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CSCI 150
Introduction to Digital and Computer System Design
Lecture 1: Digital Information Representations II

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2020 Summer Semester (S2)

Overview

- Focus: Number Systems
- Architecture: Digital Circuits
- Textbook v4: Ch1 1.3, 1.4; v5: Ch1 1.4, 1.5
- Core Ideas:
  1. Digital Number Systems
  2. Arithmetic Operations, Ranges
  3. Digital & Analog Conversion

P0 Review

Digital and Analog Circuits

- In Digital Circuits, information are represented by discrete values, usually 0/1s defined by HIGH/LOW voltages ranges
- In Analog Circuits, information representation is continuous
- Von Neumann computers: all-purpose design
  - CPU (control unit and functional/datapath unit), IO, memory
- Embedded systems: specific functions only, compact and robust

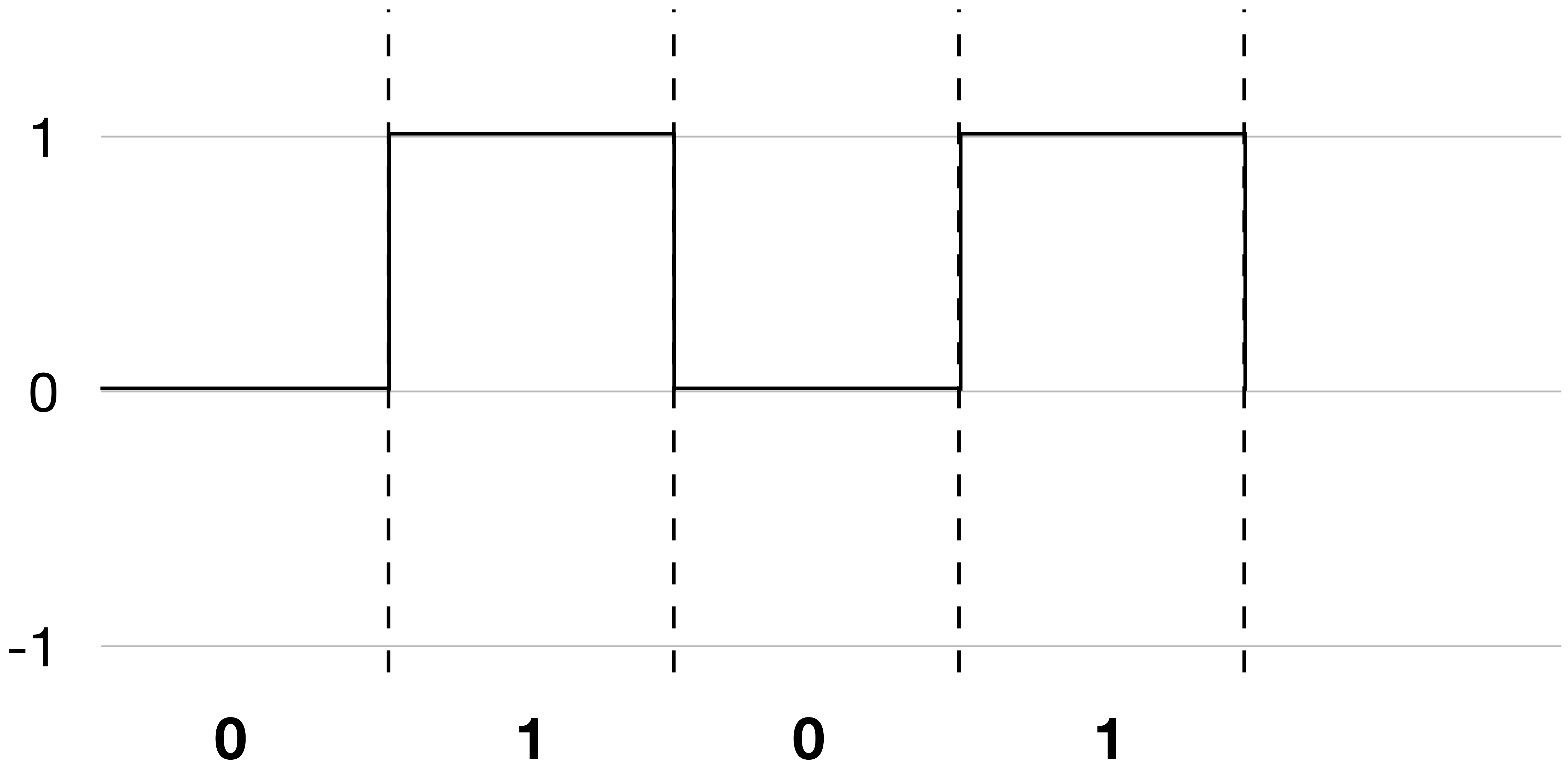
Review

Digital Number Systems

Binary, Octal and Hexadecimal Numbers;
Number Ranges

Summary

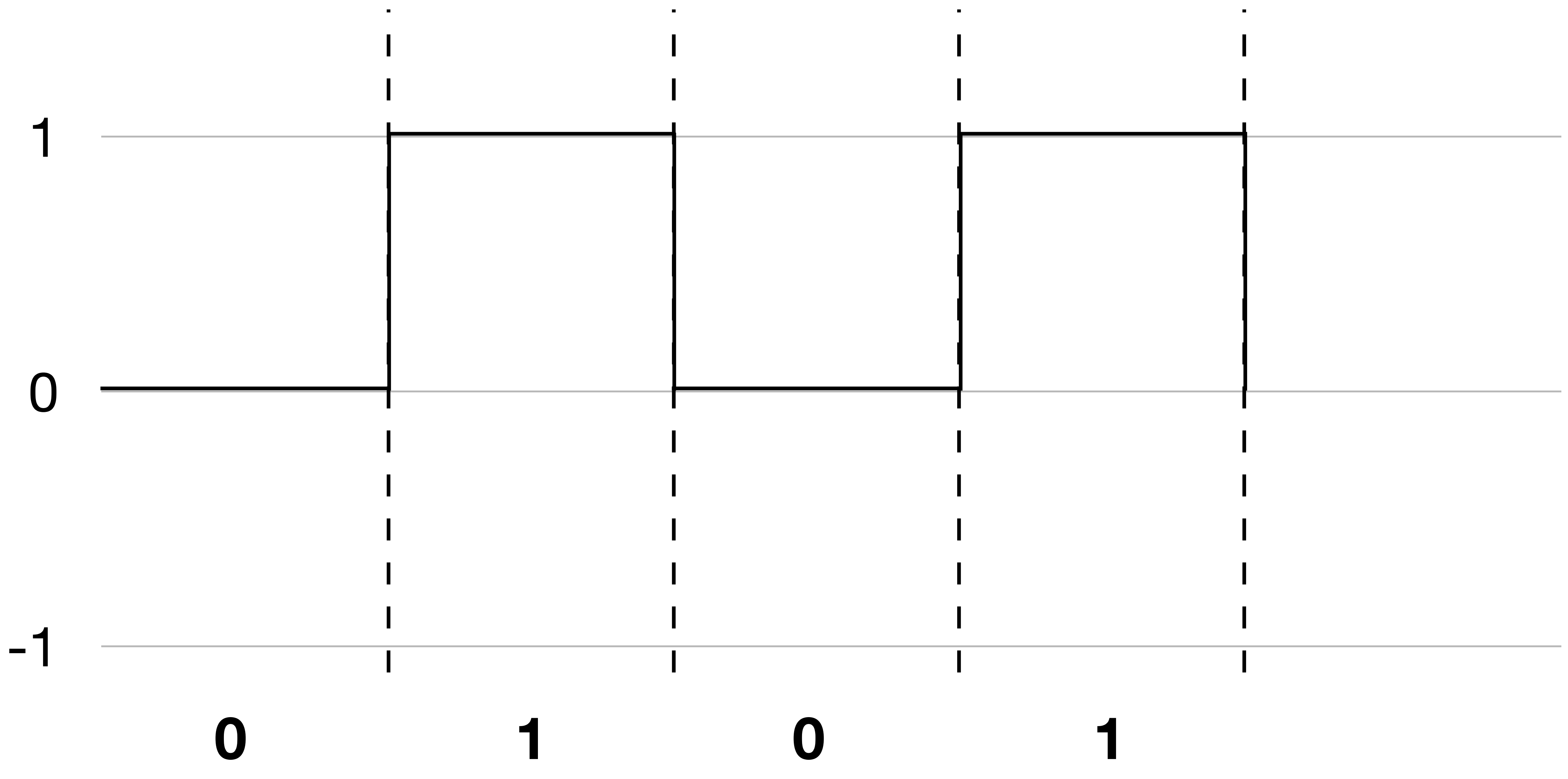
Binary System



- Base 2 system
- A number is represented with a string of 1s and 0s, each called a ***bit***
- $(0101)_2 = 5$

