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# CSCI 150

## Introduction to Digital and Computer System Design

### Lecture 2: Combinational Logical Circuits II



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2020 Summer Semester (S2)

# Overview

- Focus: Boolean Algebra
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch2 2.2, 2.3; v5: Ch2 2.2, 2.3
- Core Ideas:
  1. Boolean Algebra I
  2. In-Class Exercises

# Boolean Algebra

It's just math

# Intro. to Boolean Algebra

$$L(X_1, X_2, \dots, X_n) = Y_1, Y_2, \dots, Y_m$$

- **Boolean Expression**

An algebraic expression formed by using binary variables, the constants 0 and 1, the logic operation symbols, and parentheses

- **Boolean Function**

A Boolean equation consisting of a binary variable identifying the function, followed by an equals sign and a **Boolean Expression**

- **Single-Output / Multi-Output Boolean Function**

Multiple Boolean function variables as input, **value 0/1 (single)** or **combinations of 0/1s (multi)** as output

# Intro. to Boolean Algebra

$$L(X_1, X_2, \dots, X_n) = Y_1, Y_2, \dots, Y_m$$

- If a boolean function has  $n$  input variables and  $m$  output variables, how many rows will its Truth Table have?

# Basic Identities

1.  $X + 0 =$

2.  $X \cdot 1 =$

3.  $X + 1 =$

4.  $X \cdot 0 =$

5.  $X + X =$

6.  $X \cdot X =$

7.  $X + \bar{X} =$

8.  $X \cdot \bar{X} =$

9.  $\bar{\bar{X}} =$

# Basic Identities

- Communicative

$$10. X + Y = Y + X$$

$$11. XY = YX$$

- Associative

$$12. X + (Y + Z) = (X + Y) + Z$$

$$13. X(YZ) = (XY)Z$$

- Distributive

$$14. X(Y + Z) = XY + XZ$$

$$15. X + (YZ) = (X + Y)(X + Z)$$

- DeMorgan's

$$16. \overline{X + Y} = \bar{X} \cdot \bar{Y}$$

$$17. \overline{\bar{X} \cdot \bar{Y}} = X + Y$$

# DeMorgan's

$$\overline{X + Y} = \bar{X} \cdot \bar{Y}$$

$$\overline{X \cdot Y} = \bar{X} + \bar{Y}$$

Truth Table

$X$	$Y$	$\overline{X + Y}$	$\bar{X} \cdot \bar{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Truth Table-1

$X$	$Y$	$\overline{X \cdot Y}$	$\bar{X} + \bar{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



# Basic Identities

A.  $X + XY = X$

B.  $XY + X\bar{Y} = X$

C.  $X + \bar{X}Y = X + Y$

D.  $X(X + Y) = X$

E.  $(X + Y)(X + \bar{Y}) = X$

F.  $X(\bar{X} + Y) = XY$

# Basic Identities

- Dual: change AND to OR; OR to AND; 0 to 1; 1 to 0
- Duality principle: a Boolean equation remains valid if we take the dual of the expressions on both sides of the equals sign.

1.  $X + 0 = X$

2.  $X \cdot 1 = X$

3.  $X + 1 = 1$

4.  $X \cdot 0 = 0$

5.  $X + X = X$

6.  $X \cdot X = X$

# Basic Identities

- Prove
  - $\bar{X} \cdot \bar{Y} + \bar{X} \cdot Y + X \cdot Y = \bar{X} + Y$
  - $\bar{A} \cdot B + \bar{B} \cdot \bar{C} + A \cdot B + \bar{B} \cdot C = 1$
  - $Y + \bar{X} \cdot Z + X \cdot \bar{Y} = X + Y + Z$
  - $\bar{X} \cdot \bar{Y} + \bar{Y} \cdot Z + X \cdot Z = \bar{X} \cdot \bar{Y} + X \cdot Z$
  - $\bar{X}\bar{Y} + \bar{Y}Z + XZ + XY + Y\bar{Z} = \bar{X} \cdot \bar{Y} + X \cdot Z + Y \cdot \bar{Z}$

