

# CSCI 150 Introduction to Digital and Computer System Design Lecture 3: Combinational Logic Design III



Jetic Gū 2020 Winter Semester (S1)

#### Overview

- Focus: Logic Functions
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch3 3.6; v5: Ch3 3.1, 3.4
- Core Ideas:
  - 1. Terminologies: Value-Fixing, Transferring, Inverting, Enabler
  - 2. Decoder



### Systematic Design Procedures

- **Specification**: Write a specification for the circuit
- 2. Formulation: Derive relationship between inputs and outputs of the system e.g. using truth table or Boolean expressions
- 3. Optimisation: Apply optimisation, minimise the number of logic gates and literals required
- 4. **Technology Mapping**: Transform design to new diagram using available implementation technology
- 5. **Verification**: Verify the correctness of the final design in meeting the specifications

### Systematic Design Procedures

- Hierarchical Design
  - Divide complex designs into smaller functional blocks, then apply the same 5-step design procedures for each block
  - Reusable, easier and more efficient Implementation

# Value-Fixing, Transferring, Inverting, Enabler

Elementary Combinational Logic Functions

# Value-Fixing, Transferring, and Inverting

- 1 Value-Fixing: giving a constant value to a wire
  - F = 0; F = 1;
- 2 Transferring: giving a variable (wire) value from another variable (wire)
  - F = X;
- 3 Inverting: inverting the value of a variable
  - $F = \overline{X}$

# Value-Fixing, Transferring, and Inverting



$$0 - F = 0$$

1 Value-Fixing





1 Value-Fixing

$$X - F = X$$

2 Transferring

$$X \longrightarrow F = \overline{X}$$

3 Inverting

COUCO.

#### Vector Denotation

#### 4 Multiple-bit Function

- Functions we've seen so far has only one-bit output: 0/1
- Certain functions may have *n*-bit output
  - $F(n-1:0) = (F_{n-1}, F_{n-2}, \dots, F_0)$ , each  $F_i$  is a one-bit function
  - Curtain Motor Control Circuit:  $F = (F_{\text{Motor}_1}, F_{\text{Motor}_2}, F_{\text{Light}})$

Concept.

#### Vector Denotation

```
0—F_3
```

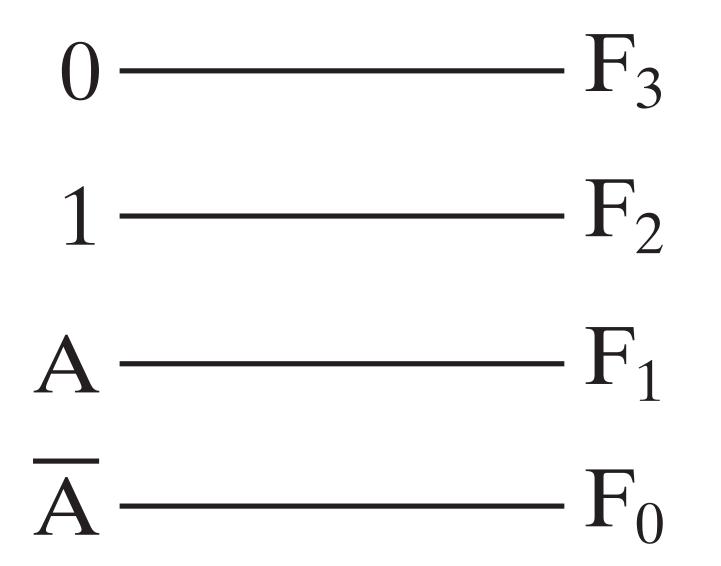
$$1 - F_2$$

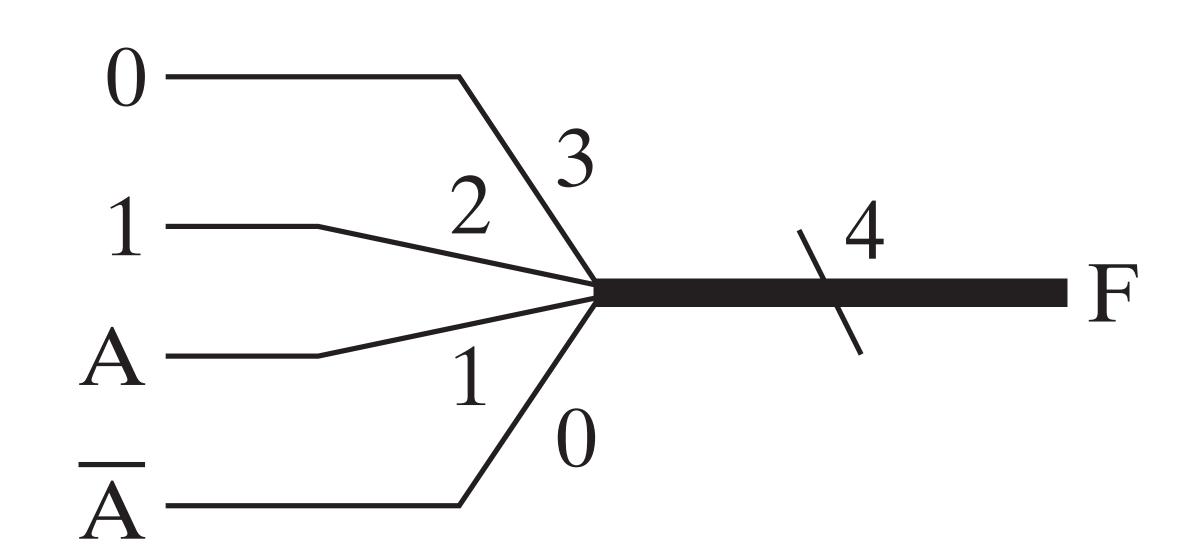
$$A$$
 —  $F_1$ 

$$\overline{A}$$
 —  $F_0$ 

P1 Elementary Func.

#### Vector Denotation

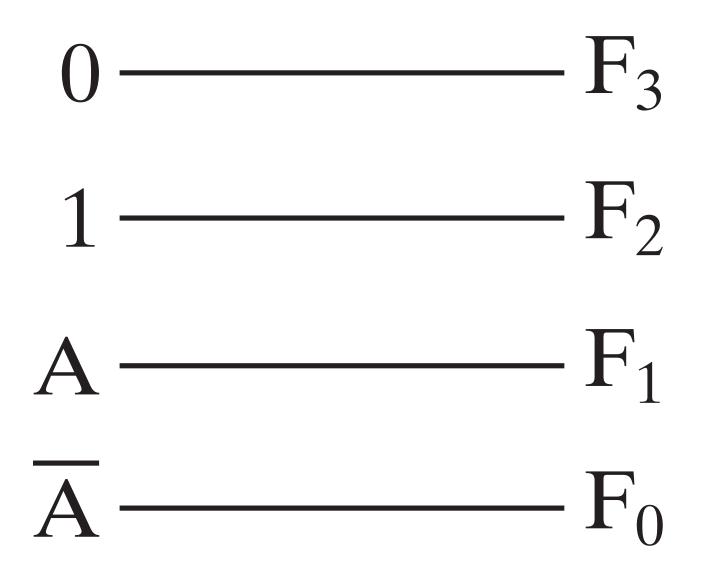


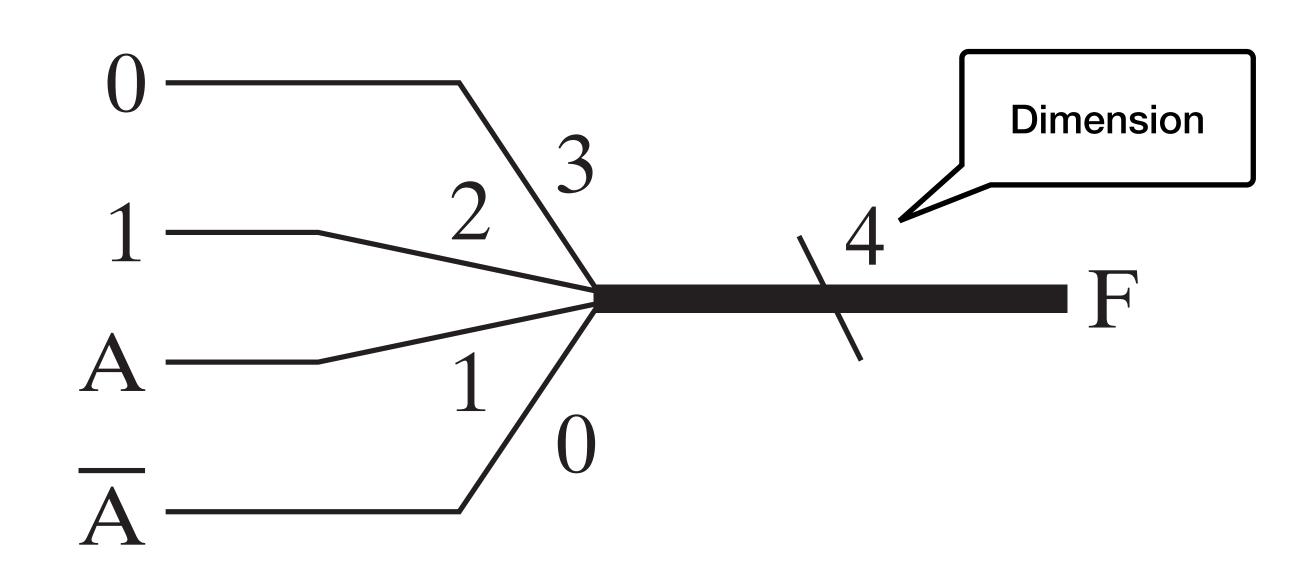


4 Multiple-bit Function

P1 Elementary Func.