Concept

Binary System

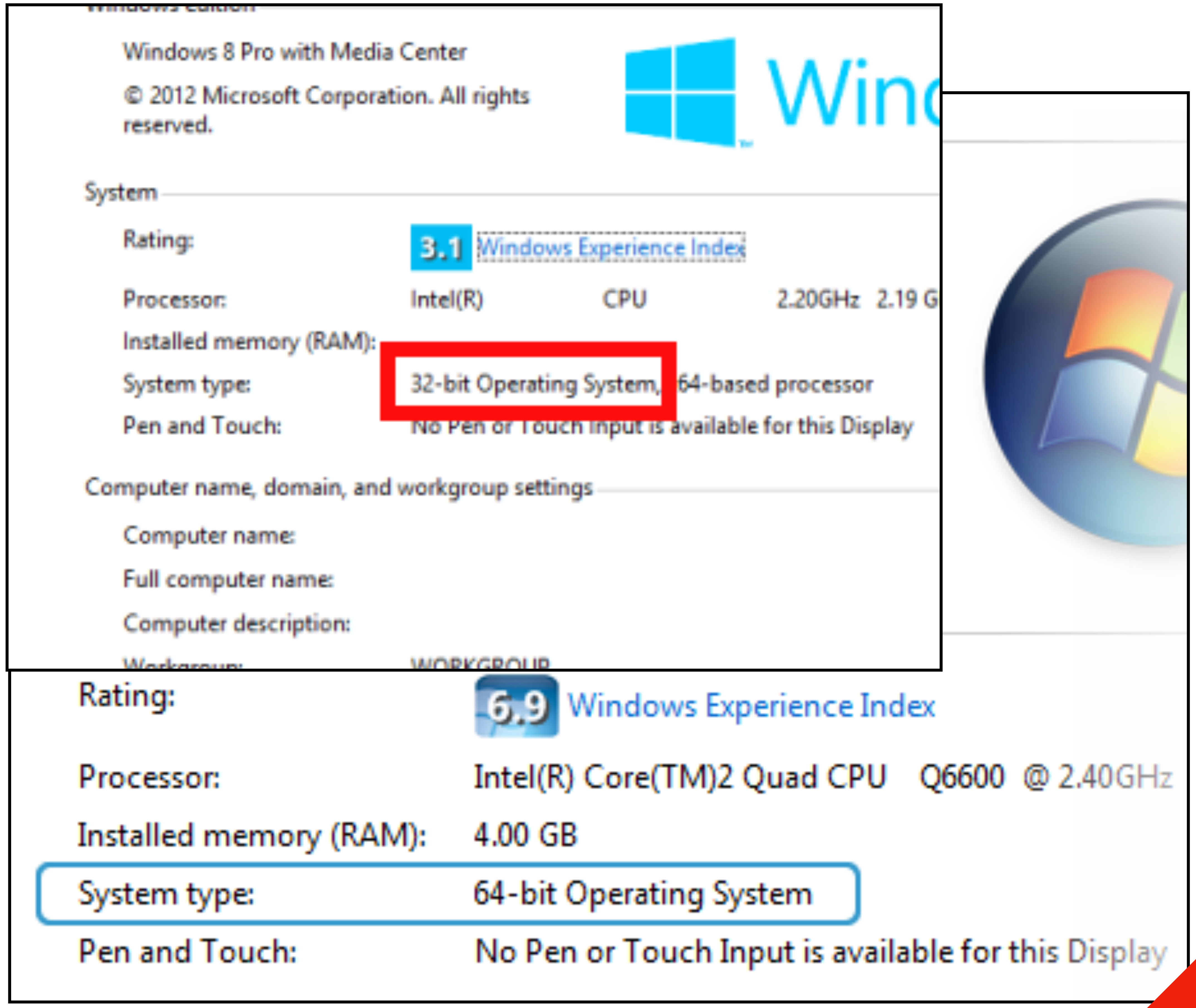


- Is it possible to use different bases in a digital circuit?
- If it is possible, why haven't we seen it very often?

P1
Number Systems

Binary Systems in Computers

- Every 8bit is called a Byte
- 32bit OS
  - A single number is represented by 32bits
  - Range (int): 1 - 4,294,967,295
- OS vs Processor?
  - Compatibility mode



