



04.02.20 14:27

# CSCI 150

## Introduction to Digital and Computer System Design

### Lecture 2: Combinational Logical Circuits V



Jetic Gū  
2020 Winter Semester (S1)

# 2 Types of Knowledge

# 2 Types of Knowledge

1. I know this by heart!
  - Important Definitions, Number Systems and Arithmetics, Boolean Algebra
2. Whatever, I can look it up when I need it.
  - The rest of lecture 2 today.
  - You need to know where to look at if you don't remember later!

# Overview

- Focus: Boolean Algebra
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch2 2.8, 2.9. 2.10; v5: Ch2 2.6, 2.7
- Core Ideas:
  1. Other Gate Types: XOR, NAND, NOR, Buffer, High-Impedance, Odd Function
  2. Lecture 2 Review

# Boolean Algebra I-III

## I. AND, OR, NOT Operators and Gates

- Simple digital circuit implementation
- Algebraic manipulation using Binary Identities

## II. Standard Forms

- Minterm & Maxterm
- Sum of Products & Product of Sums

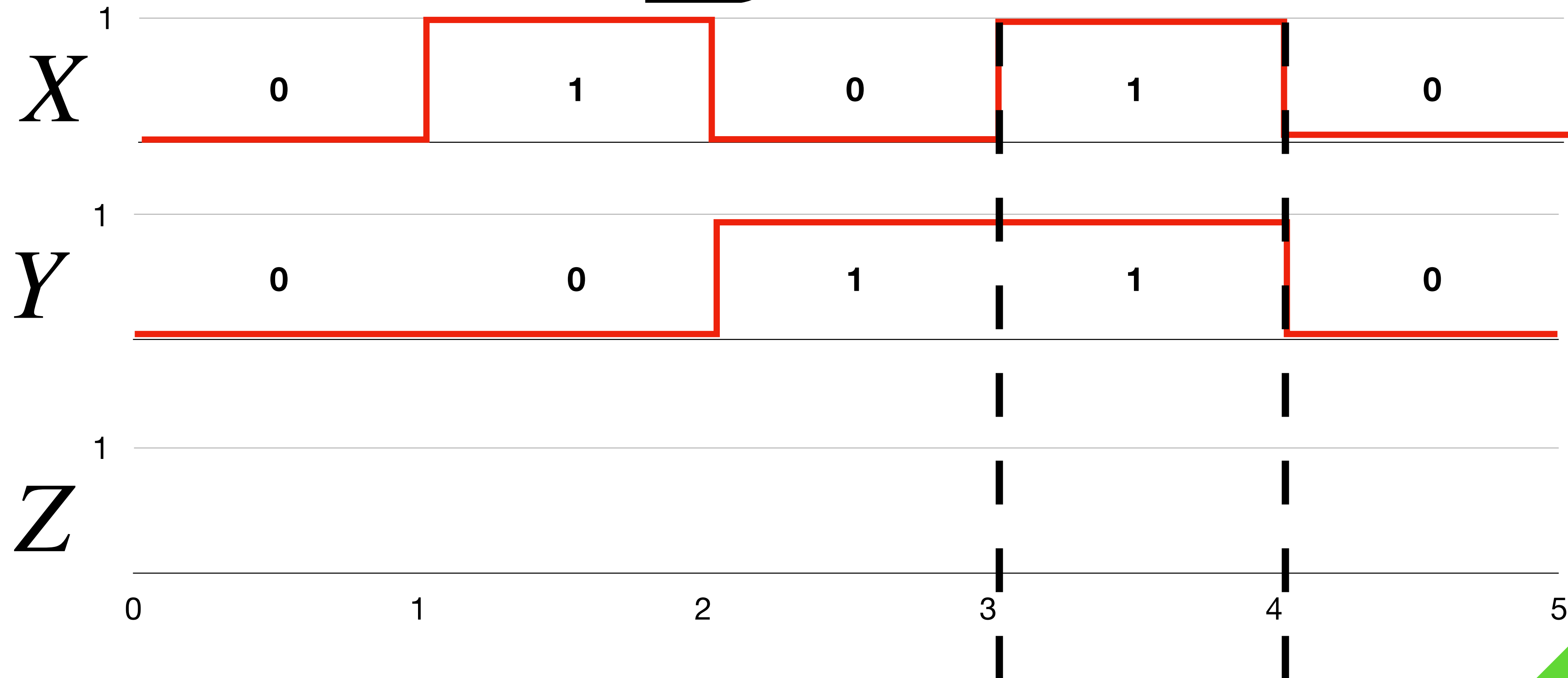
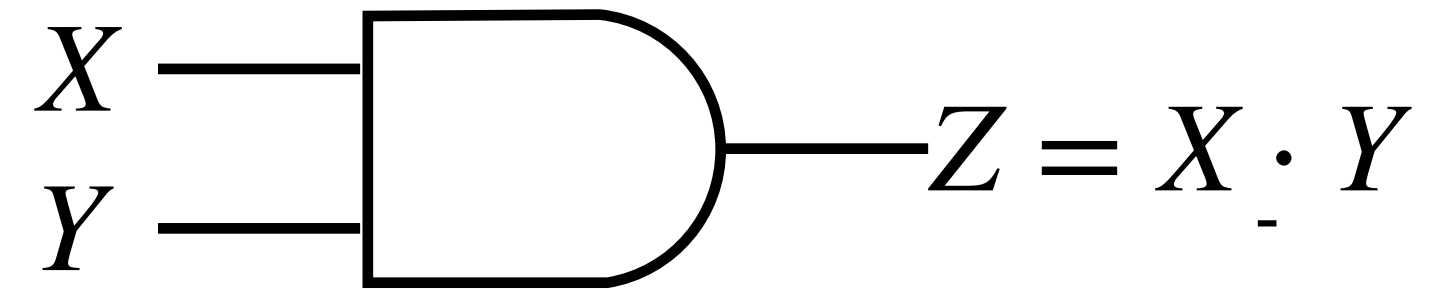
## III. Optimisation Using K-Map (For 2,3,4 Variables)