#### Vector Denotation

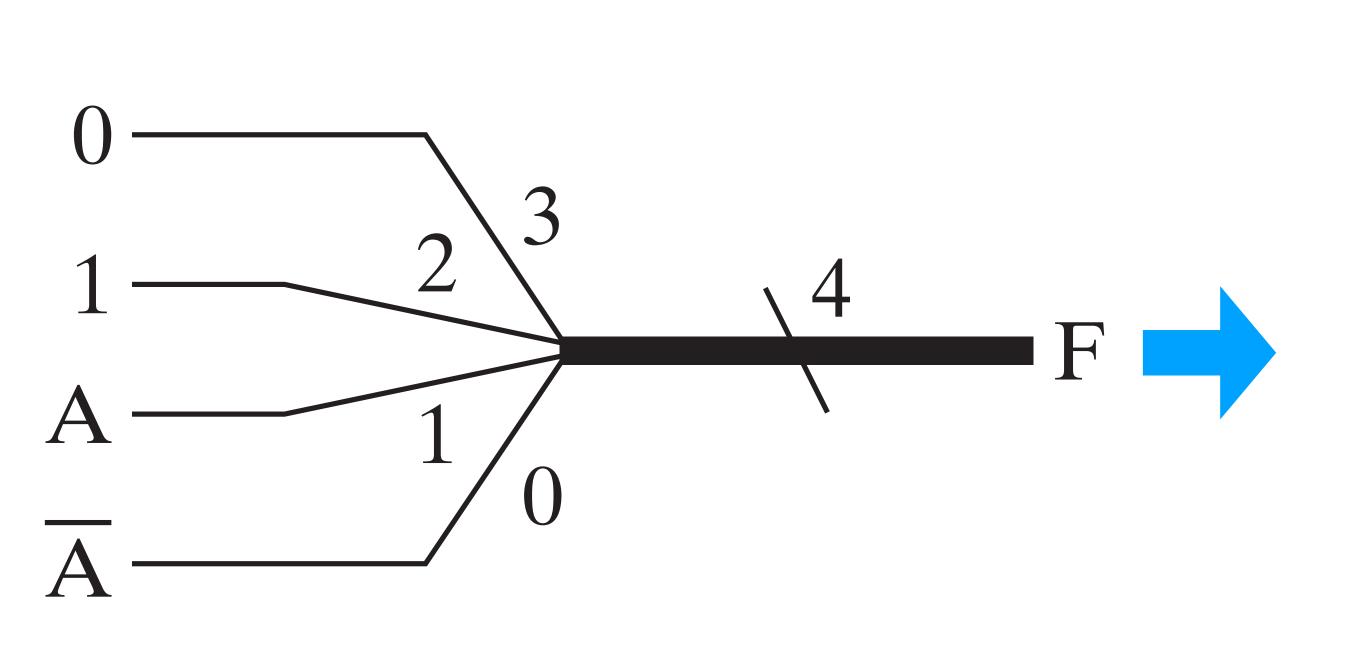




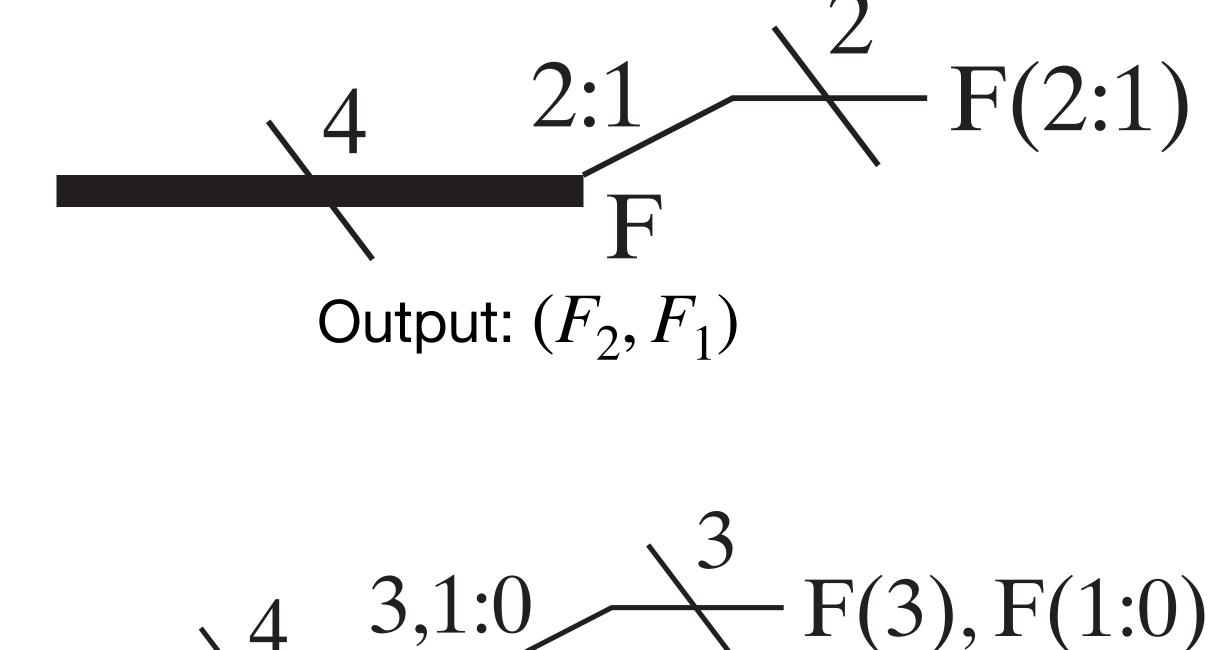
4 Multiple-bit Function

Couces

## Taking part of the Vector



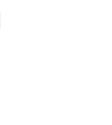
4 Multiple-bit Function



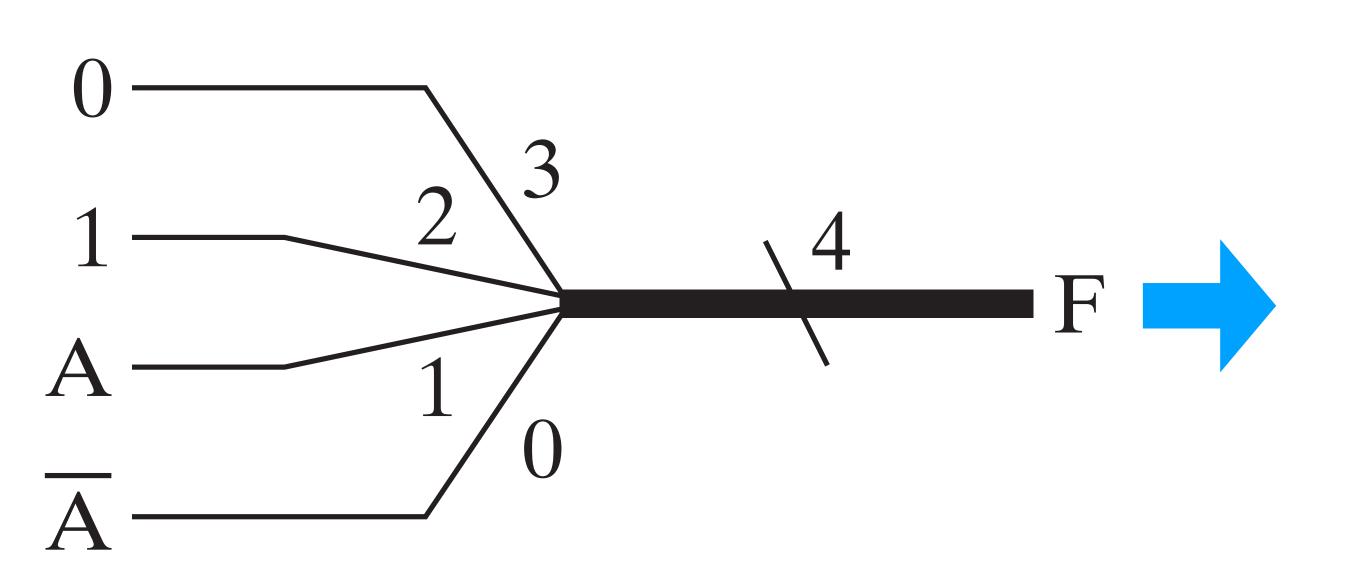
Output:  $(F_3, F_1, F_0)$ 

6000

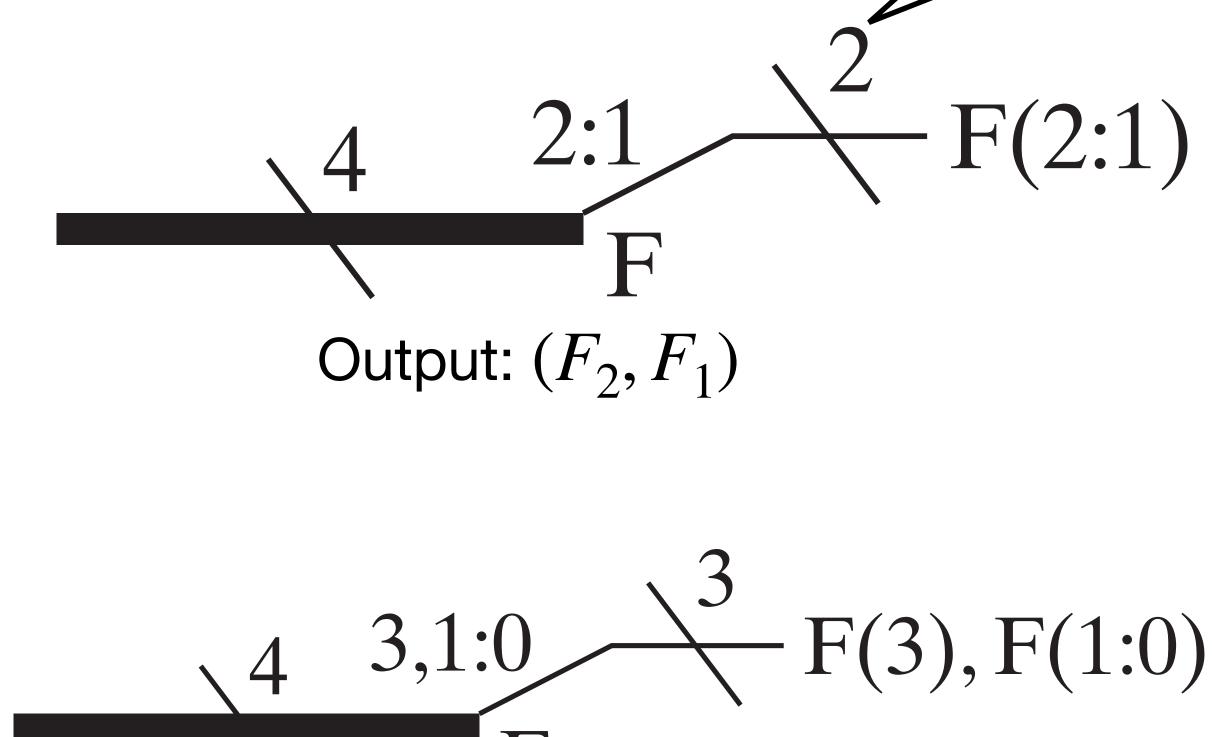
# Taking part of the Vector



**Dimension** 



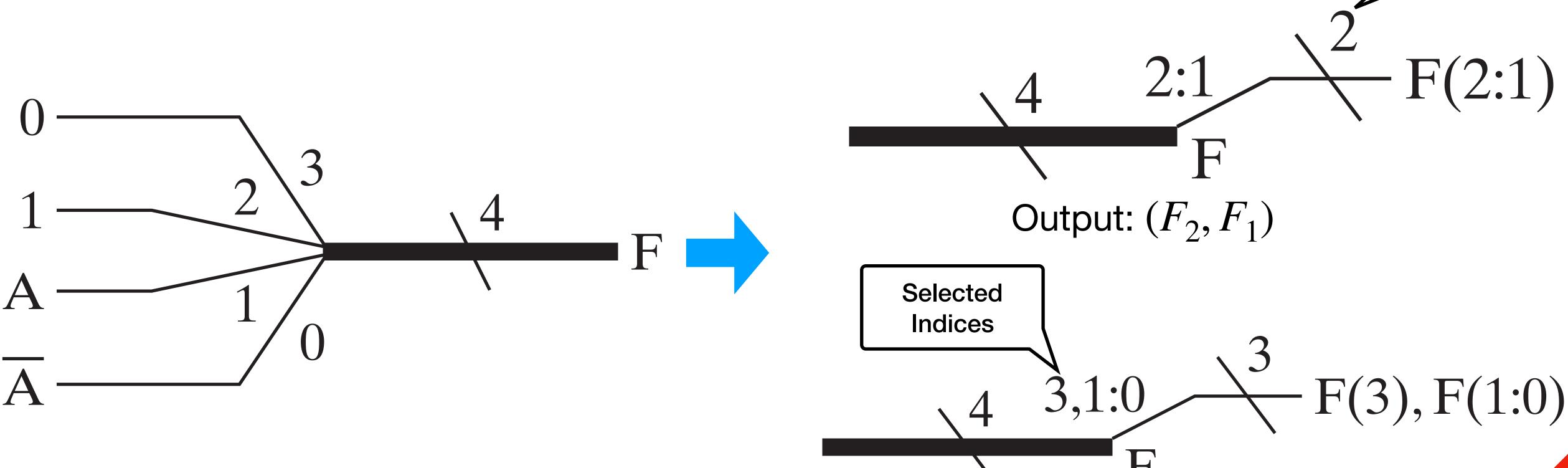
4 Multiple-bit Function



Output:  $(F_3, F_1, F_0)$ 

Elementary Func.

# Taking part of the Vector



4 Multiple-bit Function

