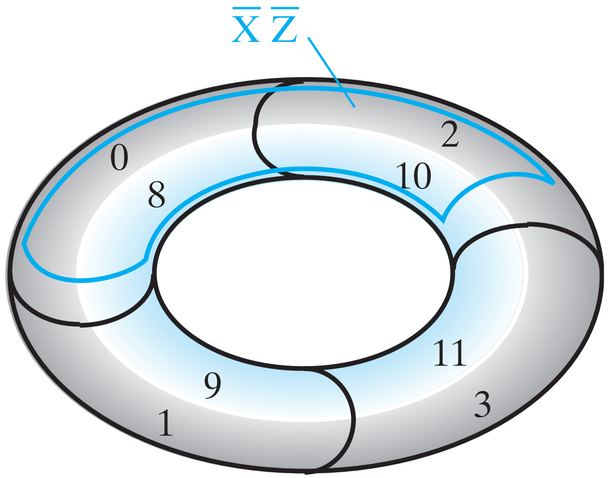
- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Exercise

P1
Optimisation

Four Variable Maps Optimisation





$$F(W,X,Y,Z) = \Sigma m(0,1,2,4,5,6,8,9,10,12,13)$$

$$= \overline{Y} + \overline{X}\overline{Z} + \overline{W}\overline{Z}$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
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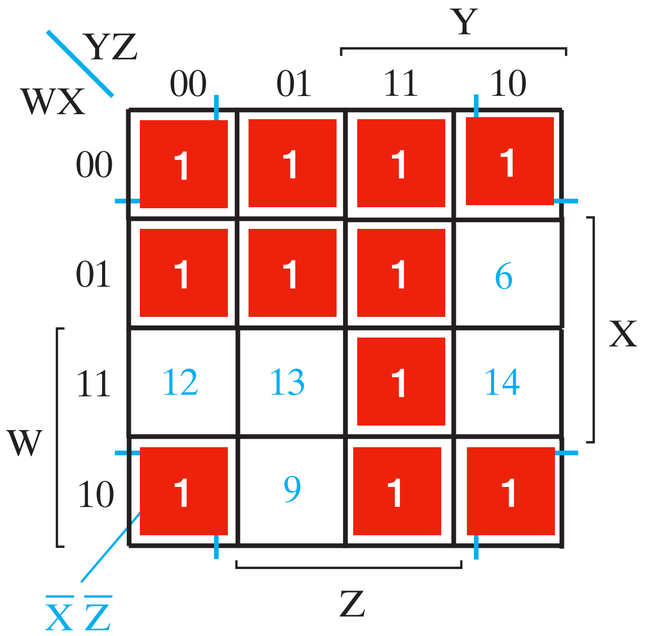
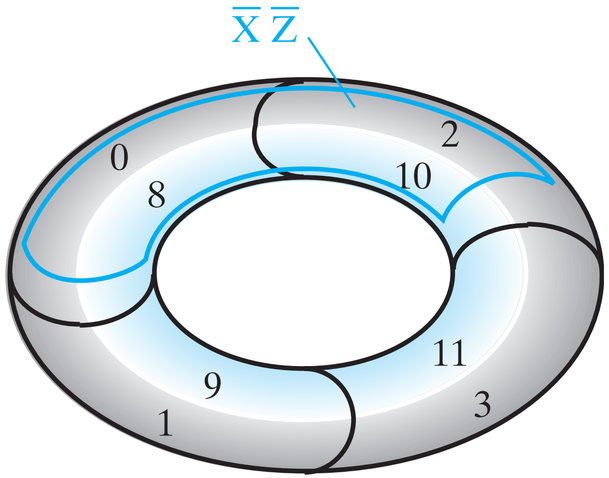
Exercise