Concept

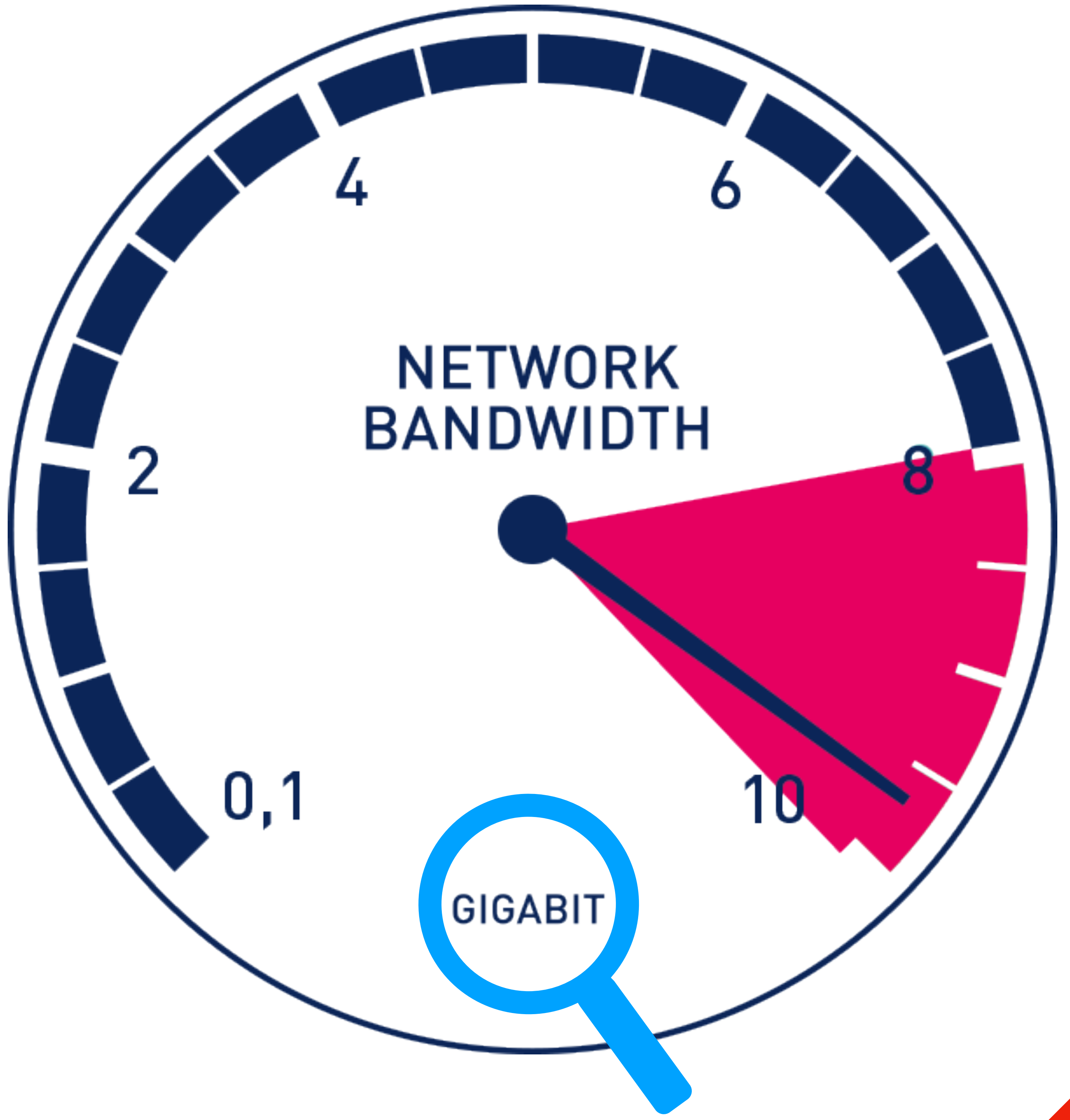
Binary Systems in Computers

- Every 8bit is called a Byte
- 1,024 = 2^{10} is called K (Kilo)
- 1,024 x 1,024 = 2^{20} is called M (Mega)
- 1,024 x 1,024 x 1,024 = 2^{40} is called G (Giga)
- Tera, Peta, Exa, Zetta, Yotta

Concept

Binary Systems in Computers

- What is the difference between MBps and Mbps?
  - MegaBytes per second vs MegaBits per second
  - 8x difference!



Binary Systems in Computers

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Concept

Octal and Hexadecimal Systems

- Octal: base 8
  - digits: 0-7
- Hexadecimal: base 16
  - digits: 0-9, A-F (10-15)

Concept

Octal and Hexadecimal Systems

Decimal (Base 10)	Binary (Base 2)	Octal (Base 8)	Hexadecimal (Base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Summary

- Digital binary systems
- Octal and Hexadecimal systems

Review

Arithmetic Operations

Additions, Subtractions, Conversions

Summary

Arithmetics

- The same as decimal (mostly)
-

$$\begin{array}{r} 0010 \\ +0011 \\ \hline 0101 \end{array} \qquad \begin{array}{r} 0101 \\ -0011 \\ \hline 0010 \end{array}$$

Example (binary)

Concept

Arithmetics

Binary additions and subtractions

Carries	
Augend	01101
Addend	+00101
Sum	

Arithmetics

Binary additions and subtractions

Carries	11010
Augend	01101
Addend	+00101
Sum	10010

Borrows	
Minuend	10110
Subtrahend	−10011
Difference	

Demo

Arithmetics

OCTAL Multiplication

$$\begin{array}{r} 762 \\ \times \quad 54 \\ \hline 4672 \\ 3710 \\ \hline 43772 \end{array}$$

$$5 \times 2 = 12$$

$$5 \times 6 + 1 = 37$$

$$5 \times 7 + 3 = 46$$

$$\cdots$$

Demo

Arithmetics

OCTAL Multiplication

Octal

$$
\begin{array}{r}
762 \\
\times \quad 54 \\
\hline
4672 \\
3710 \\
\hline
43772
\end{array}
$$

Octal	Decimal
$5 \times 2 = 12$	$10 = (12)_8$
$5 \times 6 + 1 = 37$	$31 = (37)_8$
$5 \times 7 + 3 = 46$	$38 = (46)_8$
$\cdots$	$\cdots$

Demo

Arithmetics

Convert 41 to binary

Least significant digit

$$41 \div 2 = 20 \quad \text{mod } 1$$
$$20 \div 2 = 10 \quad \text{mod } 0$$
$$10 \div 2 = 5 \quad \text{mod } 0$$
$$5 \div 2 = 2 \quad \text{mod } 1$$
$$2 \div 2 = 1 \quad \text{mod } 0$$
$$1 \div 2 = 0 \quad \text{mod } 1$$

Most significant digit

41=(1 0 1 0 0 1)

Demo

P2
Arithmetics

Arithmetics

Convert 0.6875 to binary

$$0.6875 \times 2 = \boxed{1} + 0.3750$$

$$0.3750 \times 2 = \boxed{0} + 0.7500$$

$$0.7500 \times 2 = \boxed{1} + 0.5000$$

$$0.5000 \times 2 = \boxed{1} + 0.0000$$

Most significant digit

↓

Least significant digit

0.6875= (1 0 1 1)

Demo

Signed Integers

- Integers in digital circuits have limited bits
    - e.g. 8bit for every number
- How do we represent negative values in digital circuits?

$$
\begin{array}{r}
00000101 \\
-00001011 \\
\hline
0000????
\end{array}
$$

(binary, 8bit, unsigned)

Concept

Signed & Unsigned Integers

- Unsigned 8bit:
  - $(11111111)_2 = 255$
- Signed 8bit (only in digital circuits):
  - 127 -> '01111111'
  - -127 -> '11111111'

First digit:

- **0 for positive**
- **1 for negative**

10001111

(binary, 8bit, signed)

Concept

P2 Arithmetics

Signed & Unsigned Integers

- Unsigned 8bit integer: 0 - 255
  - Signed 8bit integer: -128 - 127
- Unsigned 32bit integer: 0 - 4,294,967,295
  - Signed 32bit integer: -2,147,483,648 - 2,147,483,647
- Unless otherwise specified, treat as unsigned