Output:  $(F_3, F_1, F_0)$ 

**Dimension** 

#### **5** Enabler

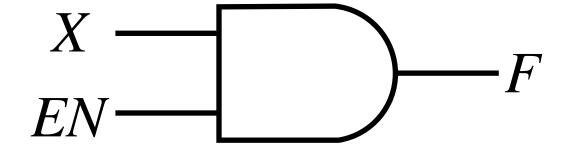
• Transferring function, but with an additional EN signal acting as switch

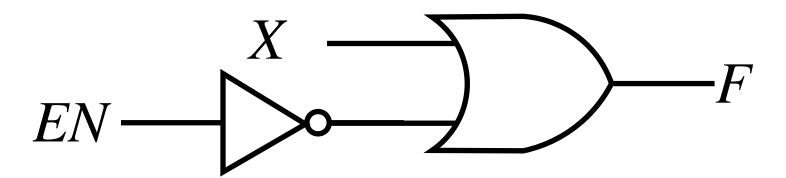
EN	X	F
0		0
1	0	0
1	1	1

Color Color

#### **5** Enabler

ullet Transferring function, but with an additional EN signal acting as switch

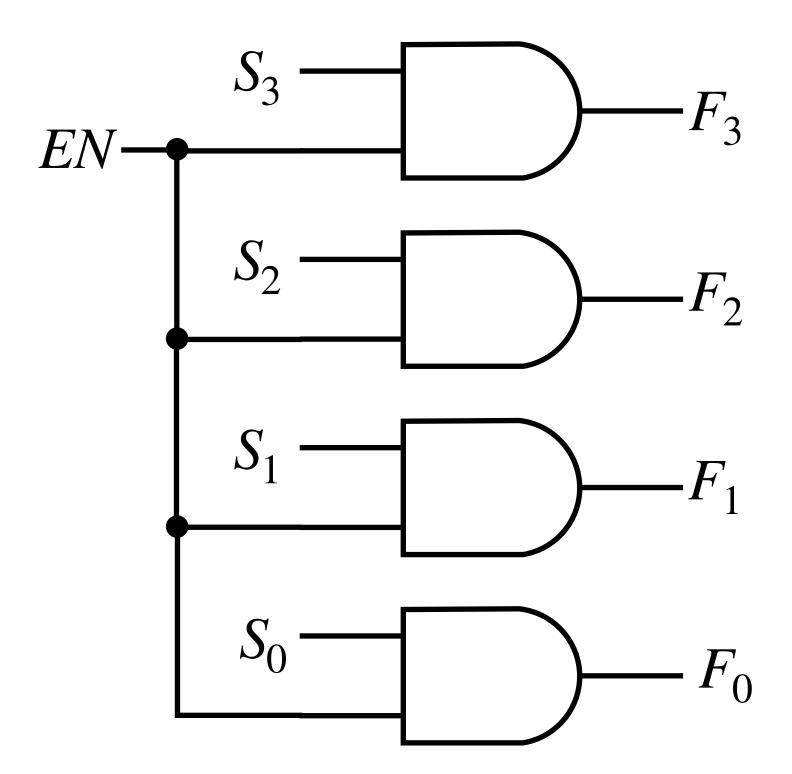




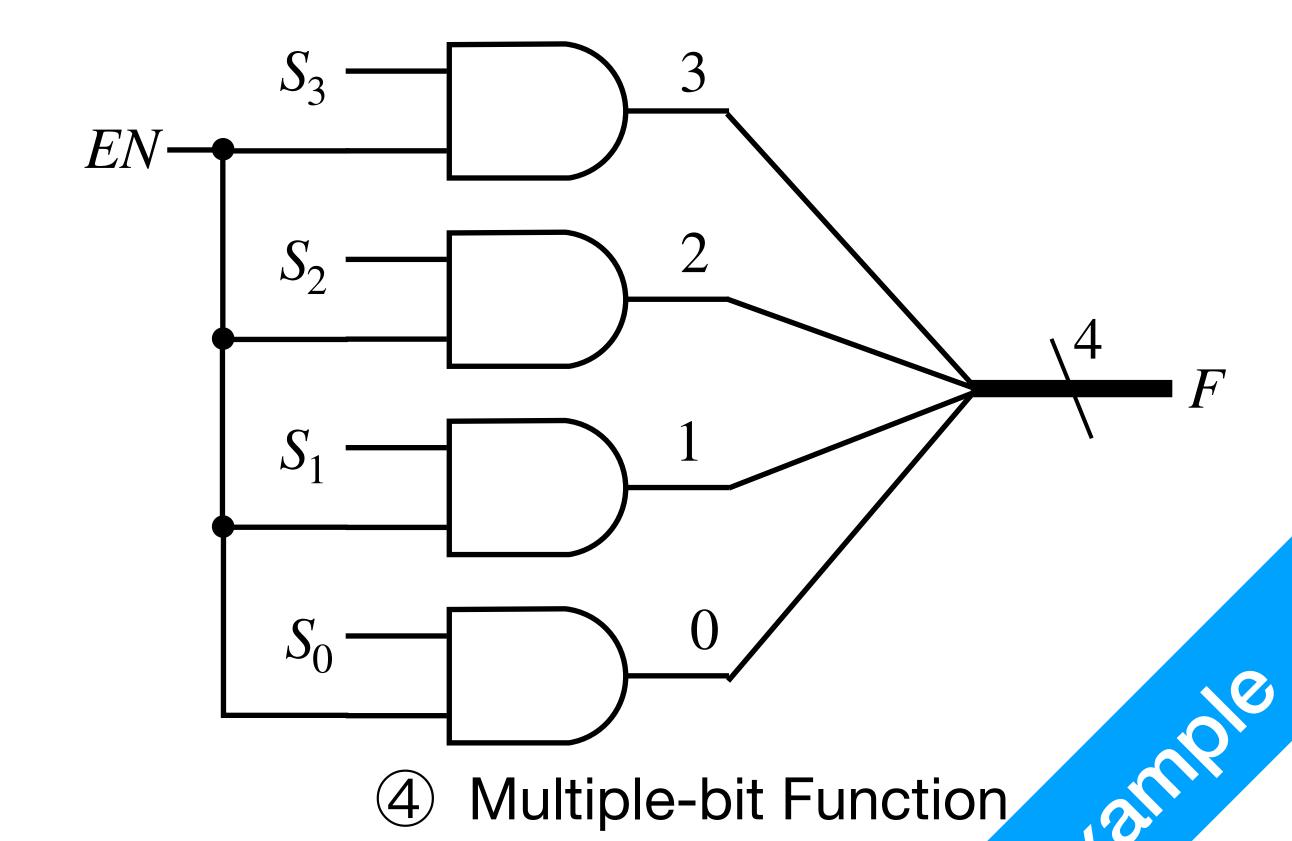
Color

- A building with individual lights F(3:0), and individual switches S(3:0)
  - $S_i$  controls  $F_i$
- Master switch: EN

