



04.02.20 14:27

# CSCI 150

## Introduction to Digital and Computer System Design

### Lecture 3: Combinational Logic Design I



Jetic Gū  
2020 Winter Semester (S1)

# Overview

- Focus: Methodology
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch3 3.1; v5: Ch3 3.1
- Core Ideas:
  1. Light Reading followed by My Lunch Break

# Lecture 1 & 2

- Lecture 1
  - What is digital (logic) circuit
  - How information is represented in digital (logic) circuit
- Lecture 2
  - Atomic components of digital (logic) circuit: Gates, I/O
  - Boolean Algebra

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  1. How to **systematically use** what you've learned in Lecture 2
  2. **practice practice practice**

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# Design Procedure

You, design this!



# Know the Problem

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2. Basic Design, a functional Beta Implementation

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1. Specification of the Problem
2. Basic Design, a functional Beta Implementation
3. Optimisation, Optimisation  
Harder, Better, Faster, Stronger

# Systematic Design Procedures

1. **Specification:** Write a specification for the circuit
2. **Formulation:** Derive relationship between inputs and outputs of the system e.g. using truth table or Boolean expressions
3. **Optimisation:** Apply optimisation, minimise the number of logic gates and literals required
4. **Technology Mapping:** Transform design to new diagram using available implementation technology
5. **Verification:** Verify the correctness of the final design in meeting the specifications

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# An Old Friend

Curtain Motors Revisited

# An Old Friend

- Curtain Motor Control
  - **Sensor1: 1 when curtain is fully closed**
  - **Sensor2: 1 when curtain is fully open**
  - Button1: 1 when user wants to open the curtain
  - Button2: 1 when user wants to close the curtain
  - Output1: 1 to make the motor open the window
  - Output2: 1 to make the motor close the window
  - Light: motor is active



- Stop the motor when the curtain is already fully opened/  
closed



# An Old Friend

## 1. Specification

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- Cond** Stop the motor when the curtain is already fully opened/closed

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- Switch • **Sensor1: 1 when curtain is fully closed**
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## 2. Formulation

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## 3. Optimisation

$$\begin{aligned}\text{FullyClosed} &= \text{Sensor1} \\ \text{FullyOpened} &= \text{Sensor2} \\ \text{Light} &= \text{Out1} + \text{Out2} \\ \text{Out1} &= \text{Bu1} \cdot \overline{\text{FullyOpened}} \cdot \overline{\text{BothPressed}} \\ \text{Out2} &= \text{Bu2} \cdot \overline{\text{FullyClosed}} \cdot \overline{\text{BothPressed}} \\ \text{BothPressed} &= \text{Bu1} \cdot \text{Bu2}\end{aligned}$$

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# An Old Friend

## 4. Technology Mapping

$$\text{Light} = \text{Out1} + \text{Out2}$$

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- Available Components (Technology)



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- Available Components (Technology)
- AND, NAND, OR, NOT gate; Switch, Prob