Concept

Summary

- Additions, Subtractions, Multiplications in Binary, Octal, Hexadecimal systems

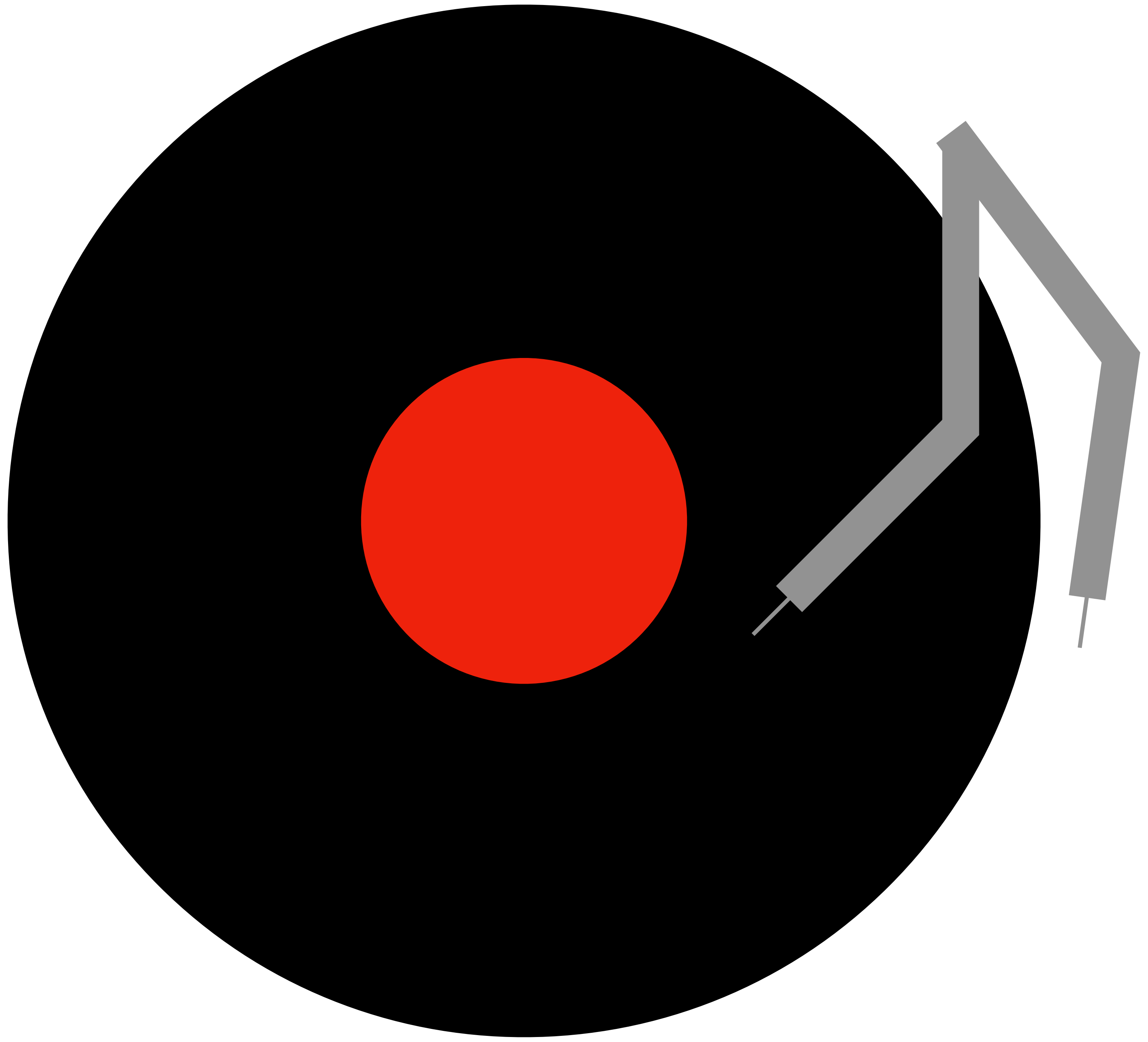
Review

Digital & Analog Conversions

Demonstration

WARNING: inbound signed Binary/Hexadecimal operations

Case Study: Audio Signal Representation



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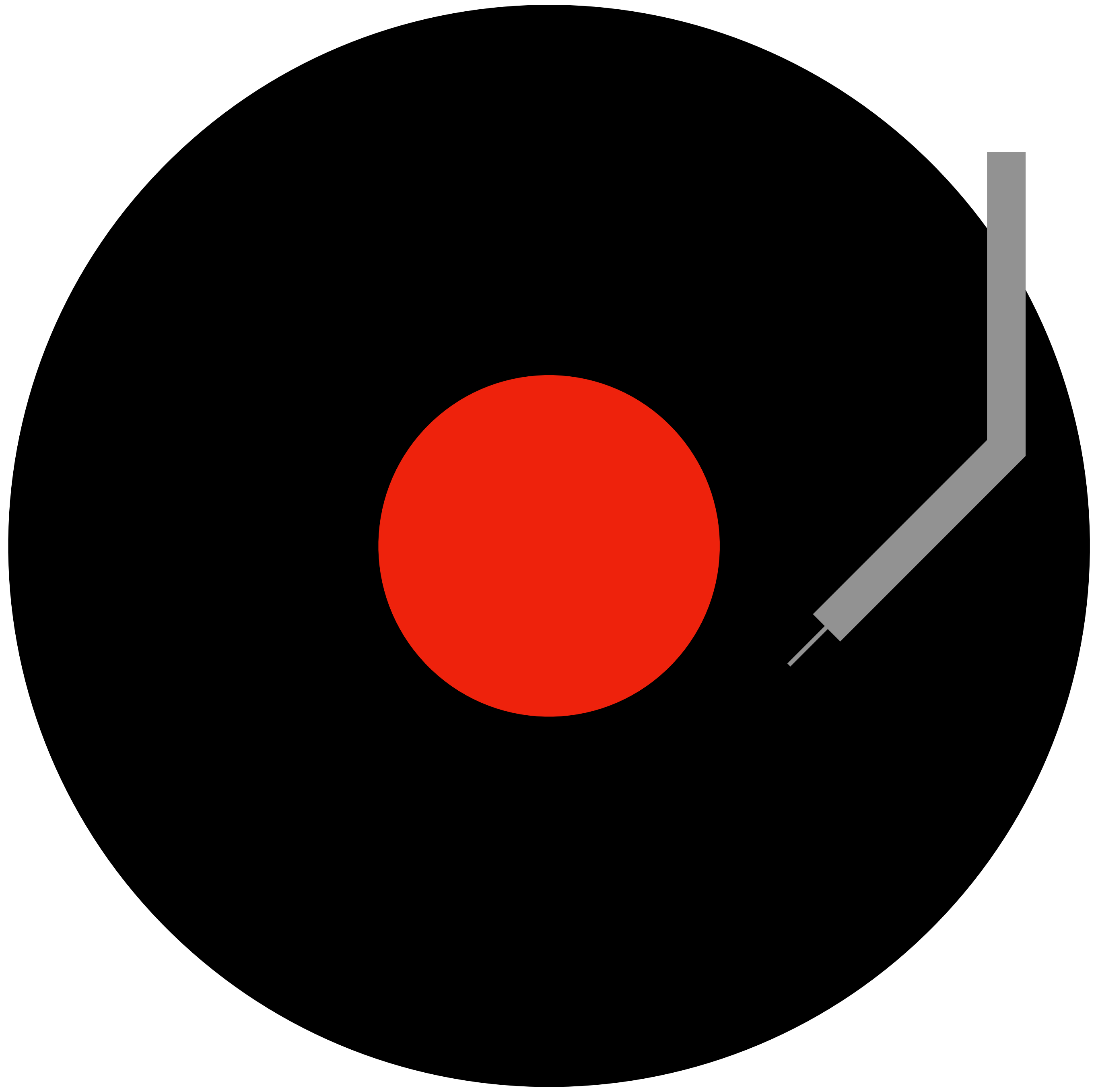
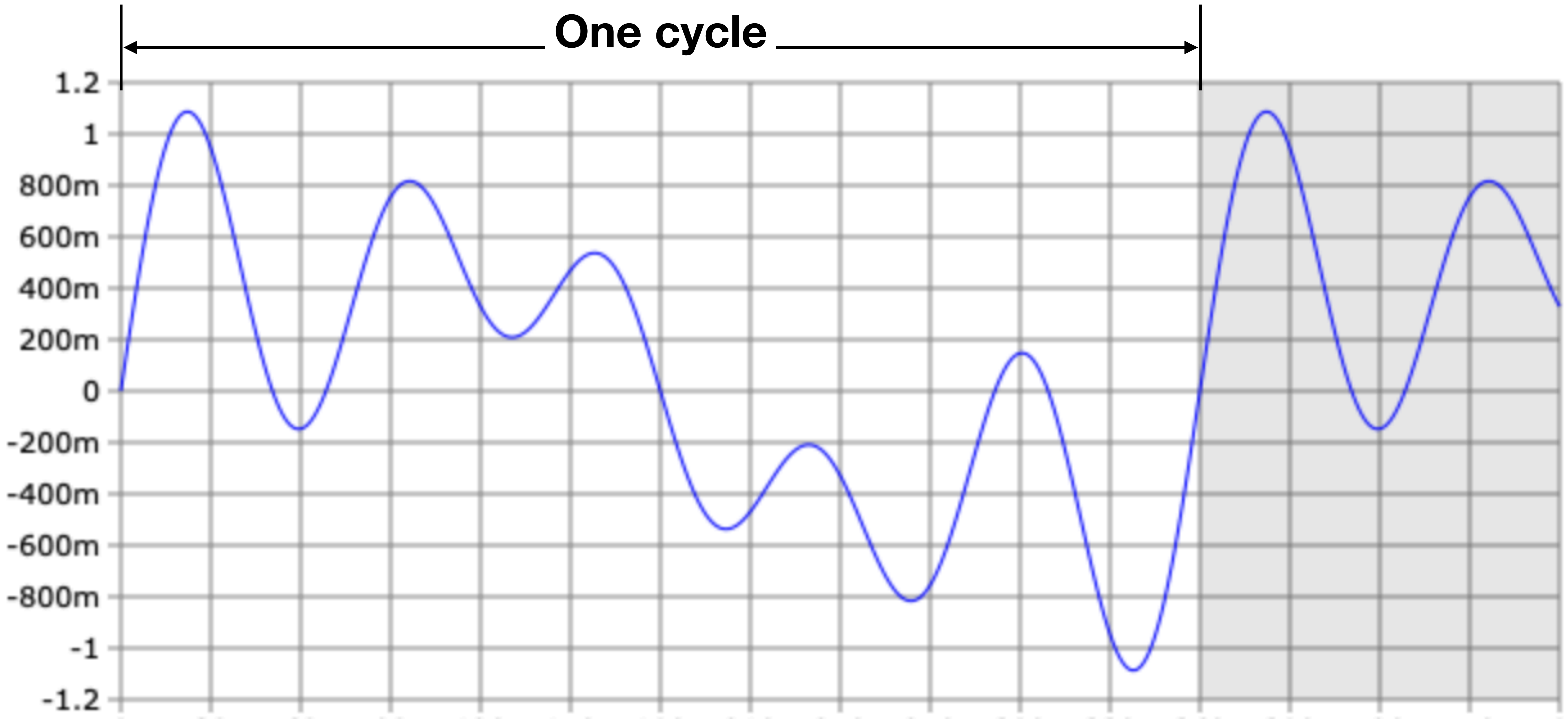
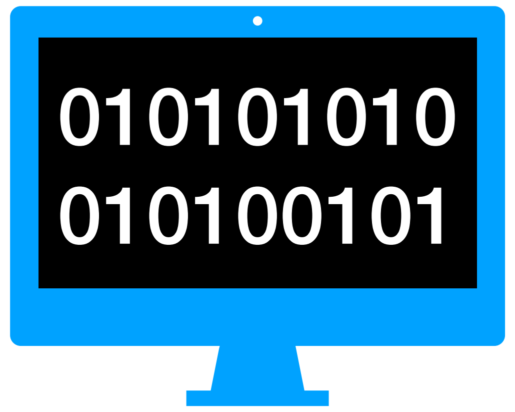