- A building with individual lights F(3:0), and individual switches S(3:0)
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- A building with individual lights F(3:0), and individual switches S(3:0)
  - $S_i$  controls  $F_i$
- Master switch: EN



# Summary

- **1** Value-Fixing
- 2 Transferring
- 3 Inverting
- 4 Multiple-bit Function
- **5** Enabler

Summan

# Decoding

n-bit input,  $2^n$ -bit output

#### Decoder

- *n*-bit input
  - 2<sup>n</sup> different combinations
- Decoder
  - n-bit input, n- $2^n$  output each unique input produces a unique output

Decoder

# 1-to-2 Decoder - 1 x NOT Gate

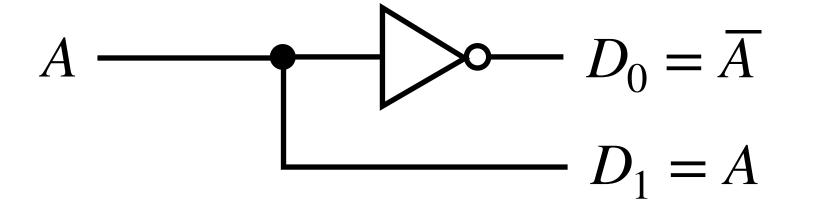
• 1bit input, 2bits output

A	D <sub>0</sub>	D <sub>1</sub>
0	1	0
1	0	

# 1-to-2 Decoder - 1 x NOT Gate

• 1bit input, 2bits output

A	D <sub>0</sub>	D <sub>1</sub>
0	1	0
1	0	1



# 2-to-4 Decoder - 2 x NOT Gate

- 4 x 2-input AND Gate

- 2bit input, 4bits output
  - $D_i = m_i$

A <sub>1</sub>	A <sub>0</sub>	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

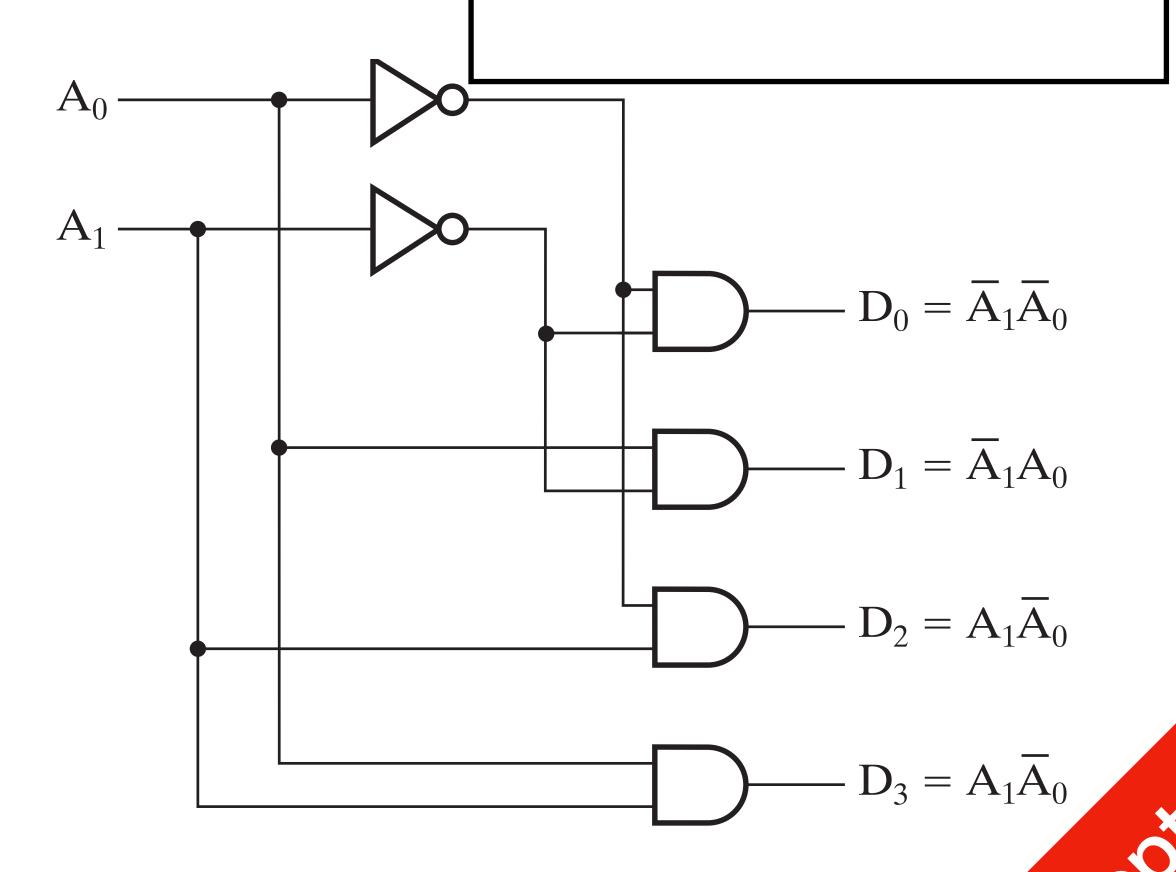
# 2-to-4 Decoder - 2 x NOT Gate

Technology

- $(k + 1)/4 \times 2$  (input) AND Gate

- 2bit input, 4bits output
  - $D_i = m_i$

A <sub>1</sub>	A <sub>0</sub>	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1



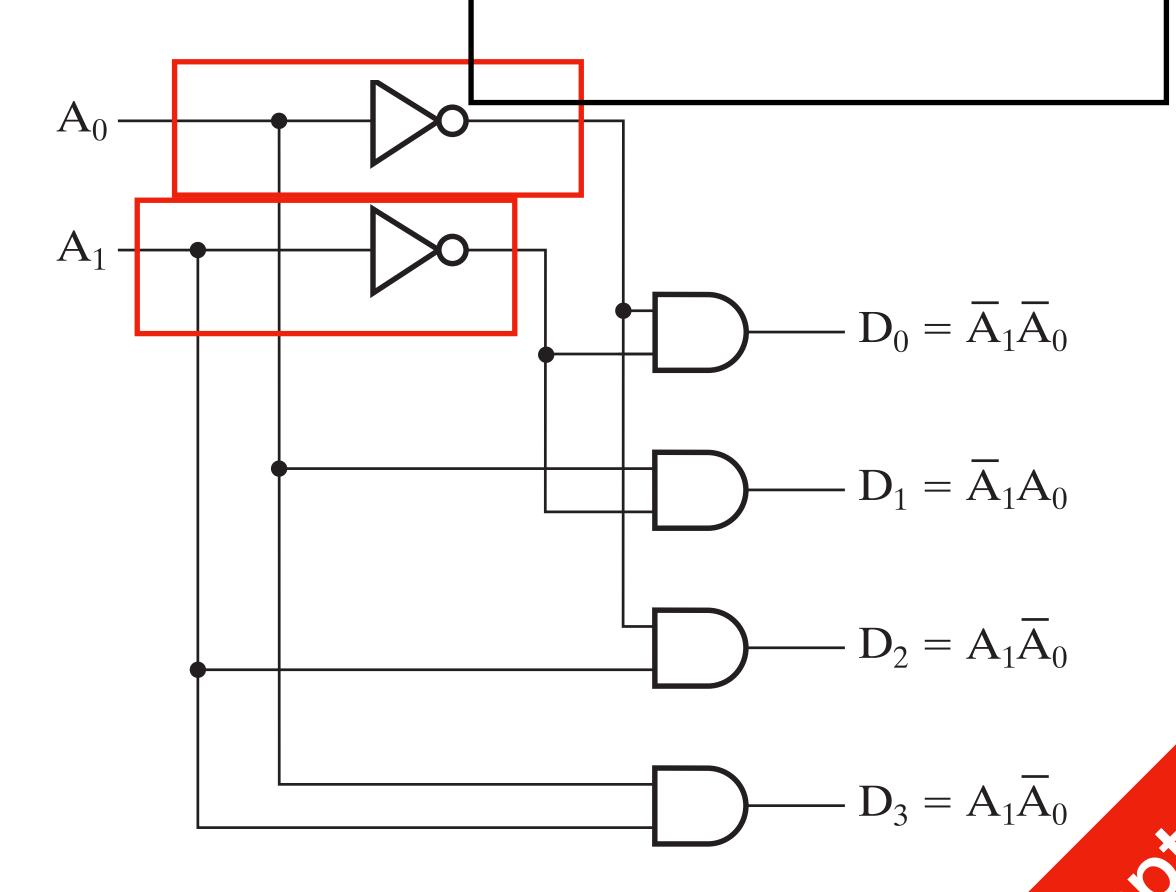
P2 Decoder

# $2^k \bar{t}o-4$ Decoder - 2 x NOT Gate

- Technology
- $(k + 1)/4 \times 2$  (input) AND Gate

- 2bit input, 4bits output
  - $D_i = m_i$

A <sub>1</sub>	A <sub>0</sub>	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