P1
Optimisation

Four Variable Maps Optimisation





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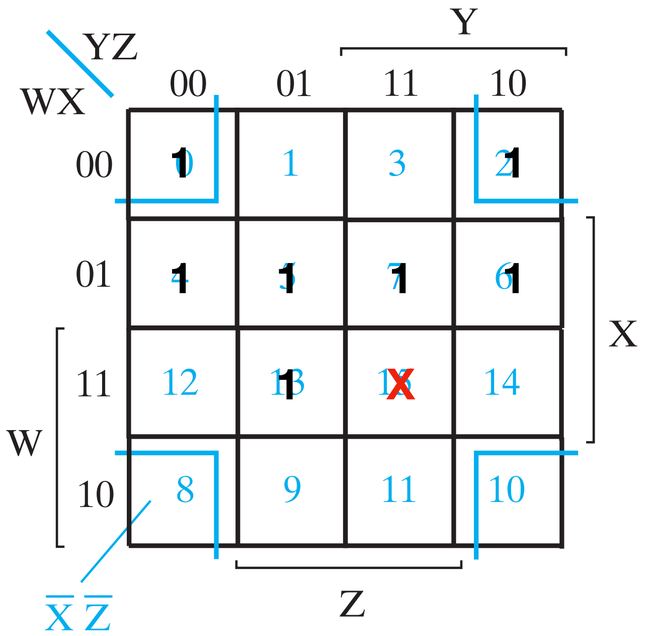
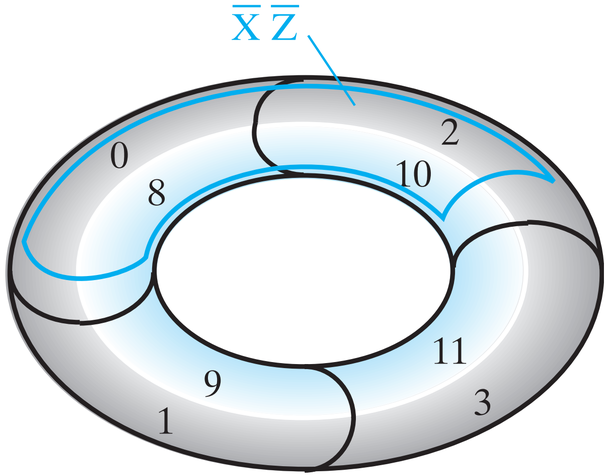
$$F(W,X,Y,Z) = \overline{W}\overline{Y}\overline{Z} + \overline{W}Z + \overline{X}Y + YZ + W\overline{X}\overline{Z}$$

Exercise

Don’t Care Condition

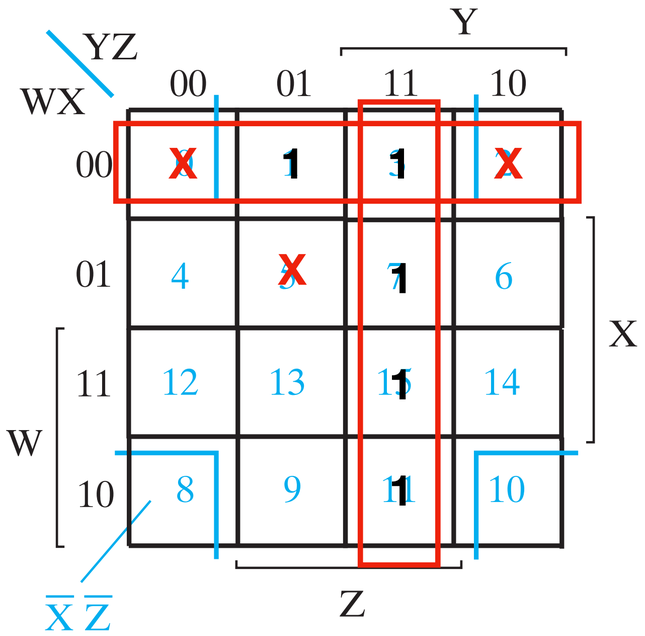


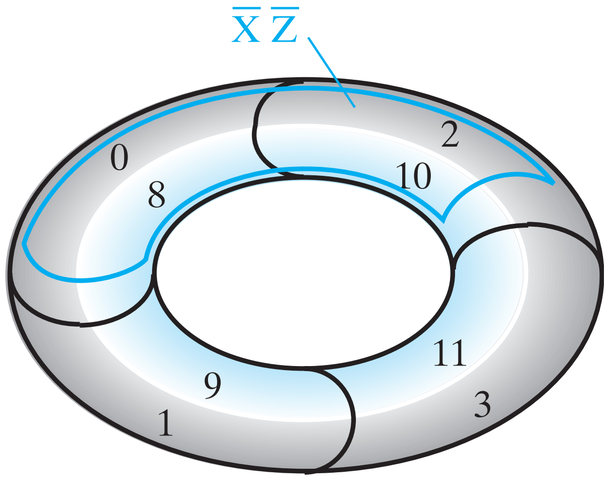


- Sometimes we don’t care what the output is when the inputs are in certain combinations