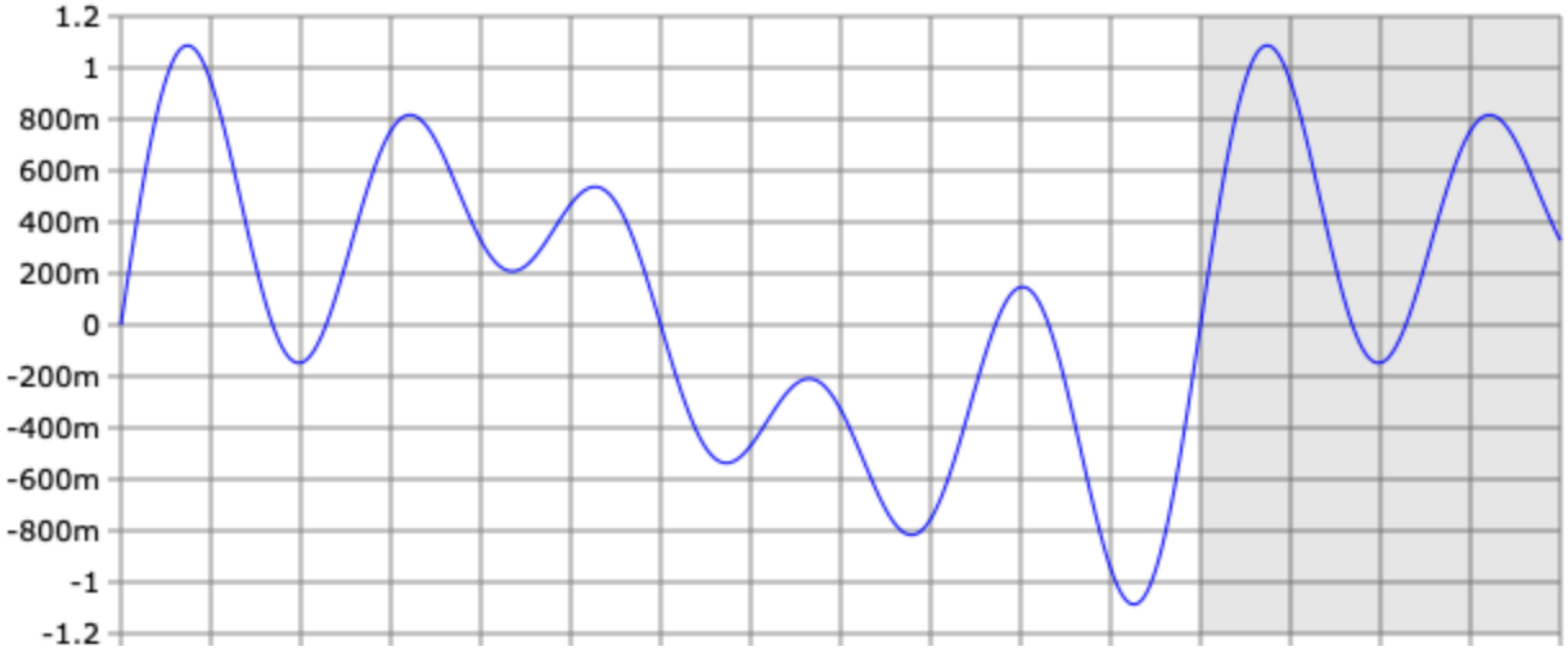


440 Hz = 440 cycles per sec

Technical

Case Study: Audio Signal Representation





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Technical

P3
DA Conversion

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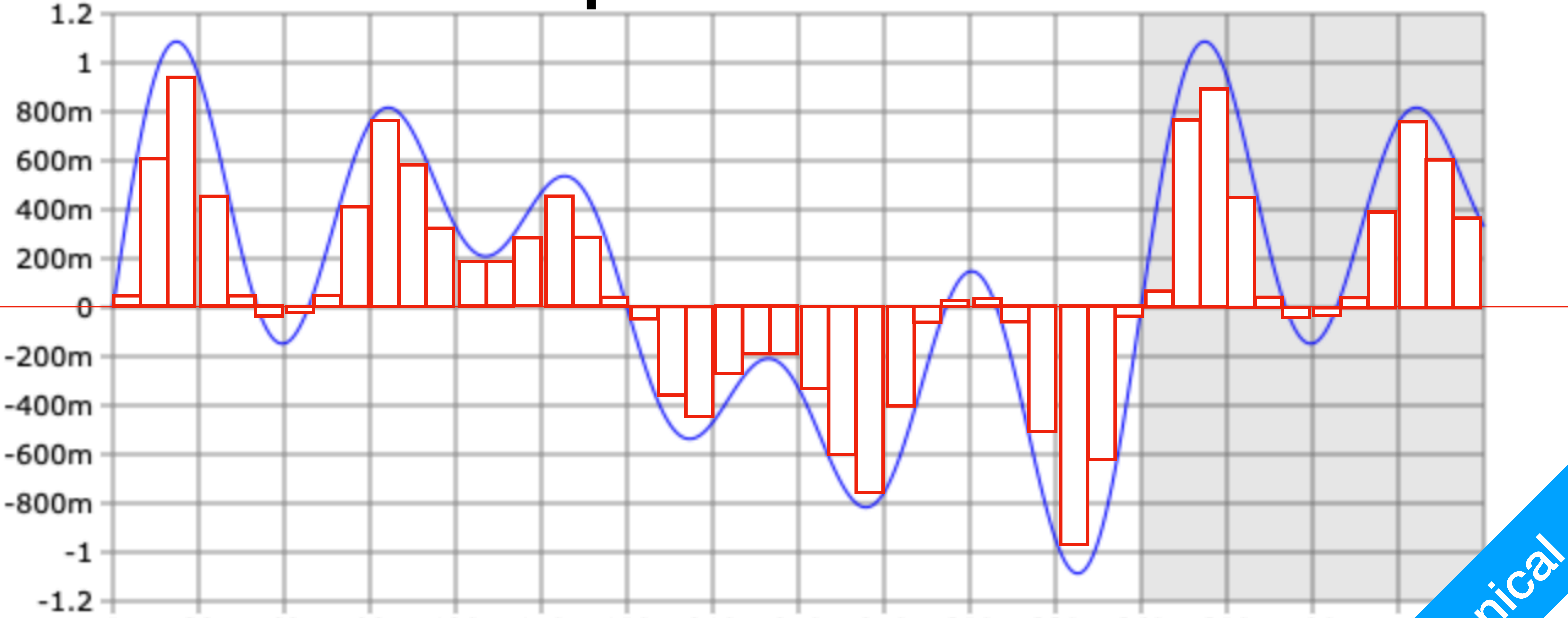