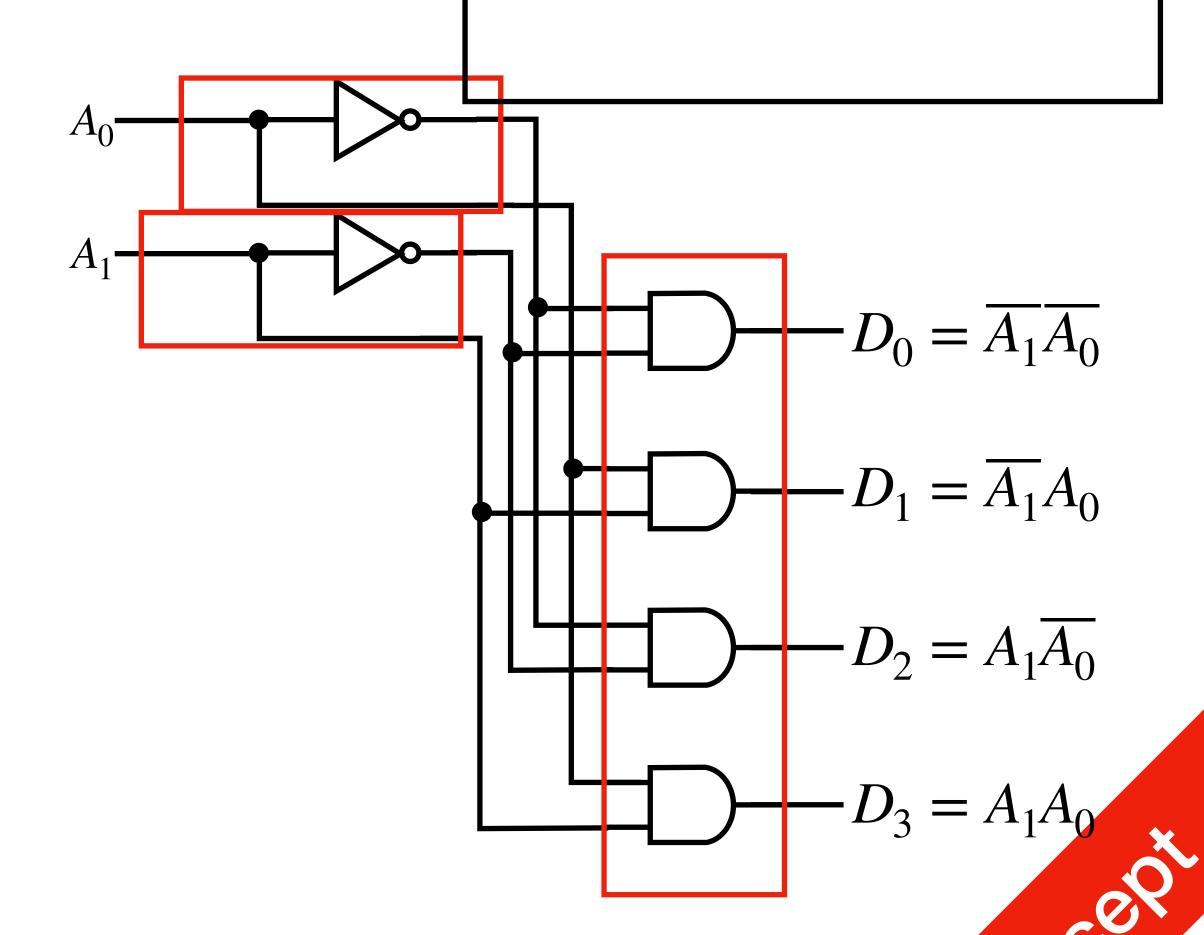


## 2-to-4 Decoder - 2 x 1-to-2 Decoder

- Technology
- 4 x 2-input AND Gate

- 2bit input, 4bits output
  - $D_i = m_i$

A <sub>1</sub>	A <sub>0</sub>	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1



# 3-to-8 Decoder - 1 x 1-to-2 Decoder

- 3bit input, 8bits output
  - $D_i = m_i$

A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

- 1 x 2-to-4 Decoder
- 8 x 2-input AND Gate

# 3-to-8 Decoder Technology 1 x 1-to-2 Decoder

- 3bit input, 8bits output
  - $D_i = m_i$

A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0			0	0			0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

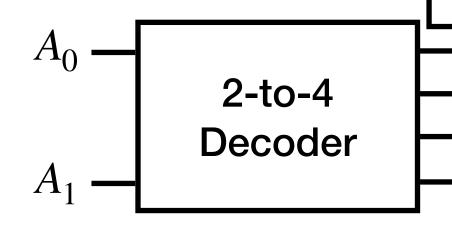
- 1 x 2-to-4 Decoder
- 8 x 2-input AND Gate

## 3-to-8 Decoder.

- Technology
- 1 x 1-to-2 Decoder
- 1 x 2-to-4 Decoder
- 8 x 2-input AND Gate

- 3bit input, 8bits output
  - $D_i = m_i$

$A_2$	A <sub>1</sub>	$A_0$	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>
	0	0	1	0	0	0	0	0	0	0
	0	1	0	1	0	0	0	0	0	0
U	1	0	0	0	1	0	0	0	0	0
	1	1	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	1	0	0	0
4	0	1	0	0	0	0	0	1	0	0
	1	0	0	0	0	0	0	0	1	0
	1	1	0	0	0	0	0	0	0	1

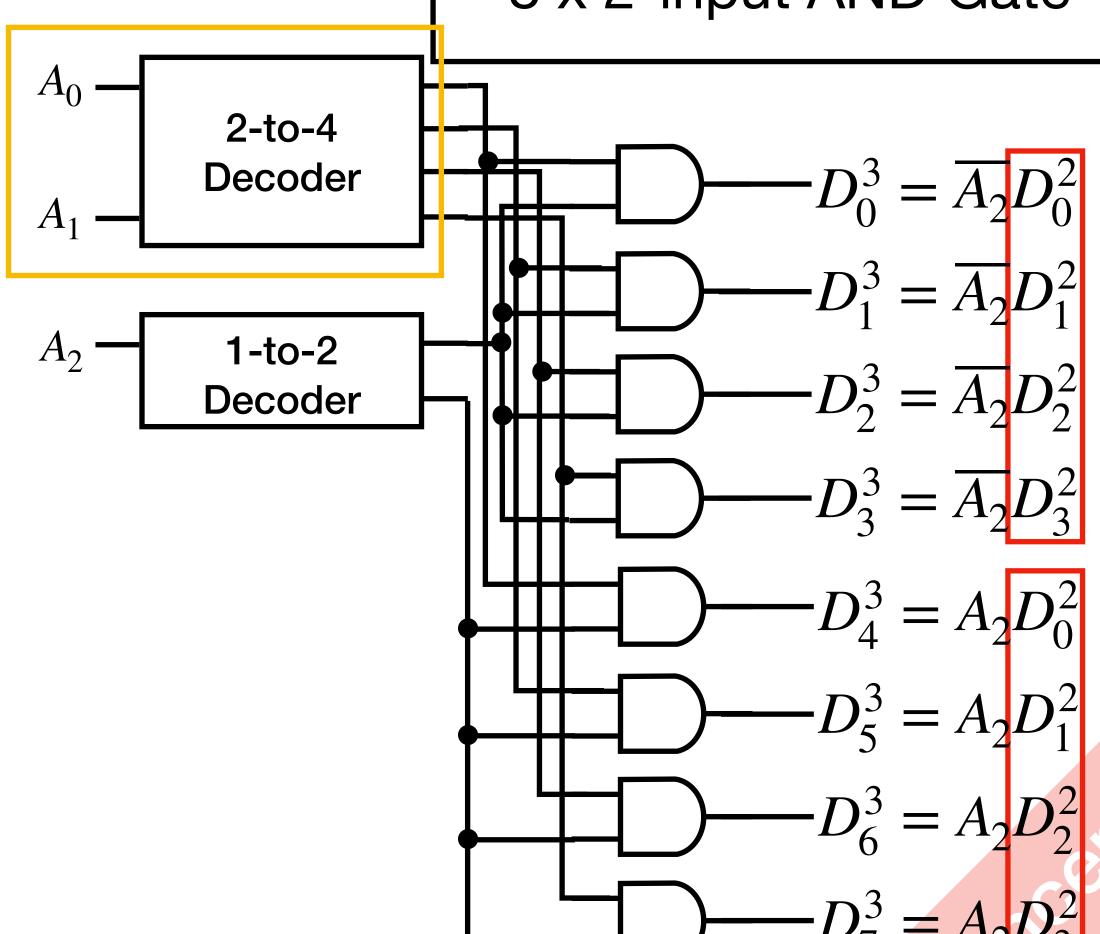


## 3-to-8 Decoder - 1 x 1-to-2 Decoder

- Technology
- 1 x 2-to-4 Decoder
- 8 x 2-input AND Gate

- 3bit input, 8bits output
  - $D_i = m_i$

A <sub>2</sub>	A <sub>1</sub>	$A_0$	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>
	0						0		0	0
							0	0	0	0
U		0						0	0	0
	1	1	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	1	0	0	0
4	0	1	0	0	0	0		1		0
	1	0	0	0	0	0	0	0	1	0
	1	1	0	0	0	0	0	0	0	1