# Basic Identities

- $\bar{X} \cdot \bar{Y} + \bar{Y} \cdot Z + X \cdot Z = \bar{X} \cdot \bar{Y} + X \cdot Z$

- Since  $\bar{Y} \cdot Z = X \cdot \bar{Y} \cdot Z + \bar{X} \cdot \bar{Y} \cdot Z$

- $\bar{X} \cdot \bar{Y} + \bar{Y} \cdot Z + X \cdot Z = \bar{X} \cdot \bar{Y} + X \cdot Z + X \cdot \bar{Y} \cdot Z + \bar{X} \cdot \bar{Y} \cdot Z$

**Rule B**

$$= \bar{X} \cdot \bar{Y} + \bar{X} \cdot \bar{Y} \cdot Z + X \cdot Z + X \cdot \bar{Y} \cdot Z$$

**Rule A x 2**

$$= \bar{X} \cdot \bar{Y} + X \cdot Z$$

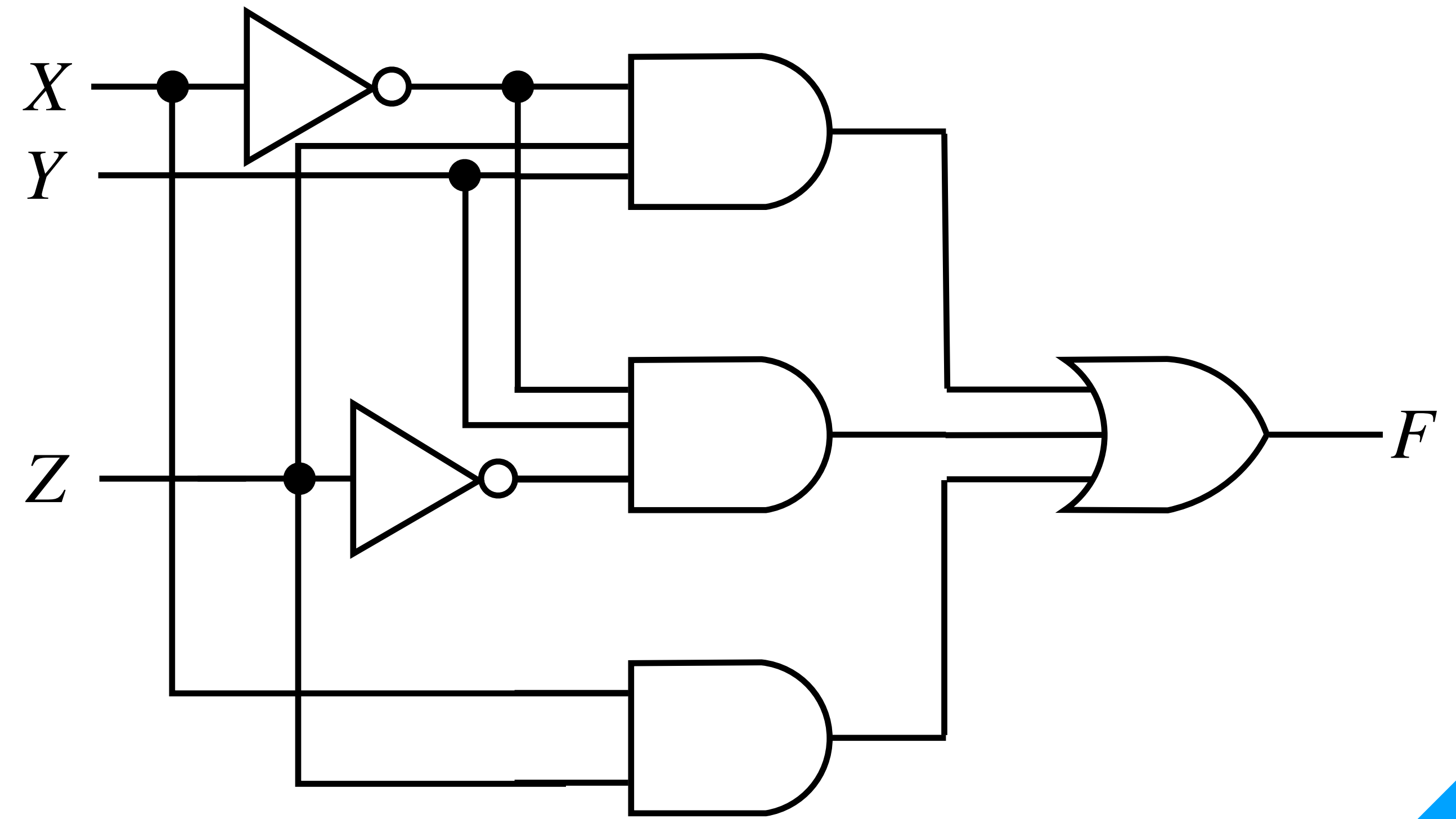
# Basic Identities

- Boolean Algebra solving