440 Hz = 440 cycles per sec, Sample rate 36 x 440 = 15,840 = 16kps

Technical

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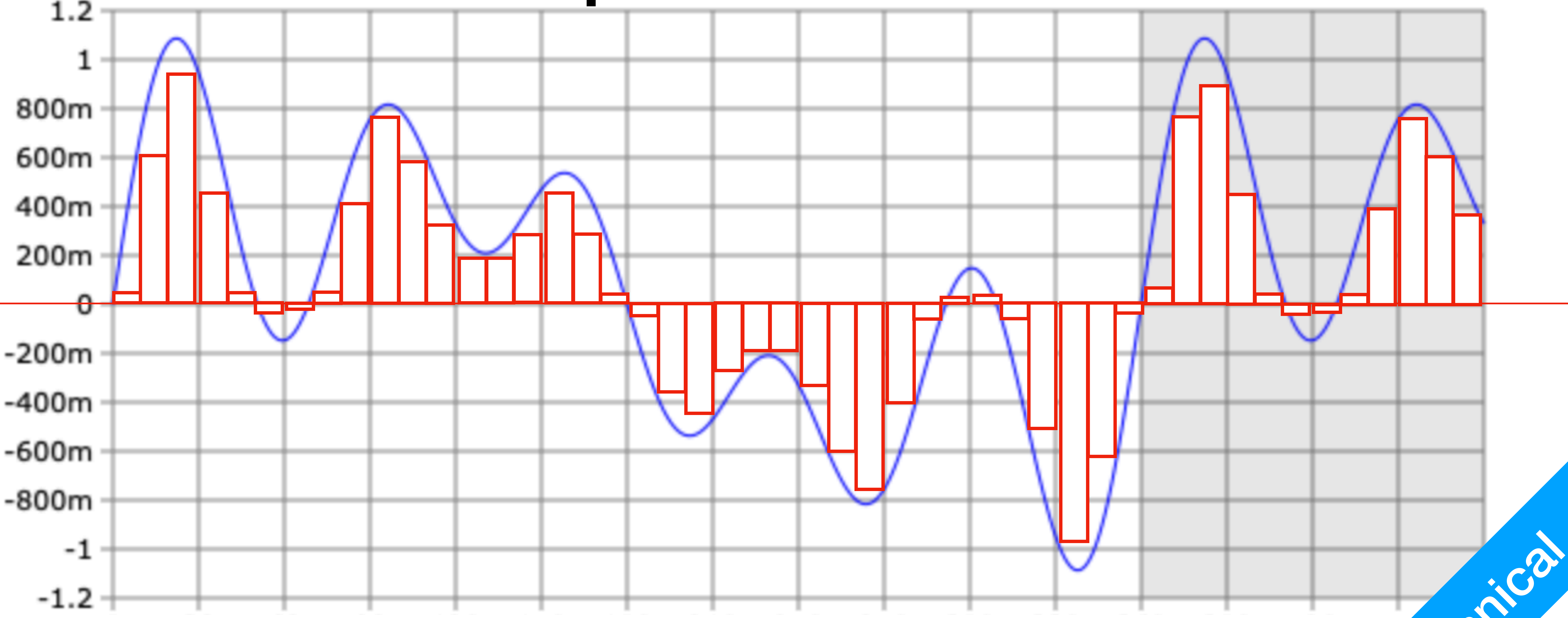

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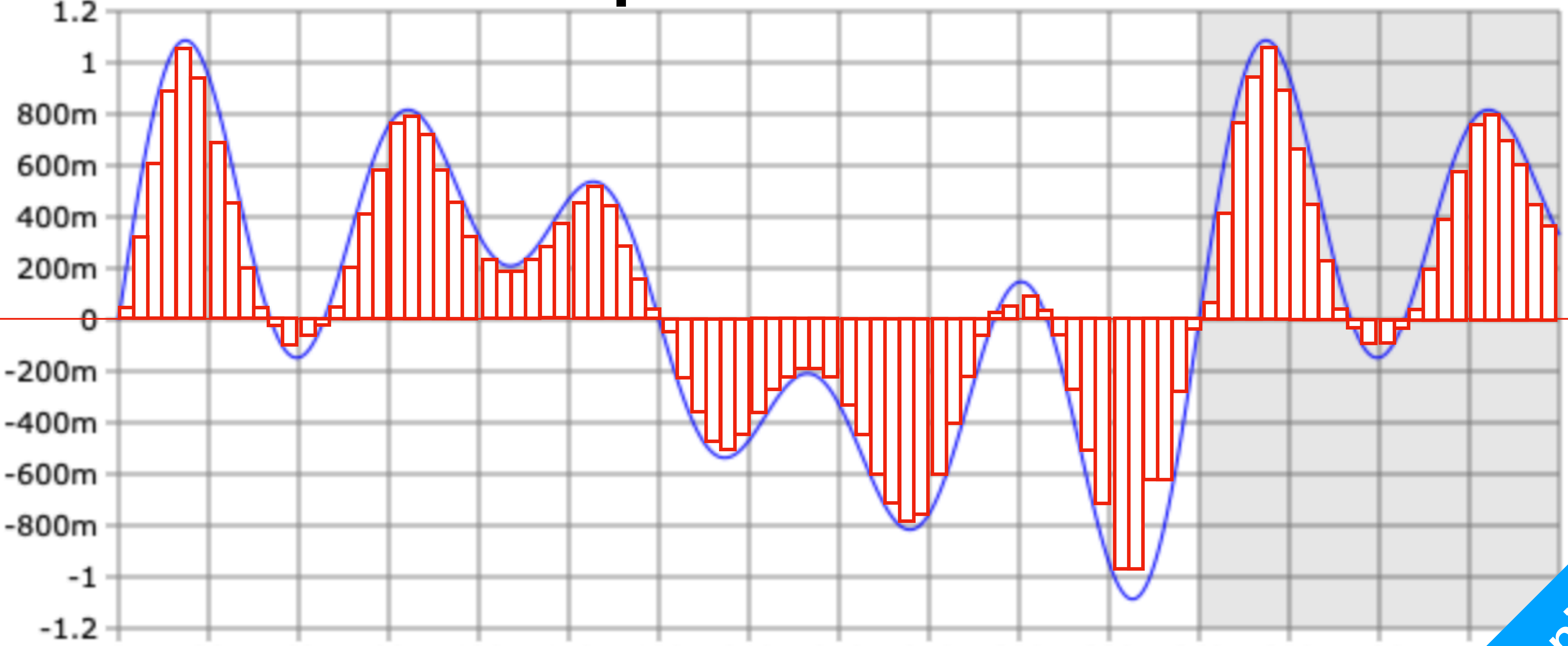
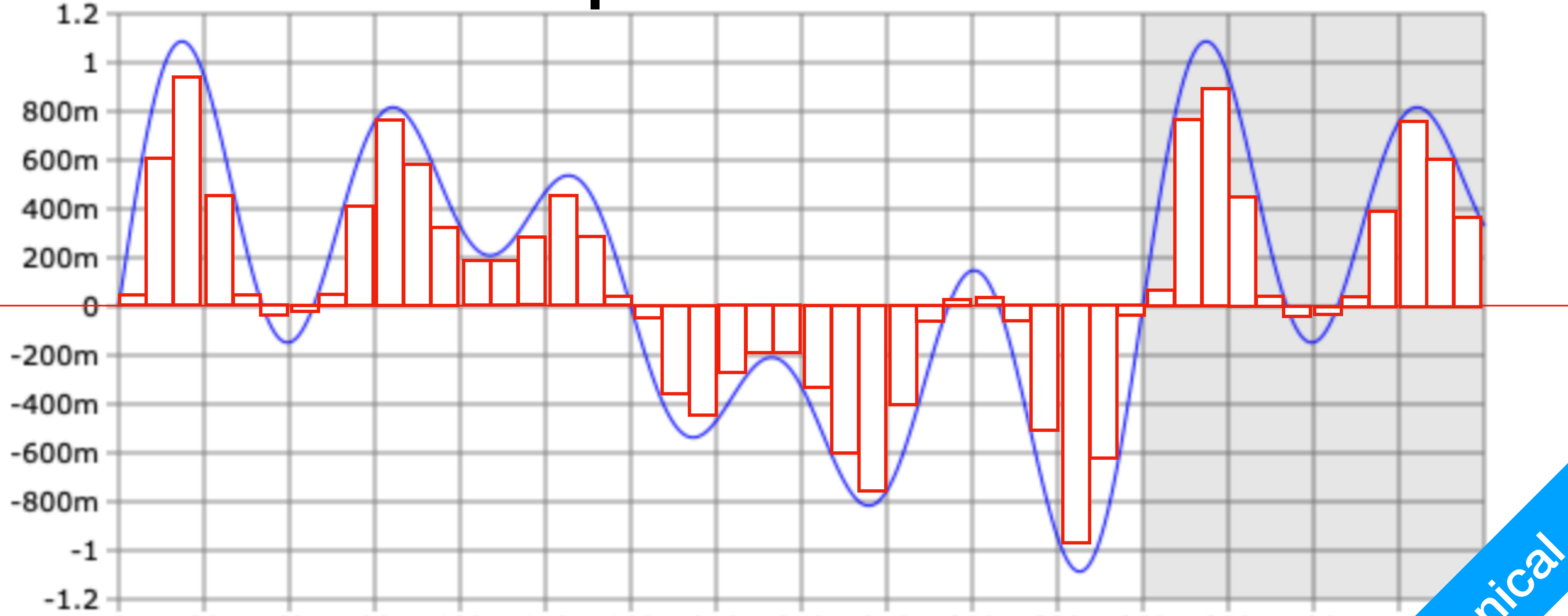