### 3-to-8 Decoder.

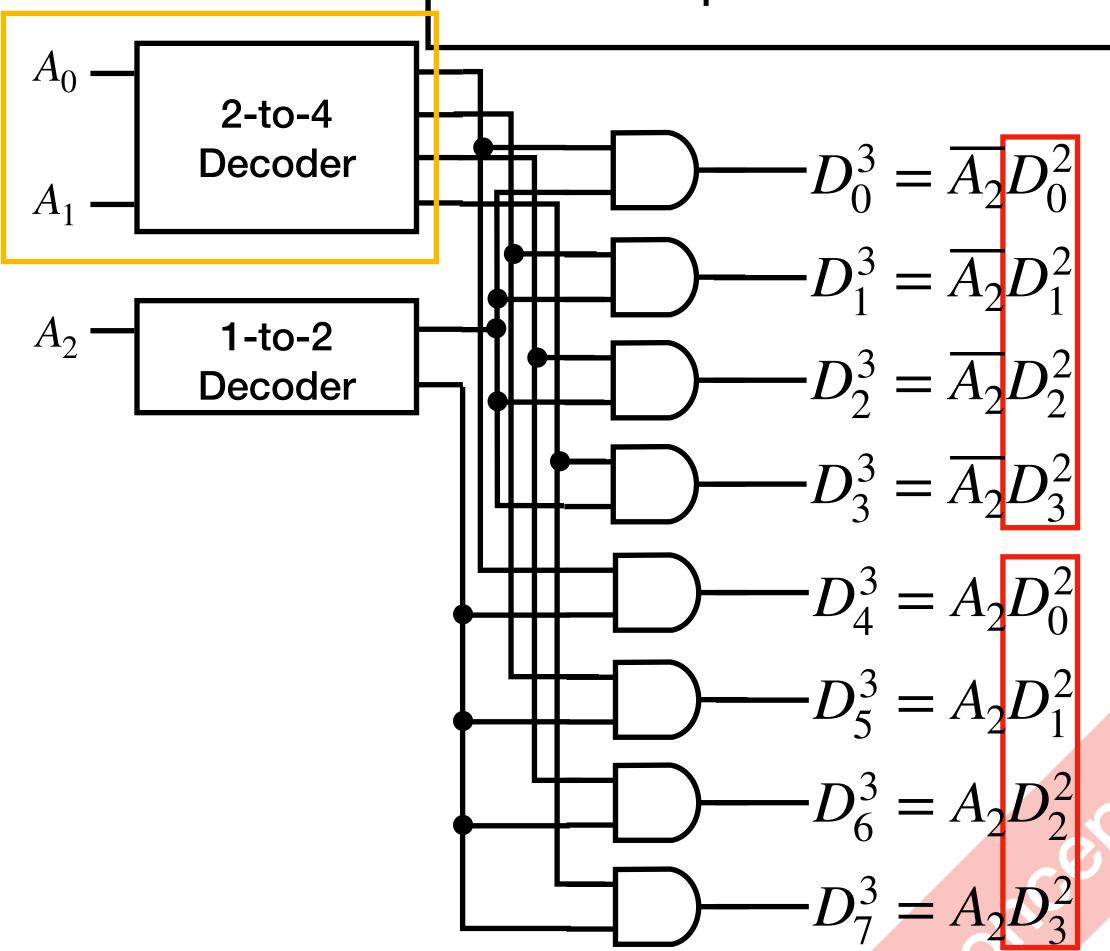
#### **Incremental Design**

- 3bit input, 8bits output
  - $D_i = m_i$

A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>
	<u> </u>	en e					0	0	0	0
	0	1	0		0	0	0	0	0	0
U	1	0			1	0	0	0	0	0
	1	1	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	1	0	0	0
	0	1	0	0	0	0	0	1	0	0
	1	0	0	0	0	0	0	0	1	0
	1	1	0	0	0	0	0	0	0	0

#### Technology

- 1 x 1-to-2 Decoder
- 1 x 2-to-4 Decoder
- 8 x 2-input AND Gate



Decoder

# 4-to-16 Decoder Technology 1 x 1-to-2 Decoder

**Incremental Design** 

• 4bit input, 16bits output

• 
$$D_i = m_i$$

<b>A</b> <sub>3</sub>				D <sub>0</sub>							
0				1							
	0	0	1	0	1	0	0	0	0	0	0
	0	1	0	0	0	1	0	0	0	0	0
	0	1	1	0	0	0	1	0	0	0	0
	1			0							
	1			0							
	1	1	0	0	0	0	0	0	0	1	0
	1	1	1	0	0	0	0	0	0	0	1

1 x 3-to-8 Decoder

• 16 x 2-input AND Gate

<b>A</b> <sub>3</sub>	A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>	D <sub>8</sub>	D <sub>9</sub>	D <sub>10</sub>	D <sub>11</sub>	D <sub>12</sub>	<b>D</b> <sub>13</sub>	D <sub>14</sub>	D <sub>15</sub>
1			0	1	0	0	0	0	0	0	0
	0	0	1	0	1	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0
	0	1	1	0	0	0	1	0	0	0	0
	1	0	0	0	0	0	0	1	0	0	0
	1	0	1	0	0	0	$\mathbf{O}$	0	1	0	0
	1	1	0	0	0	0	0	0	0	1	0
	1	1	1	0	0	0	0	0	0	0	_1

# 4-to-16 Decoder 1 x 1-to-2 Decoder

**Incremental Design** 

4bit input, 16bits output

• 
$$D_i = m_i$$

$$D_{0:7}^4 = \overline{A_3} D_{0:7}^3$$

$$D_{0:7}^4 = \overline{A_3}D_{0:7}^3 \quad D_{8:15}^4 = A_3D_{0:7}^3$$

Technology

- 1 x 3-to-8 Decoder
- 16 x 2-input AND Gate

<b>A</b> <sub>3</sub>	A <sub>2</sub>	A <sub>1</sub>	$A_0$	$D_0$	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>
								0			0
	0	0	1	0	1	0	0	0	0	0	0
	0	1	0	0	0	1	0	0	0	0	0
	0	1	1	0	0	0	1	0	0	0	0
U	1	0	0	0	0	0	0	1	0	0	0
			1					0			0
	1	1	0	0	0	0	0	0	0	1	0
	1	1	1	0	0	0	0	0	0	0	1

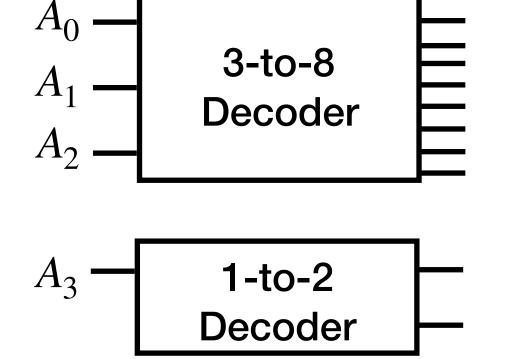
<b>A</b> <sub>3</sub>	A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>	D <sub>8</sub>	D <sub>9</sub>	<b>D</b> <sub>10</sub>	D <sub>11</sub>	D <sub>12</sub>	<b>D</b> <sub>13</sub>	D <sub>14</sub>	<b>D</b> <sub>15</sub>
1			0	1	0	0	0	0	0	0	0
	0	0	1	0	1	0	0	0	0	0	0
	0	1	0	0	0	1	0	0	0	0	0
	0	1	1	0	0	0	1	0	0	0	0
	1	0	0	0	0	0	0	1	0	0	0
	1	0	1	0	0	0	0	0	1	0	0
	1	1	0	0	0	0	0	0	0	1	0
	1	1	1	0	0	0	0	0	0	0	1

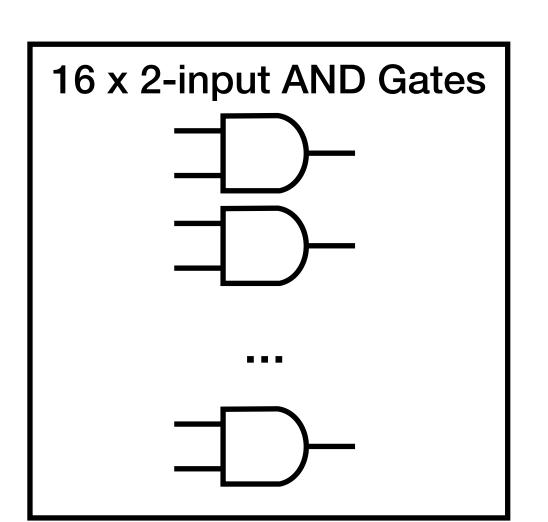
### 4-to-16 Decoder 1 x 1-to-2 Decoder

**Incremental Design** 

• 4bit input, 16bits output

• 
$$D_i = m_i$$





#### Technology

- 1 x 3-to-8 Decoder
- 16 x 2-input AND Gate

$$D_{0:7}^4 = \overline{A_3} D_{0:7}^3$$

$$D_{8:15}^4 = A_3 D_{0:7}^3$$

# 4-to-16 Decoder Technology 2 x 2-to-4 Decoder

16 x 2-input AND Gate

#### **Recursive Design**

• 4bit input, 16bits output

• 
$$D_i = m_i$$

$$D_{0:7}^4 = \overline{A_3} D_{0:7}^3$$

$$D_{0:7}^4 = \overline{A_3}D_{0:7}^3 \quad D_{8:15}^4 = A_3D_{0:7}^3$$

<b>A</b> <sub>3</sub>	A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>
		0	0	1	0	0	0	0	0	0	0
	0	0	1	0	1	0	0	0	0	0	0
	U	1	0	0	0	1	0	0	0	0	0
		1	1	0	0	0	1	0	0	0	0
U		0	0	0	0	0	0	1	0	0	0
		0	1	0	0	0	0	0	1	0	0
		1	0	0	0	0	0	0	0	1	0
		1	1	0	0	0	0	0	0	0	1

<b>A</b> <sub>3</sub>	A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>	D <sub>8</sub>	D <sub>9</sub>	D <sub>10</sub>	D <sub>11</sub>	D <sub>12</sub>	D <sub>13</sub>	<b>D</b> <sub>14</sub>	D <sub>15</sub>
1		0	0	1	0	0	0	0	0	0	0
		0	1	0	1	0	0	0	0	0	0
	U	1				1					0
		1	1	0	0	0	1	0	0	0	0
		0	0	0	0	0	0	1	0	0	0
	1	0	1	0	0	0	0	0	1	0	0
		1	0	0	0	0	0	0	0	1	0
		1	1	0	0	0	0	0	0	0	0 0 1