  - **Identify** rules **applicable** to the expression
  - **Apply** rules that can help you **simplify** the expression
    - **Simplification:** reducing the number of variables and operators in an expression without changing its truth table values
    - **Atomic element:** an element that can't have the number of its variables and operators reduced any further

# Algebraic Manipulation

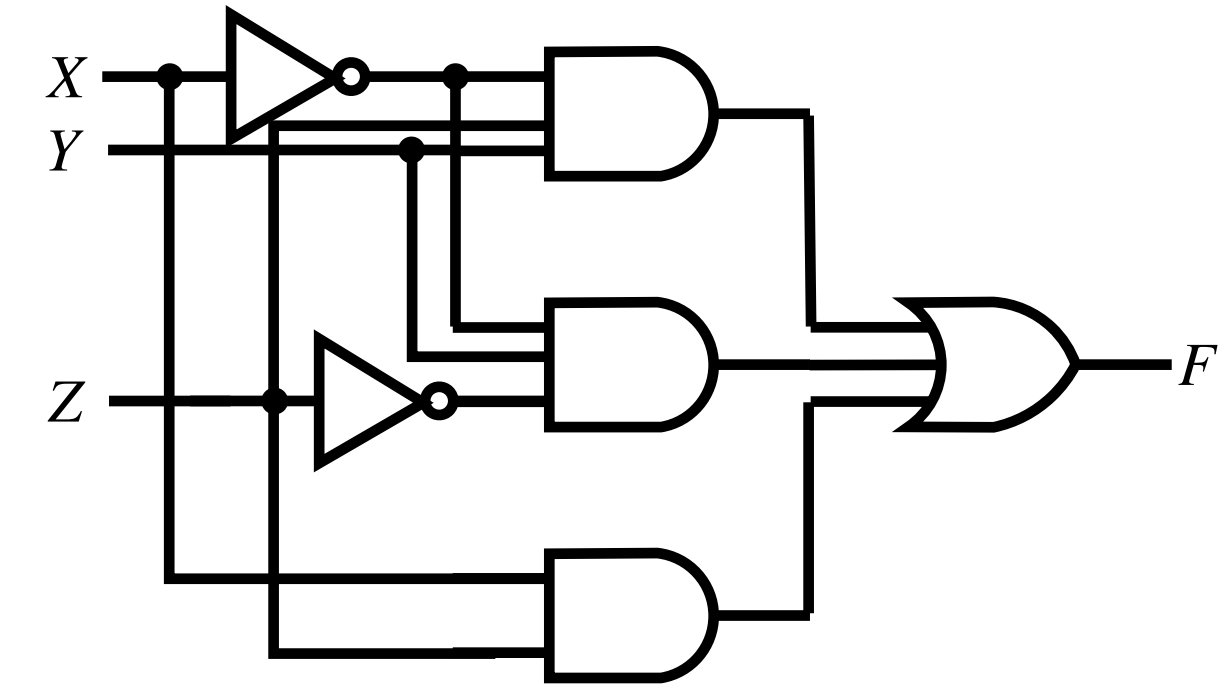
Truth Table

$X$	$Y$	$Z$	$F = \bar{X}YZ + \bar{X}Y\bar{Z} + XZ$
0	0	0	0
0	1	0	1
1	0	0	0
1	1	0	0
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1