Concept

Don’t Care Condition





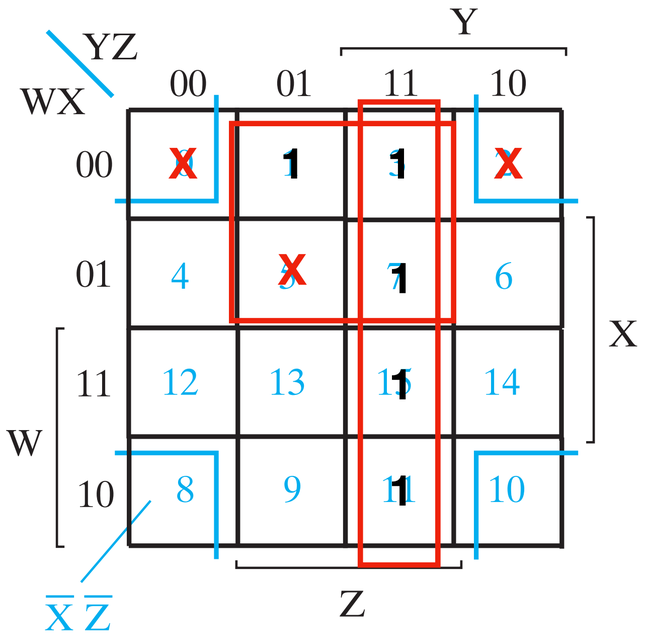
- Sometimes we don’t care what the output is when the inputs are in certain combinations

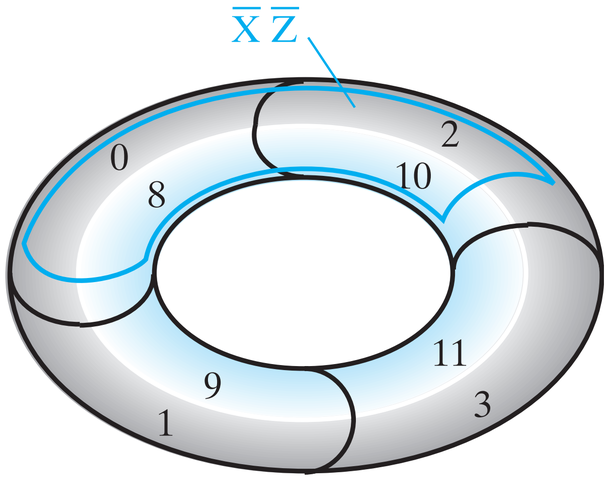
$$F = YZ + \overline{W}\overline{X}$$

Demo

P1
Optimisation

Don’t Care Condition





- Sometimes we don’t care what the output is when the inputs are in certain combinations