Technology

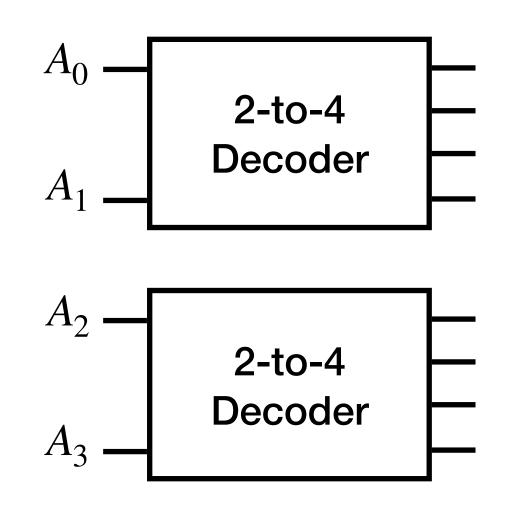
16 x 2-input AND Gate

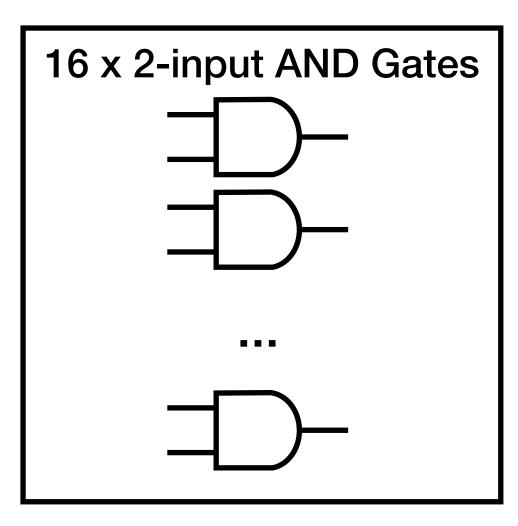
#### **Recursive Design**

• 4bit input, 16bits output

• 
$$D_i = m_i$$

$$D_{0:7}^4 = \overline{A_3}D_{0:7}^3 \quad D_{8:15}^4 = A_3D_{0:7}^3$$





$$D_{0:3}^{4} = \overline{A_3} \overline{A_2} D^2$$

$$D_{4:7}^{4} = \overline{A_3} A_2 D^2$$

$$D_{8:11}^{4} = A_3 \overline{A_2} D^2$$

$$D_{12:15}^{4} = A_3 A_2 D^2$$

# 4-to-16 Decoder Technology 2 x 2-to-4 Decoder

**Recursive Design** 

- 4bit input, 16bits output
  - $D_i = m_i$

$$D_{0:7}^4 = \overline{A_3} D_{0:7}^3$$

$$D_{0:7}^4 = \overline{A}_3 D_{0:7}^3$$

$$D_{8:15}^4 = A_3 D_{0:7}^3$$

# 4-to-16 Decoder Technology 2 x 2-to-4 Decoder

**Recursive Design** 

- 4bit input, 16bits output
  - $D_i = m_i$

$$D^{4} = D_{0:15}^{4} = [\overline{A_3}D_{0:7}^{3}; A_3D_{0:7}^{3}] = [\overline{A_3}D^{3}; A_3D^{3}]$$

**Bracket: concatenation of vectors** 

# 4-to-16 Decoder Example 16 Technology 2 x 2-to-4 Decoder

**Recursive Design** 

- 4bit input, 16bits output
  - $D_i = m_i$

$$D^4 = [\overline{A_3}D^3; A_3D^3]$$

**P2** Decoder

### 4-to-16 Decoder 2 x 2-to-4 Decoder

Technology

- 16 x 2-input AND Gate

#### **Recursive Design**

• 4bit input, 16bits output

$$D^4 = [\overline{A_3}D^3; A_3D^3]$$

•  $D_i = m_i$ 

$$D^{3} = \begin{bmatrix} \overline{A_{2}}D_{0}^{2} \\ \overline{A_{2}}D_{1}^{2} \\ \overline{A_{2}}D_{2}^{2} \\ \overline{A_{2}}D_{3}^{2} \end{bmatrix}; \begin{bmatrix} A_{2}D_{0}^{2} \\ A_{2}D_{1}^{2} \\ A_{2}D_{2}^{2} \\ A_{2}D_{3}^{2} \end{bmatrix}] = [\overline{A_{2}} \begin{bmatrix} D_{0}^{2} \\ D_{1}^{2} \\ D_{2}^{2} \\ D_{3}^{2} \end{bmatrix}; A_{2} \begin{bmatrix} D_{0}^{2} \\ D_{1}^{2} \\ D_{2}^{2} \\ D_{3}^{2} \end{bmatrix}] = [\overline{A_{2}}D^{2}; A_{2}D^{2}]$$

### Technology

16 x 2-input AND Gate

#### **Recursive Design**

• 4bit input, 16bits output

$$D^4 = [\overline{A_3}D^3; A_3D^3]$$

• 
$$D_i = m_i$$

$$D^3 = [\overline{A_2}D^2; A_2D^2]$$

#### **Recursive Design**

- 4bit input, 16bits output
  - $D_i = m_i$

### 4-to-16 Decoder 2 x 2-to-4 Decoder

### Technology

- 16 x 2-input AND Gate

$$D^{4} = [\overline{A_{3}}D^{3}; A_{3}D^{3}]$$

$$D^{3} = [\overline{A_{2}}D^{2}; A_{2}D^{2}]$$

**Recursive Design** 

- 4bit input, 16bits output
  - $D_i = m_i$

$$D^3 = [\overline{A_2}D^2; A_2D^2]$$

$$D^4 = [\overline{A_3}D^3; A_3D^3]$$

### Technology

- 16 x 2-input AND Gate

Technology

16 x 2-input AND Gate

#### **Recursive Design**

- 4bit input, 16bits output
  - $D_i = m_i$

$$D^3 = [\overline{A_2}D^2; A_2D^2]$$

$$D^{4} = [\overline{A_{3}}D^{3}; A_{3}D^{3}] = [\overline{A_{3}}[\overline{A_{2}}D^{2}; A_{2}D^{2}]; A_{3}[\overline{A_{2}}D^{2}; A_{2}D^{2}]]$$

Technology

- 16 x 2-input AND Gate

#### **Recursive Design**

- 4bit input, 16bits output
  - $D_i = m_i$

$$D^3 = [\overline{A_2}D^2; A_2D^2]$$

$$D^{4} = [\overline{A_{3}}D^{3}; A_{3}D^{3}] = [\overline{A_{3}}[\overline{A_{2}}D^{2}; A_{2}D^{2}]; A_{3}[\overline{A_{2}}D^{2}; A_{2}D^{2}]]$$
$$= [\overline{A_{3}}\overline{A_{2}}D^{2}; \overline{A_{3}}A_{2}D^{2}]; [A_{3}\overline{A_{2}}D^{2}; A_{3}A_{2}D^{2}]]$$

Technology

16 x 2-input AND Gate

### **Recursive Design**

- 4bit input, 16bits output
  - $D_i = m_i$

$$D^3 = [\overline{A_2}D^2; A_2D^2]$$

$$D^{4} = [\overline{A_{3}}D^{3}; A_{3}D^{3}] = [\overline{A_{3}}[\overline{A_{2}}D^{2}; A_{2}D^{2}]; A_{3}[\overline{A_{2}}D^{2}; A_{2}D^{2}]]$$
$$= [\overline{A_{3}}\overline{A_{2}}D^{2}; \overline{A_{3}}A_{2}D^{2}]; [A_{3}\overline{A_{2}}D^{2}; A_{3}A_{2}D^{2}]]$$

= 
$$[\overline{A_3}\overline{A_2}D^2; \overline{A_3}A_2D^2; A_3\overline{A_2}D^2; A_3A_2D^2]$$

**Recursive Design** 

- 4bit input, 16bits output
  - $D_i = m_i$

$$D^3 = [\overline{A_2}D^2; A_2D^2]$$

$$D^{4} = [\overline{A_{3}}D^{3}; A_{3}D^{3}] = [\overline{A_{3}}[\overline{A_{2}}D^{2}; A_{2}D^{2}]; A_{3}[\overline{A_{2}}D^{2}; A_{2}D^{2}]]$$

= 
$$[\overline{A_3}\overline{A_2}D^2; \overline{A_3}A_2D^2]; [A_3\overline{A_2}D^2; A_3A_2D^2]]$$

= 
$$[\overline{A_3}\overline{A_2}D^2; \overline{A_3}A_2D^2; A_3\overline{A_2}D^2; A_3A_2D^2]$$

$$= [\overline{A_3}\overline{A_2}; \overline{A_3}A_2; A_3\overline{A_2}; A_3\overline{A_2}; A_3A_2]D^2$$

Technology

### Summary

- What is a decoder
- Truth table of a decoder
- Implementation of 1-to-2, 2-to-4, *n*-to-2<sup>n</sup> decoder
  - Incremental design
  - Recursive design

## 1-bit Binary Adder

Using decoder

## Binary Addition

```
Carries Z 11010
Augend X 01101
Addend Y +00101
Sum 10010
```

## Binary Addition

Carries 
$$Z$$
  $11010$ 
Augend  $X$   $01101$ 
Addend  $Y$   $+00101$ 
Sum  $10010$ 