Example

# Algebraic Manipulation



Truth Table

$X$	$Y$	$Z$	$F = \bar{X}YZ + \bar{X}Y\bar{Z} + XZ$
0	0	0	0
0	1	0	1
1	0	0	0
1	1	0	0
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1

$$\begin{aligned}
 F &= \bar{X}YZ + \bar{X}Y\bar{Z} + XZ \\
 &= \bar{X}Y(Z + \bar{Z}) + XZ \\
 &= \bar{X}Y \cdot 1 + XZ \\
 &= \bar{X}Y + XZ
 \end{aligned}$$

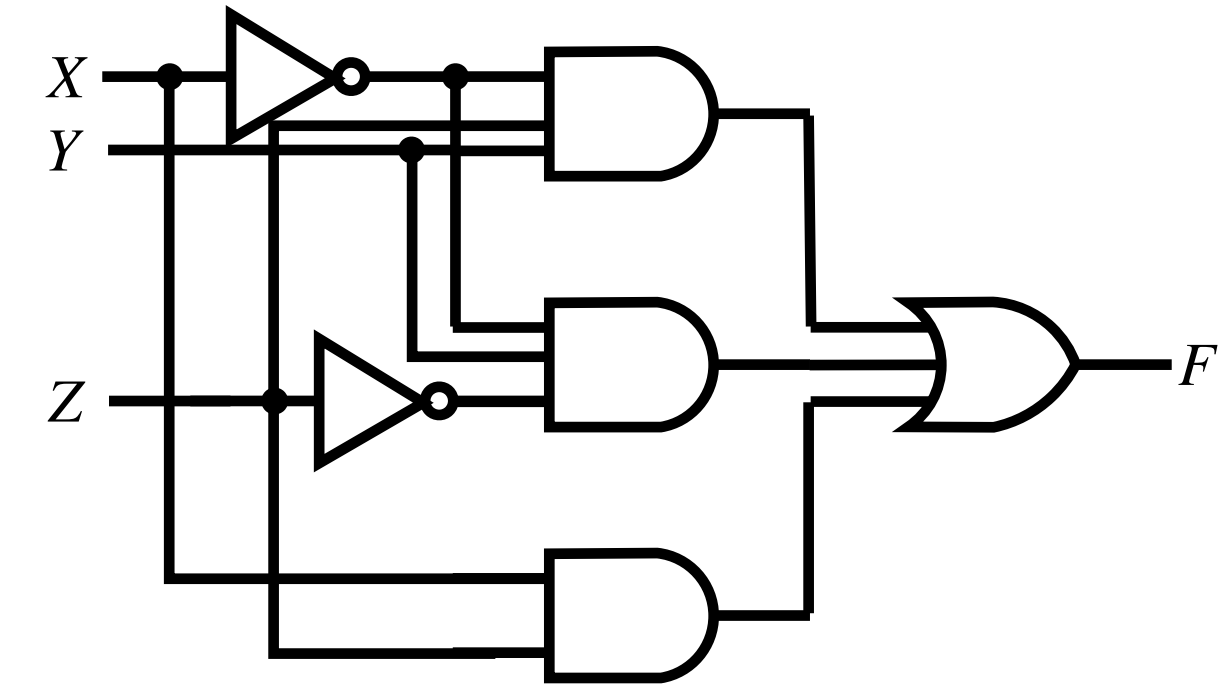
**Rule 14**

**Rule 7**

**Rule 2**

Example

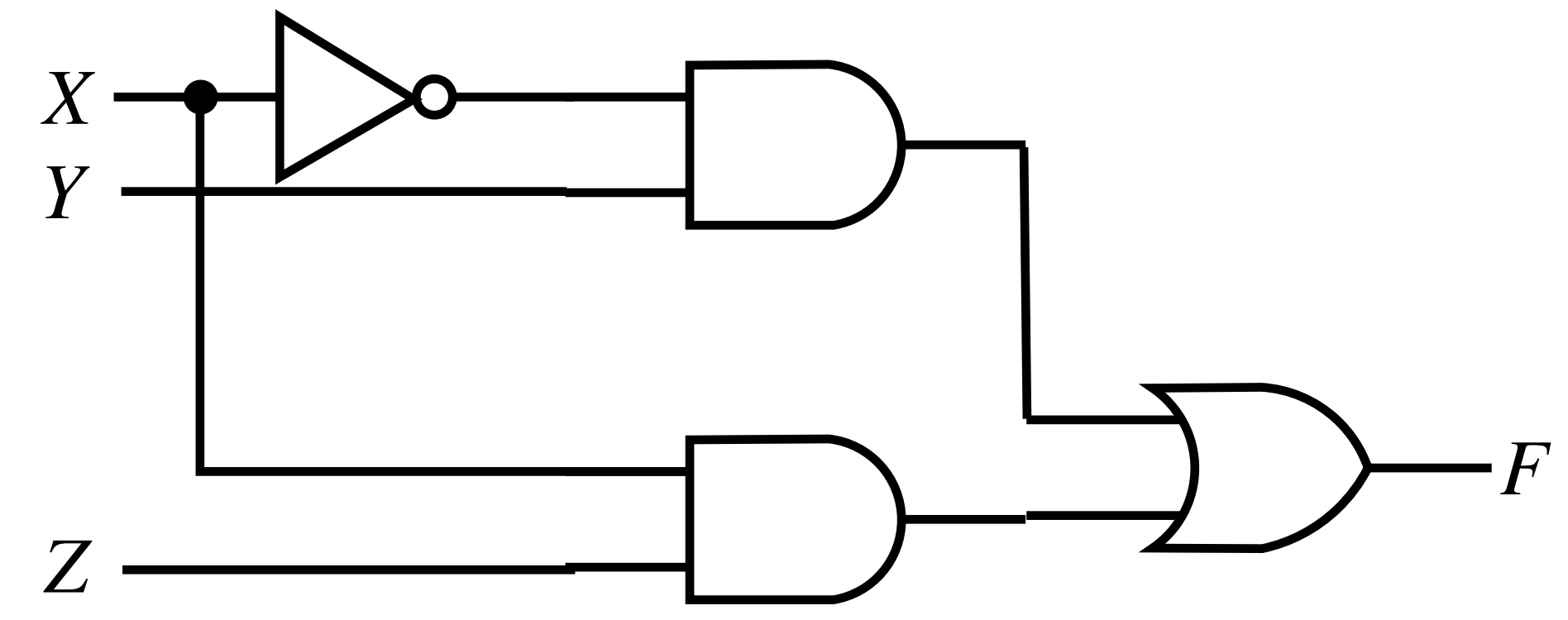
# Algebraic Manipulation



Truth Table

$X$	$Y$	$Z$	$F = \bar{X}YZ + \bar{X}Y\bar{Z} + XZ$
0	0	0	0
0	1	0	1
1	0	0	0
1	1	0	0
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1

$$\begin{aligned}
 F &= \bar{X}YZ + \bar{X}Y\bar{Z} + XZ \\
 &= \bar{X}Y(Z + \bar{Z}) + XZ && \text{Rule 14} \\
 &= \bar{X}Y \cdot 1 + XZ && \text{Rule 7} \\
 &= \bar{X}Y + XZ && \text{Rule 2}
 \end{aligned}$$



Example



# Algebraic Manipulation

- Algebraic Manipulation can help reduce the number of gates in a circuit
  - easier to implement and debug
  - more efficient