$$F = YZ + \overline{W}Z$$

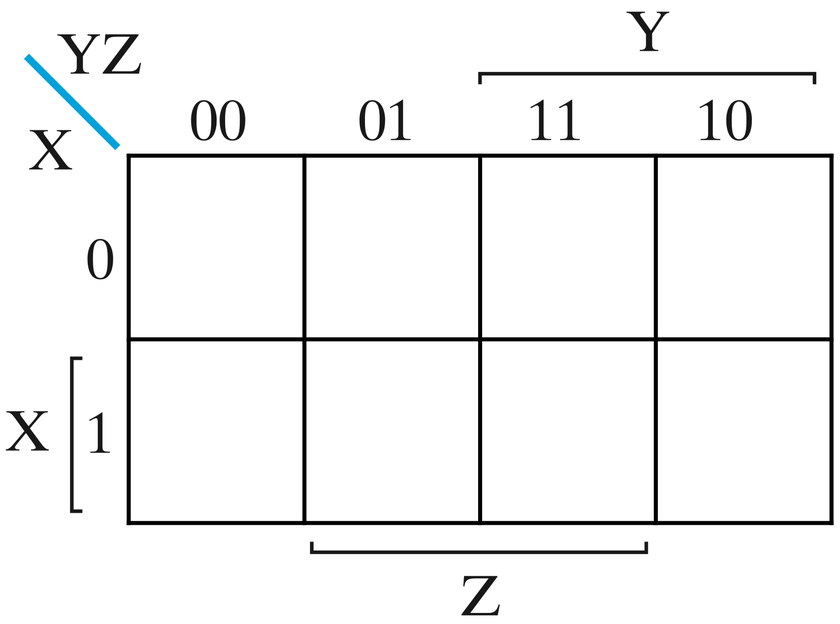
Demo

Summary

- Boolean Algebra III: K-Map
  - Two Variable K-Map
  - Three Variable K-Map
  - Four Variable K-Map
  - Don’t care optimisation

Review

Exercises



$$F(X, Y, Z) = \Sigma m(0,2,6,7)$$

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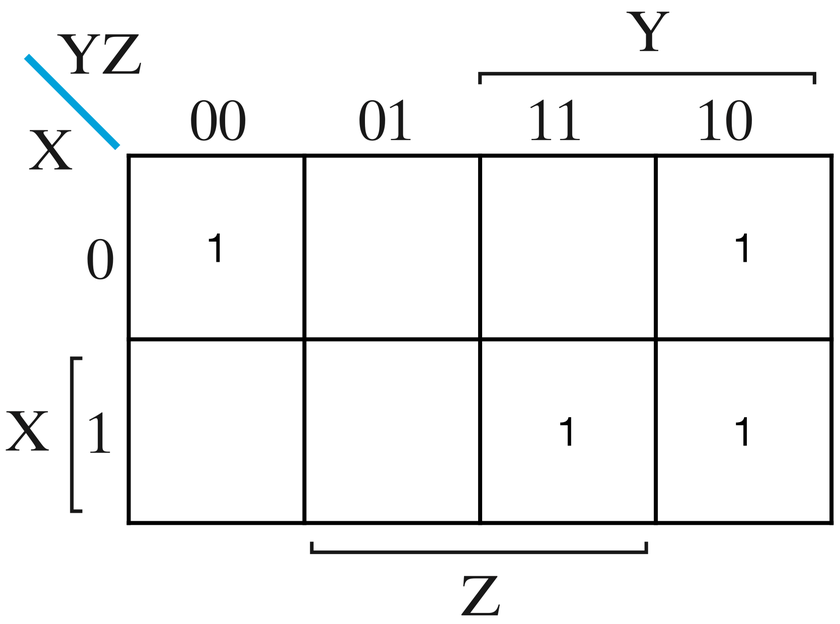
Exercise

Exercises



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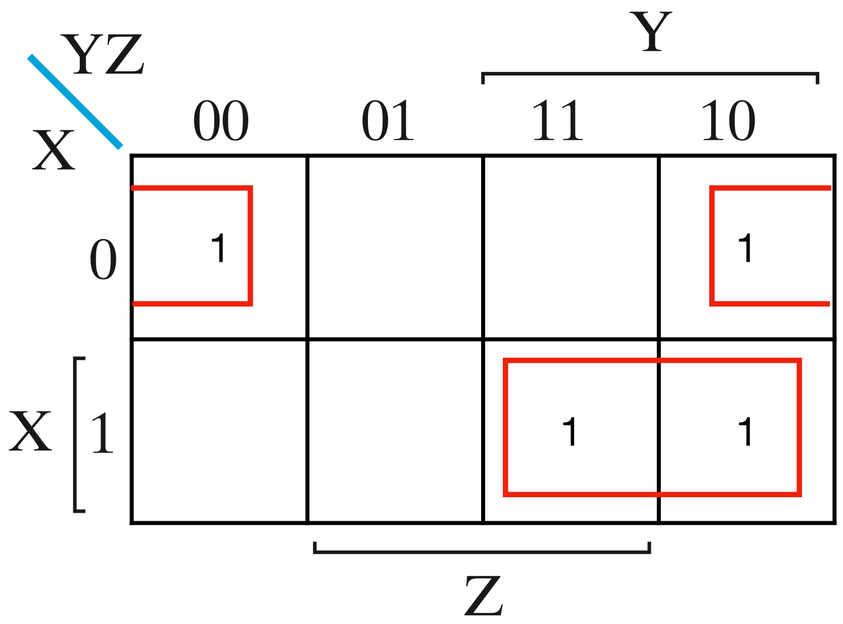