440 Hz = 440 cycles per sec, Sample rate 64 x 440 = 31,680 = 32kps

Case Study: Audio Signal Representation



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Technical

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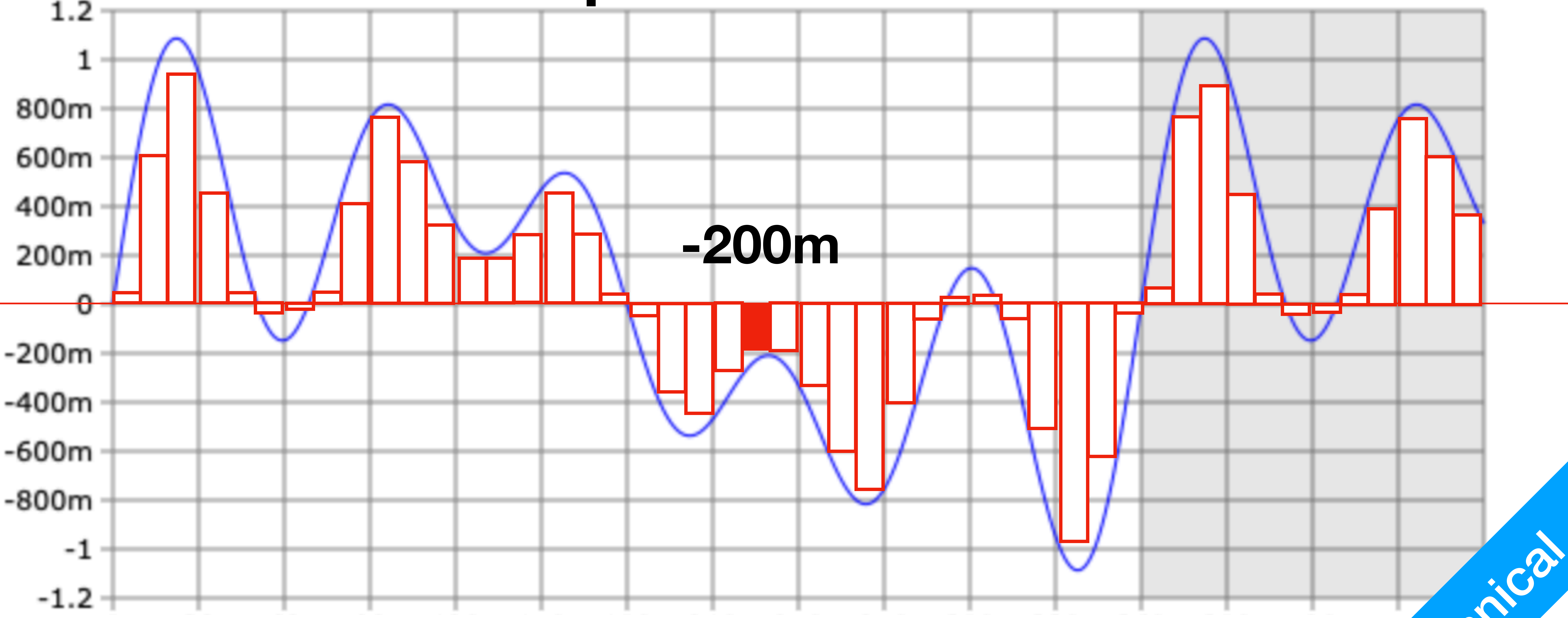

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P3
DA Conversion

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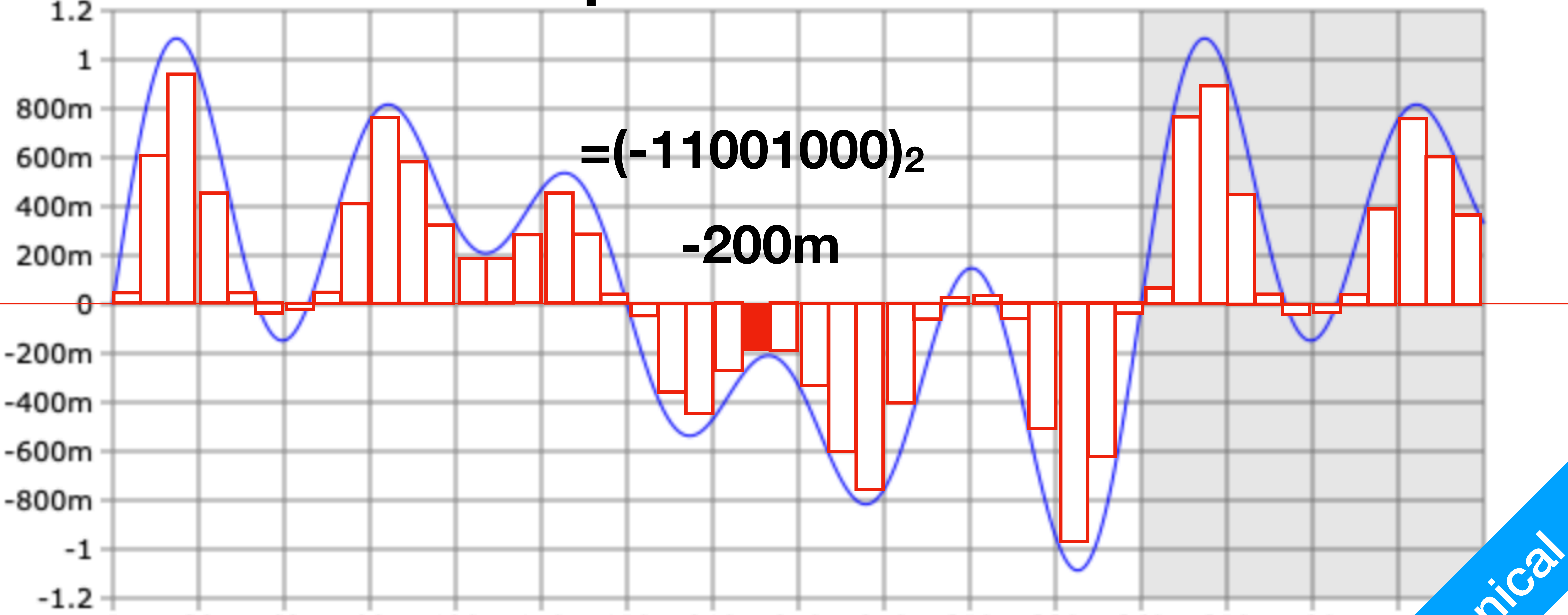