# An Old Friend

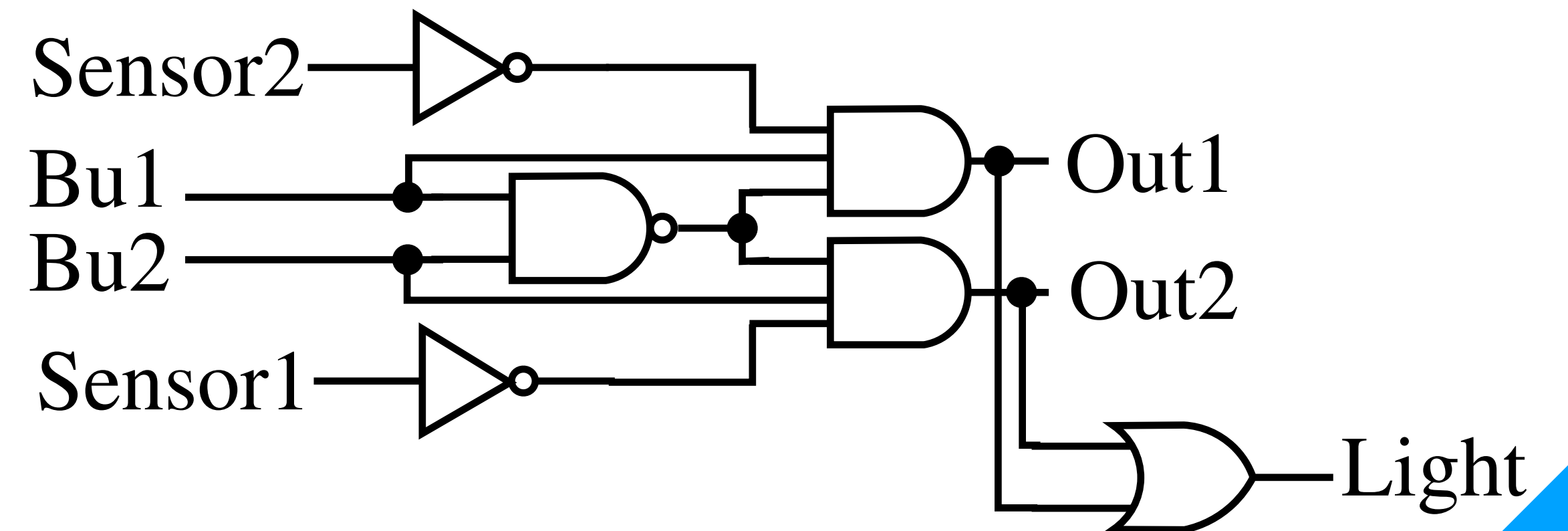
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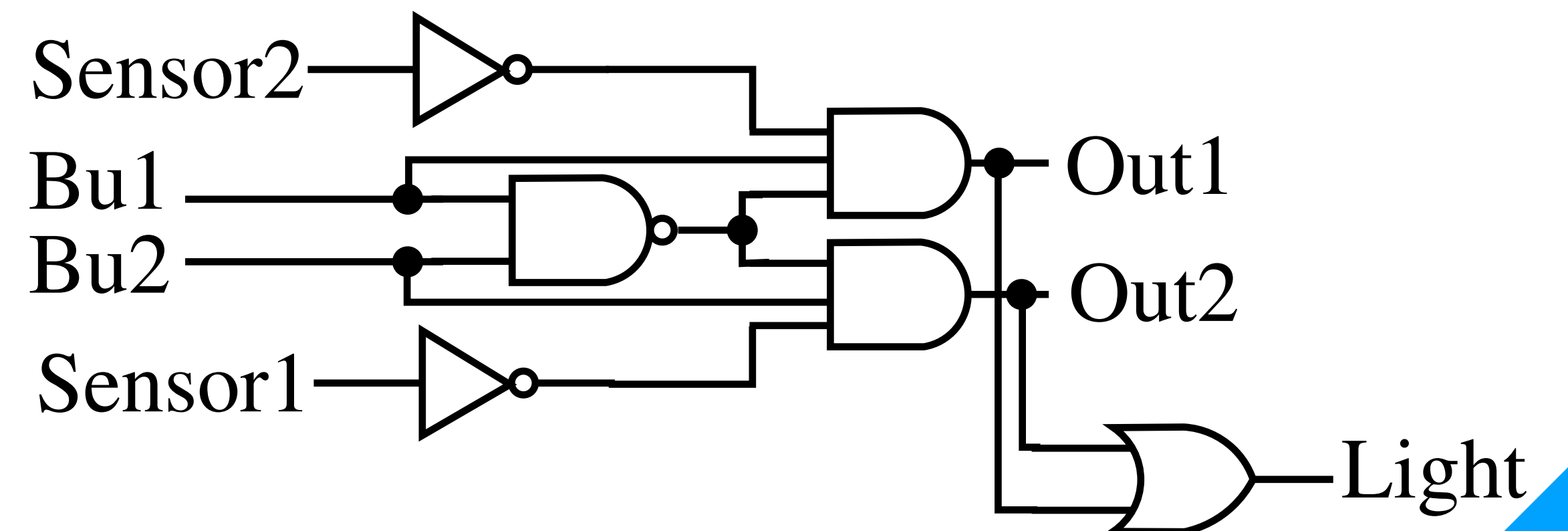
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**NO NOT GATE!**