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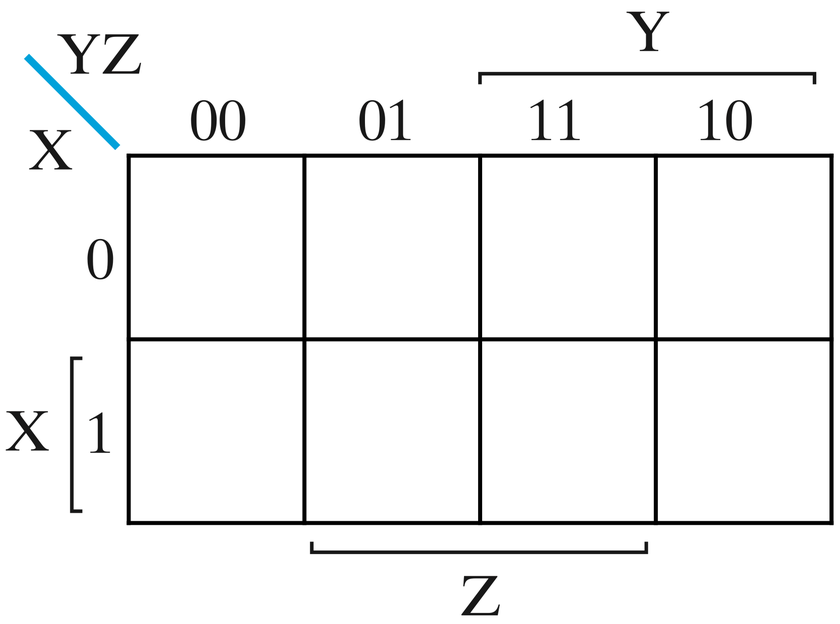
Exercises



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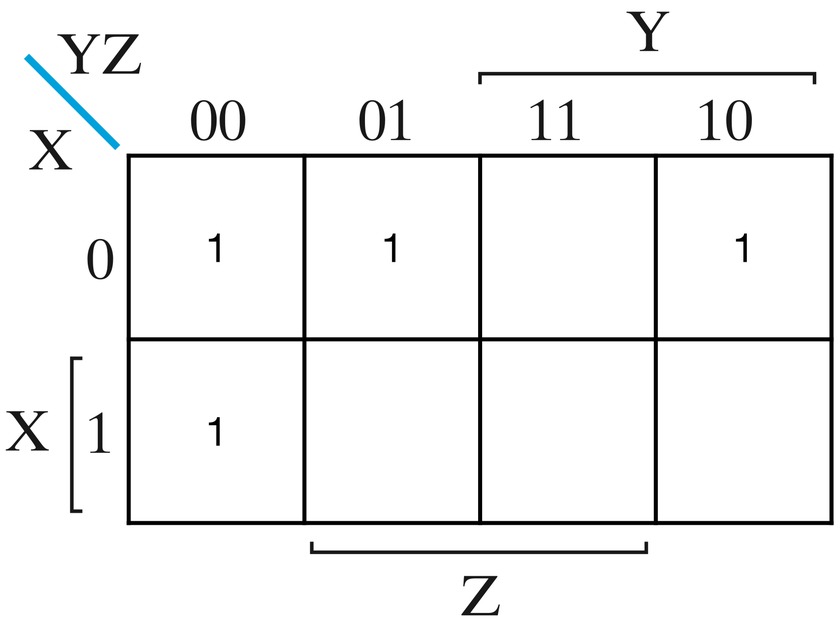
Exercise