## 1-bit Binary Adder

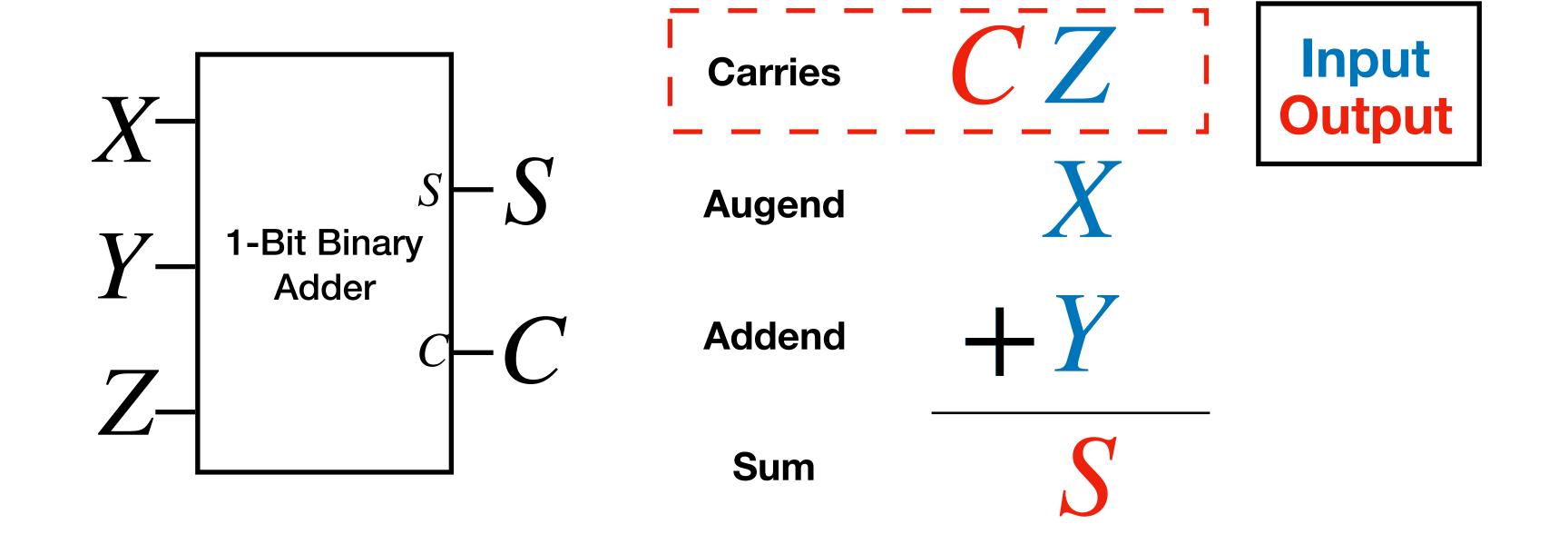
- Binary Adder
  - Logical functional block that performs addition
- 1-Bit Binary Adder: smallest building block of a multi-bit adder
  - Inputs

Augend: X; Addend: Y; Previous Carry: Z

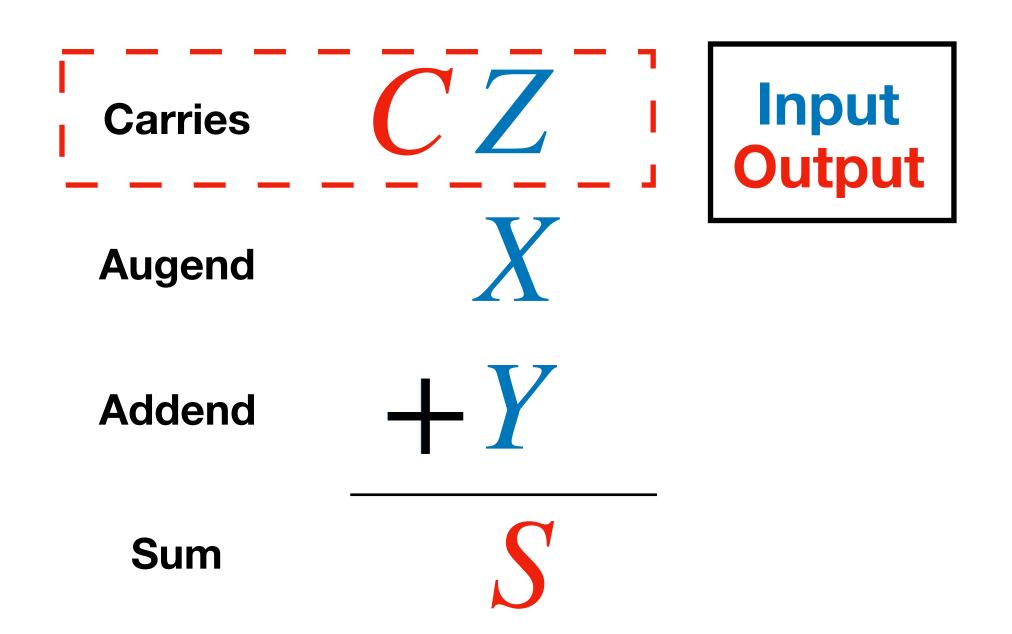
Outputs

Sum bit: S; next carry: C

### 1. Specification

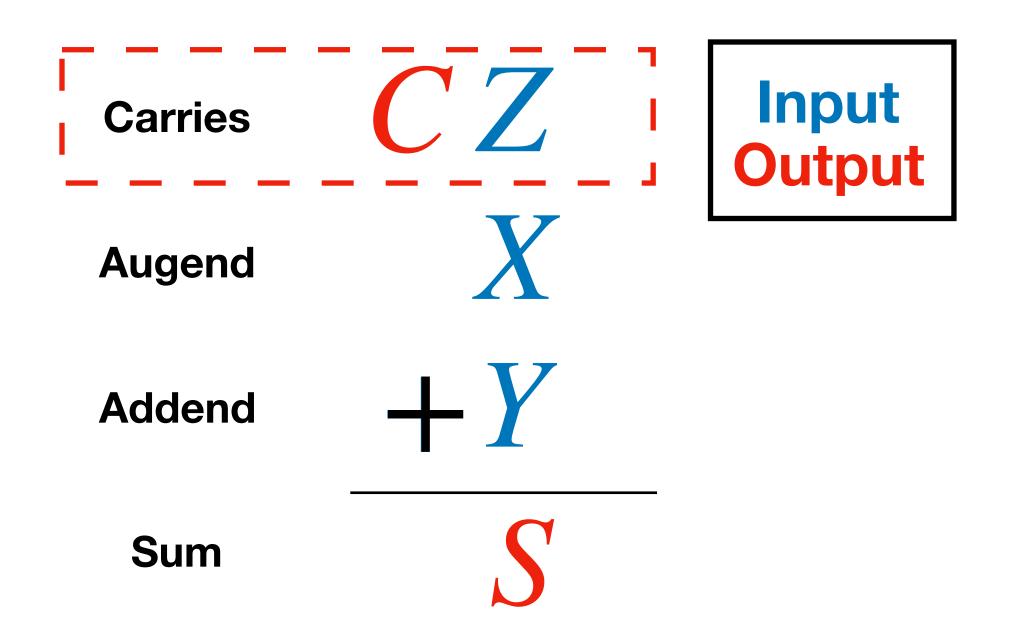


### 2. Formulation



X	Y	Z	S	C
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

### 2. Formulation



X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

### 2. Formulation

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$C = \Sigma m(3,5,6,7)$$

$$S = \Sigma m(1,2,4,7)$$

### 3. Optimisation

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

#### **Sum of Minterms**

$$C = \sum m(3,5,6,7)$$

$$S = \Sigma m(1,2,4,7)$$

We can use decoders to implement the adder!

Sign P

### 3. Optimisation

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

#### **Sum of Minterms**

$$C = \sum m(3,5,6,7)$$

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We can use decoders to implement the adder!

Sign P

# 4. Technology Mapping : 1 x 3-to-8 OR Gates

Technology

$$C = \Sigma m(3,5,6,7)$$

$$S = \Sigma m(1,2,4,7)$$

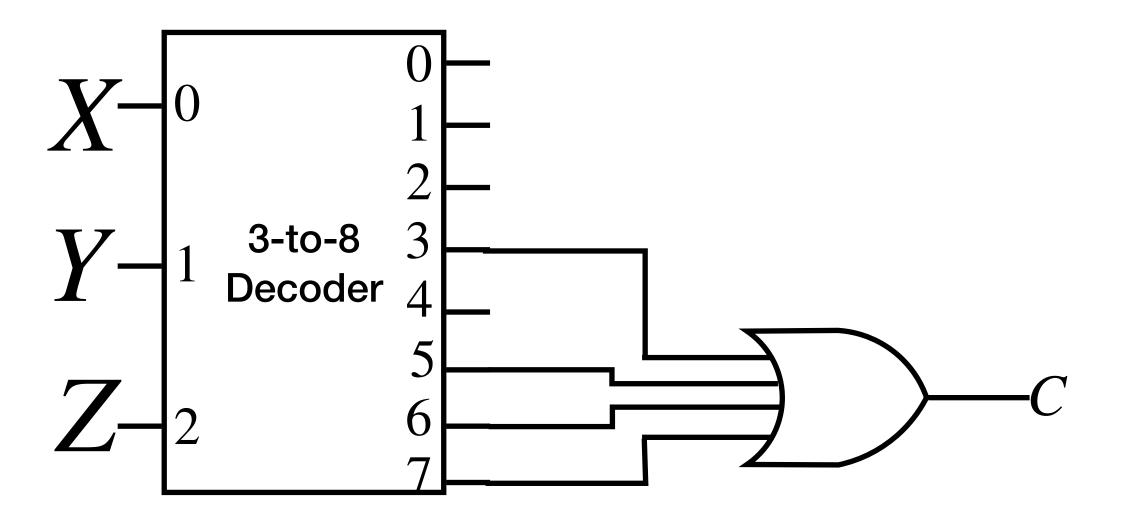
$$X$$
 = 0 1 - 2 - 2  $Y$  = 1 3-to-8 3 - 2  $Z$  = 2 5 - 2  $Z$  = 2

# 4. Technology Mapping : 1 x 3-to-8 Decoder OR Gates

Technology

$$C = \Sigma m(3,5,6,7)$$

$$S = \Sigma m(1,2,4,7)$$



# 4. Technology Mapping : 1 x 3-to-8 Decoder OR Gates

Technology

$$C = \Sigma m(3,5,6,7)$$

$$S = \Sigma m(1,2,4,7)$$