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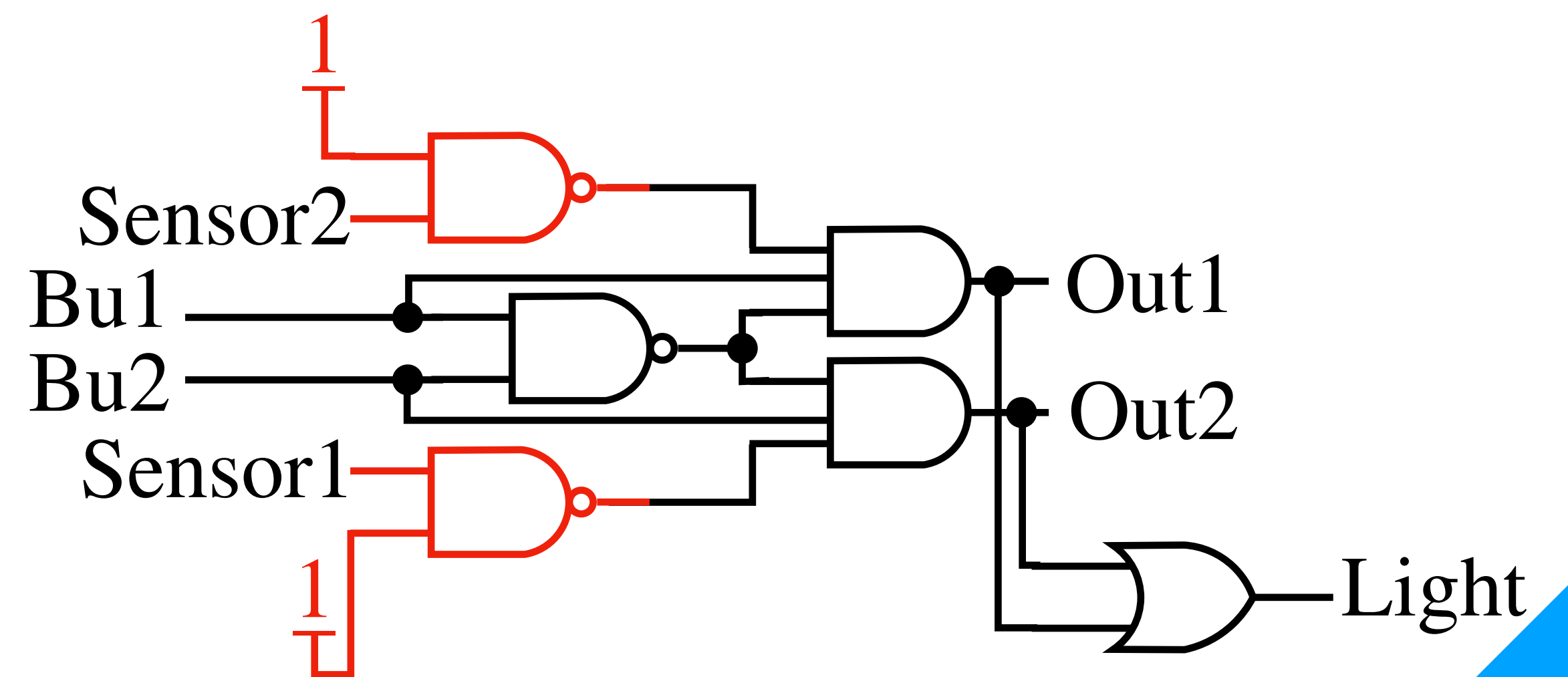
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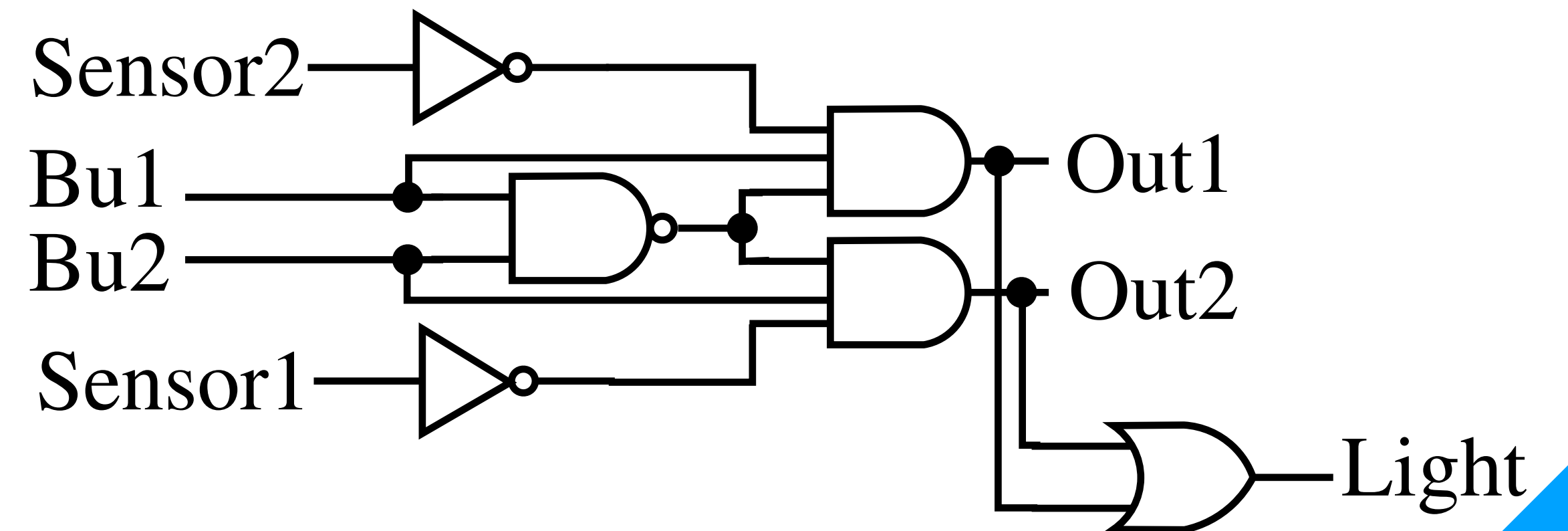
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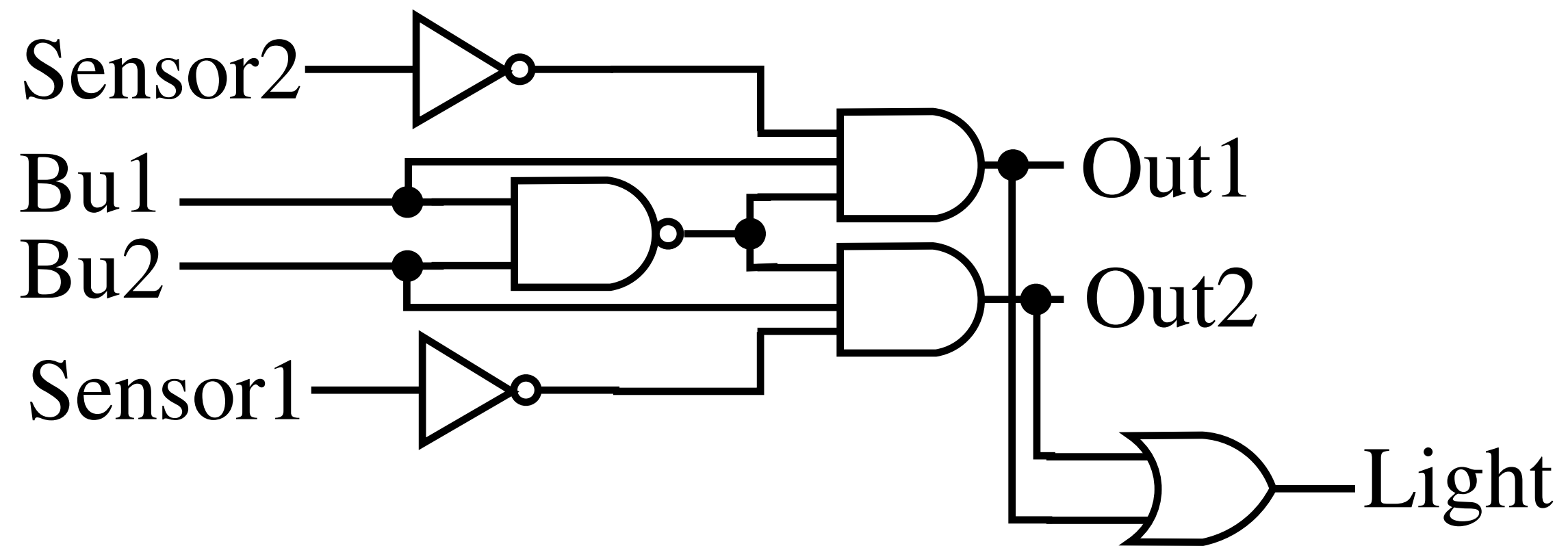
$$\text{Out2} = \text{Bu2} \cdot \overline{\text{Sensor1}} \cdot \overline{\text{Bu1}} \cdot \text{Bu2}$$

- Available Components (Technology)
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# An Old Friend

## 5. Verification



- Design test cases
  - Sensor2, Bu1 on
  - Sensor1, Bu2 on
  - Bu1, Bu2 on
  - Bu1 on
  - Bu2 on
  - etc...