Exercises



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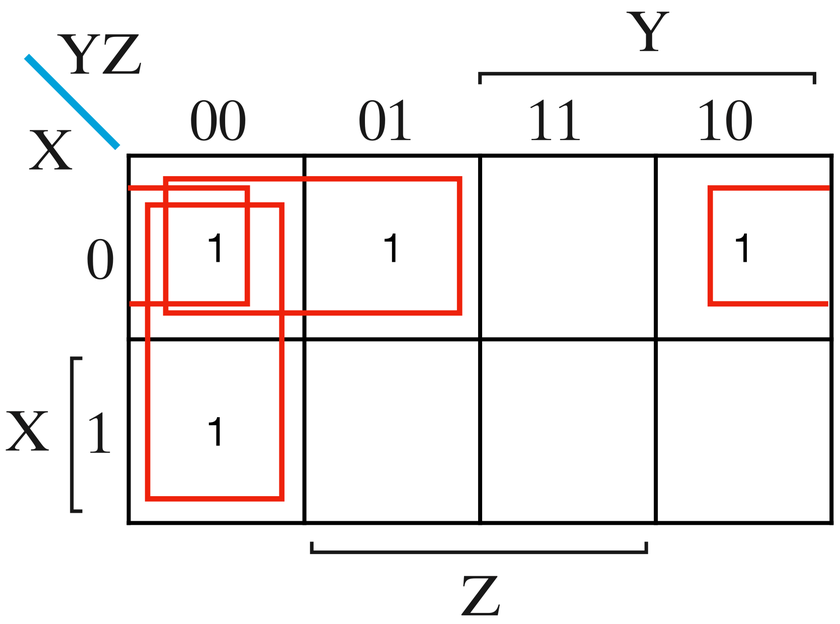
Exercises



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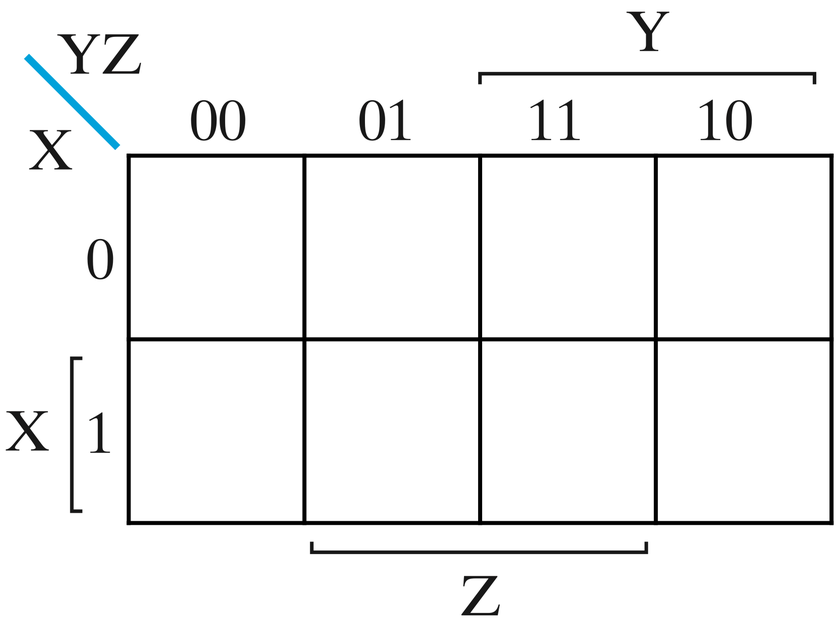