# Complementation

- $\bar{F}$ : complement (invert) representation for a function  $F$ , obtained from an interchange of 1s to 0s and 0s to 1s for the values of  $F$  in the truth table
- Apply DeMorgan's Rule

$$16. \overline{X_1 + X_2 + \dots + X_n} = \bar{X}_1 \cdot \bar{X}_2 \cdot \dots \cdot \bar{X}_n$$

$$17. \overline{X_1 \cdot X_2 \cdot \dots \cdot X_n} = \bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_n$$

# Complementation

- $F_1 = \bar{X}Y\bar{Z} + \bar{X}\bar{Y}Z$
- $F_2 = X(\bar{Y}\bar{Z} + YZ)$

# Algebra Solving 1

- Known  $BC + D = 1; \bar{C} = 1$
- Calculate:  $\bar{D}(\bar{B} + \bar{C})$
- Calculate:  $(D + B)(D + C)$

# Algebra Solving 2

- Known  $AB + C = 1; A + \bar{D} = 0$
- Calculate:  $C + AB$
- Calculate:  $\bar{C}\bar{A} + \bar{C}\bar{B}$

# Algebra Solving 3

- Known  $A + B + \bar{C} = 1$ ;  $\bar{A}\bar{C} + \bar{A}D = 1$
- Calculate:  $(\bar{C} + B)(\bar{C} + D)$
- Calculate:  $\bar{C} + BD$

# Boolean Algebra

Exercises! Use #04-2020-1000-401 to help you!

# Boolean Algebra

Difficulty: Simple

Prove by truth table that

- $\bar{X}Y + \bar{Y}Z + X\bar{Z} = X\bar{Y} + Y\bar{Z} + \bar{X}Z$



# Algebraic Manipulation

Difficulty: Simple

Use DeMorgan's Rules to transform the following expression to one WITHOUT AND operator

- $\overline{ABC} + CD$
- $A\overline{B}C + \overline{A} \cdot \overline{C} + AB$

# Logic Diagram

Difficulty: Simple

Draw the logic diagram for the following expression (you don't have to perform any manipulation/transformation)

- $XYZ + \overline{X}\overline{Y} + \overline{X}\overline{Z}$
- $B(\overline{A} \cdot \overline{C} + AC) + \overline{B}(A + \overline{B}C)$

# Algebraic Manipulation

Difficulty: Simple

Simplify the following expressions

- $\bar{X} \cdot \bar{Y} + XYZ + \bar{X}Y$
- $X + Y(Z + \overline{X + Z})$

# Algebraic Manipulation

Difficulty: Mid

Simplify the following expressions

- $\bar{W}X(\bar{Z} + \bar{Y}Z) + X(W + \bar{W}YZ)$
- $(AB + \bar{A}\bar{B})(\bar{C}\bar{D} + CD) + AC$

# Algebraic Manipulation

Difficulty: Mid

Simplify the following expressions

- $\bar{A} \cdot \bar{C} + \bar{A}BC + \bar{B}C$

- $\overline{A + B + C} \cdot \overline{ABC}$

# Algebraic Manipulation

Difficulty: Mid

Simplify the following expressions

- $ABC\bar{C} + AC$

- $\bar{A} \cdot \bar{B}D + \bar{A} \cdot \bar{C}D + BD$

# Algebraic Manipulation

Difficulty: HARDCORE

Given that  $AB = 0$  and  $A + B = 1$ , prove that

- $(A + C)(\bar{A} + B)(B + C) = BC$

# Algebraic Manipulation

Difficulty: HARDCORE

Prove the identity of each of the following Boolean equations

- $ABC\bar{C} + BC\bar{C} \cdot \bar{D} + BC + \bar{C}D = B + \bar{C}D$
- $WY + \bar{W}Y\bar{Z} + WXZ + \bar{W}X\bar{Y} = WY + \bar{W}X\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}Z$
- $A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C = (\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + B + C + D)$