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Technical

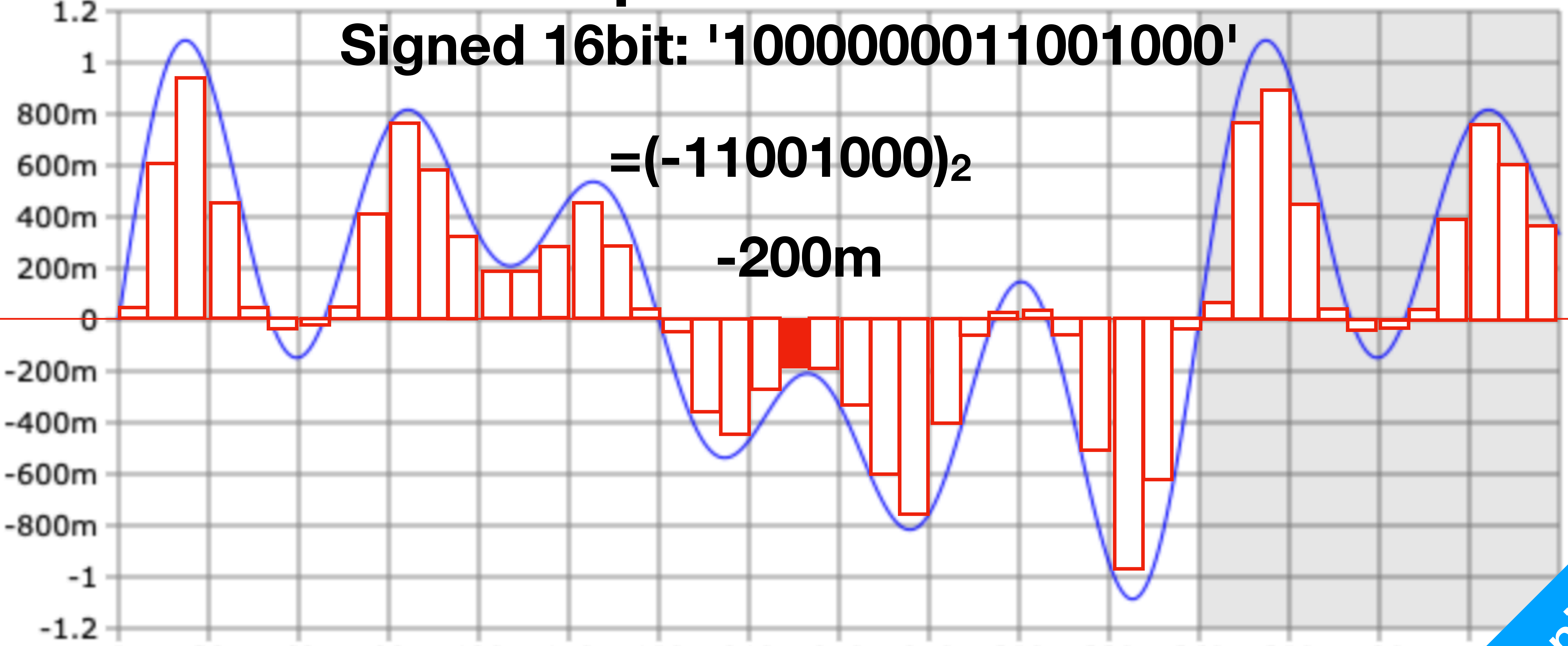
P3
DA Conversion

Case Study: Audio Signal Representation

Signed 16bit: '1000000011001000'

$=(-11001000)_2$

-200m



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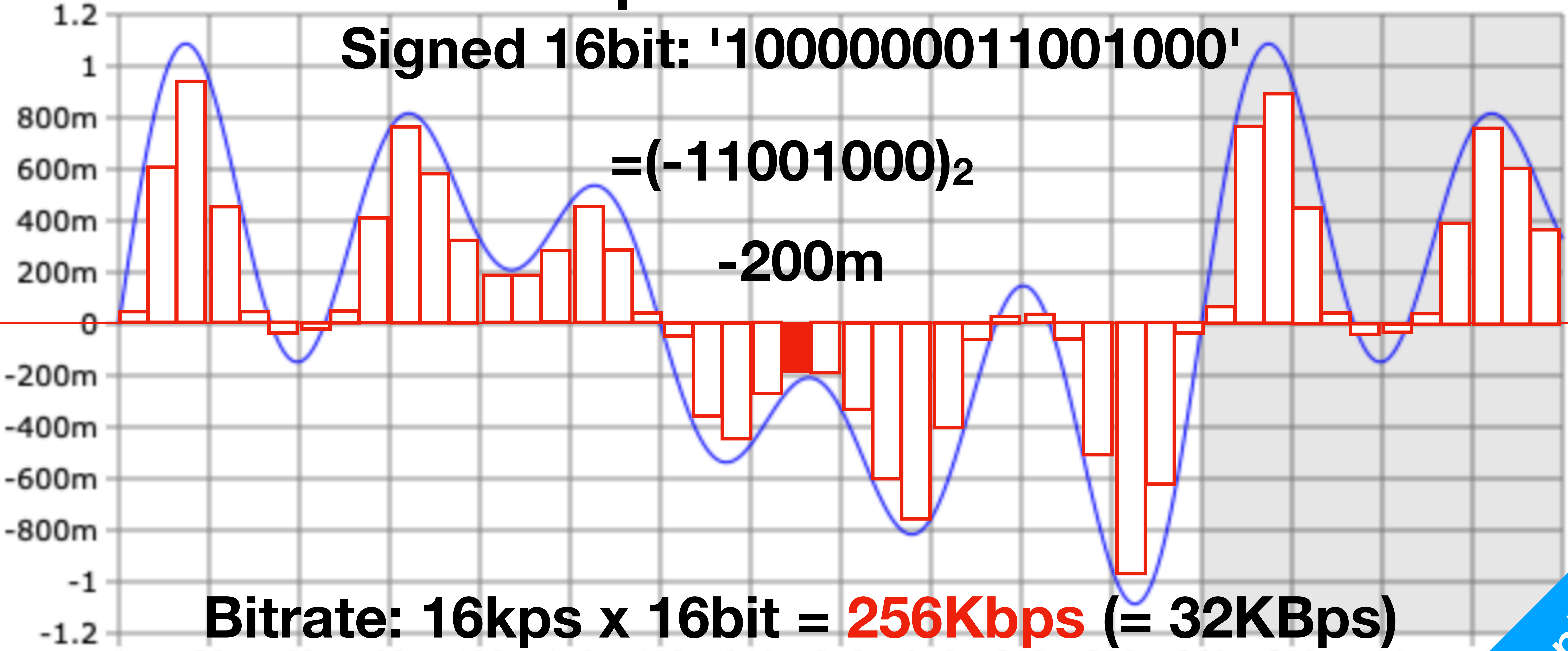

Bitrate: 16kps x 16bit = 256Kbps (= 32KBps)

440 Hz = 440 cycles per sec, Sample rate 36 x 440 = 15,840 = 16kps

Technical

P3
DA Conversion

Case Study: Audio Signal Representation

- Standard Mp3 audio file
  - Sampling rate: 44,100
  - Bitrate: 256 kBit/s (Kbps)
  - 4 min audio file size? $> 4 \times 60 \times 256 \div 8 = 7680\text{KB}$

Technical

P3
DA Conversion

Summary

- Digital to Analog Conversion
  - Frequency: number of cycles per second
  - Sample rate: number of samples per unit time
  - Bitrate: number of bits per second

Review