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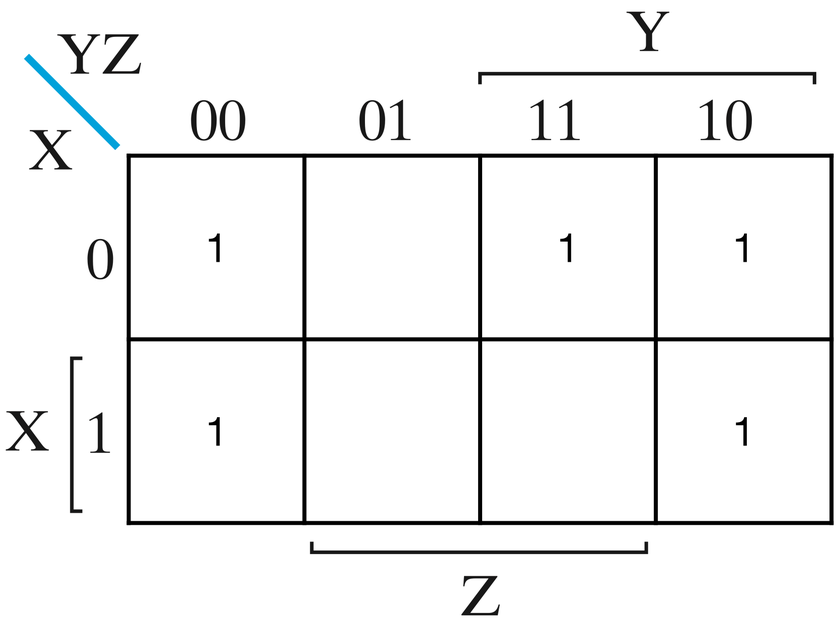
Exercises



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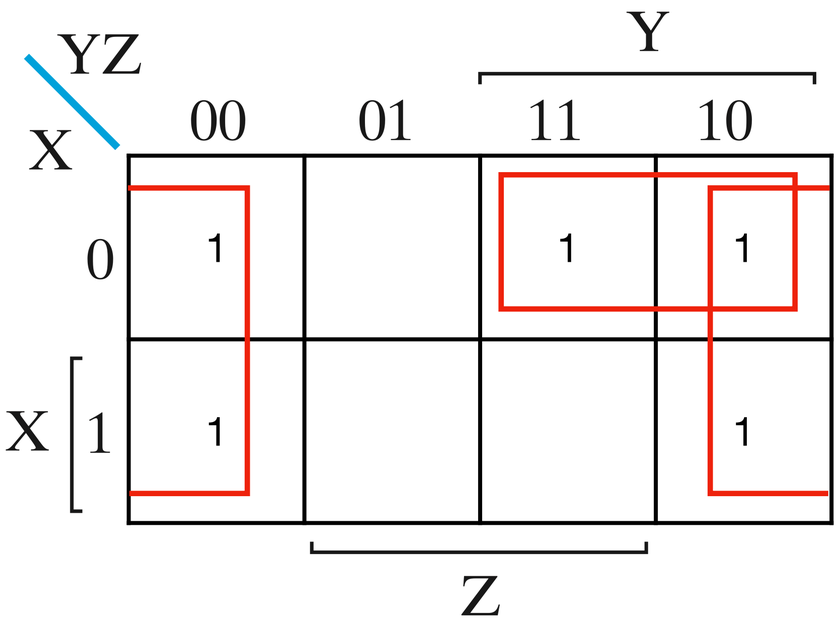

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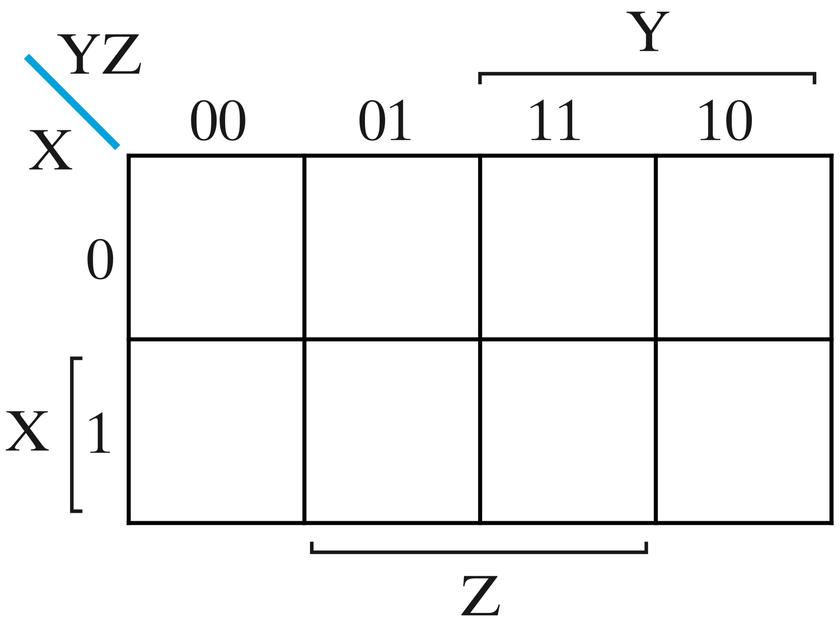

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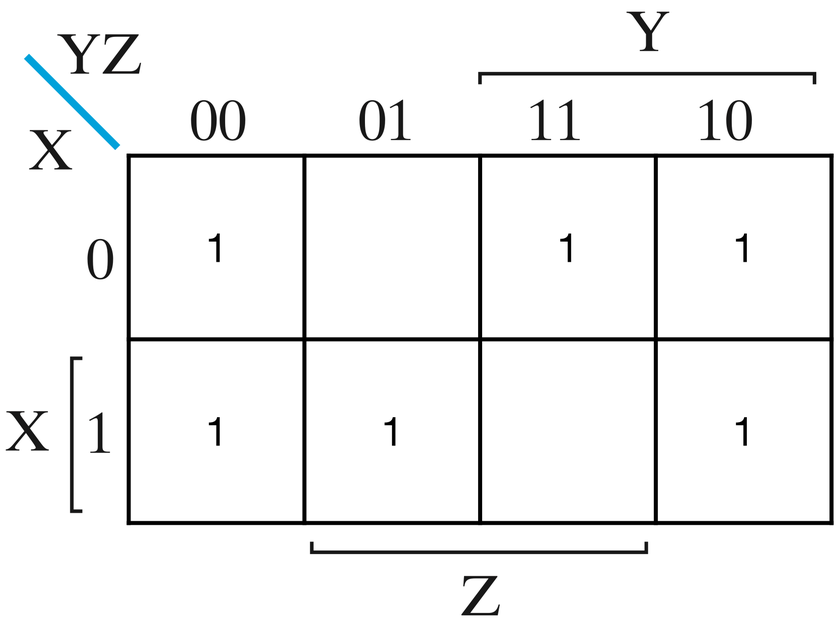
Exercises



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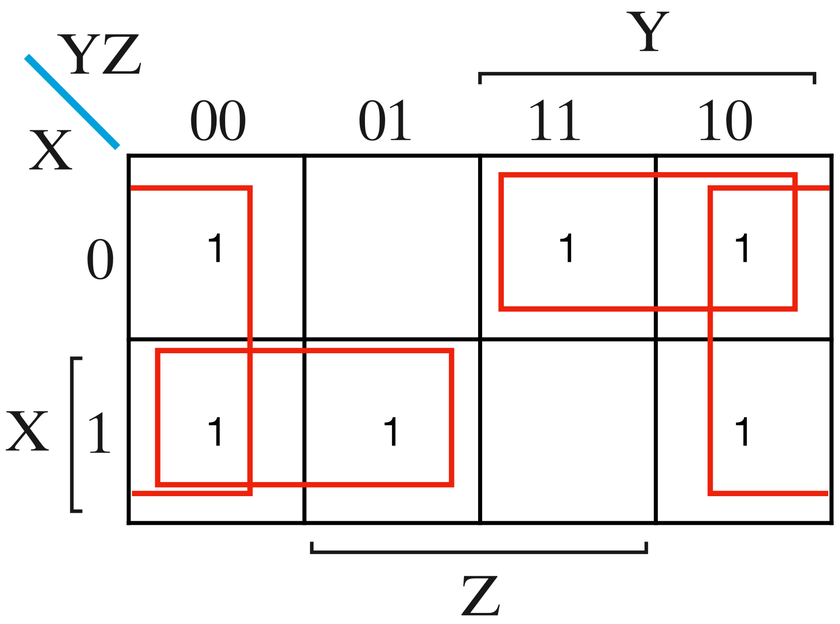

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Exercise