# Other Gate Types

Delay Propagation  
XOR, NAND, NOR, Buffer

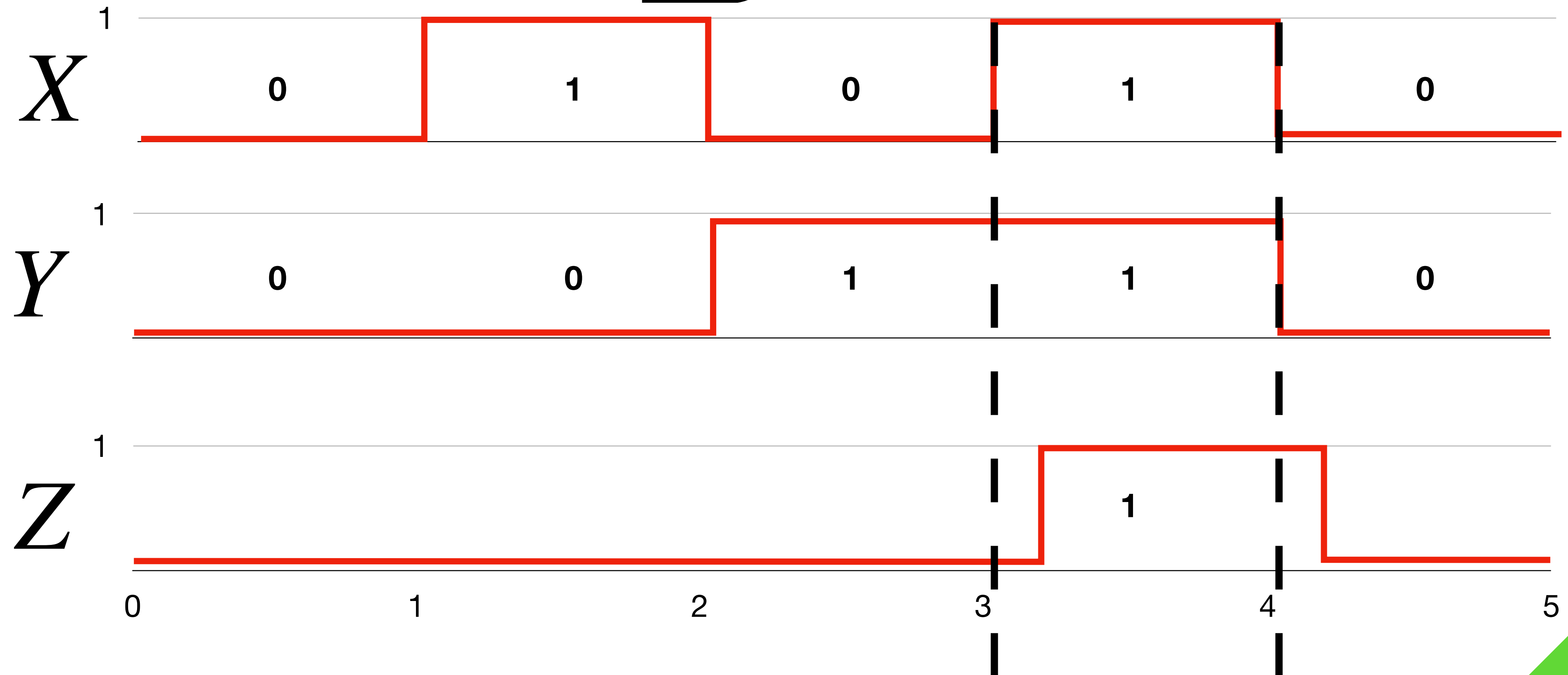
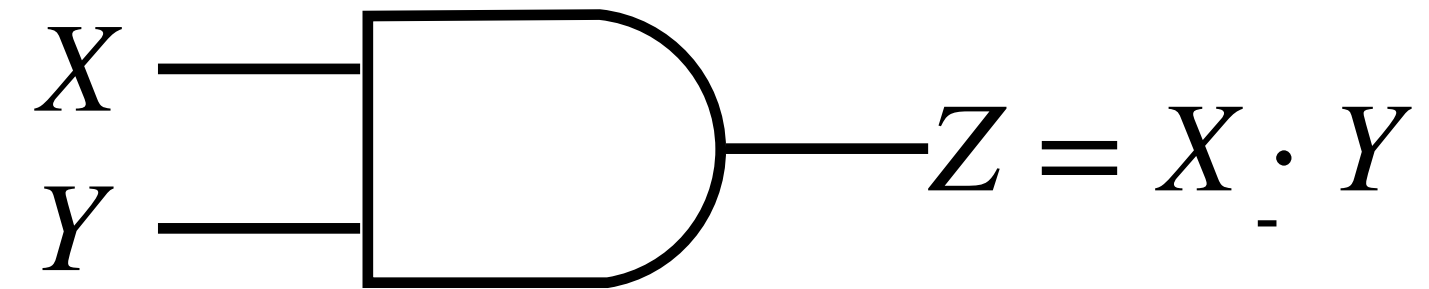
# Gate Delay

Timing Diagram (**REALITY**)



# Gate Delay

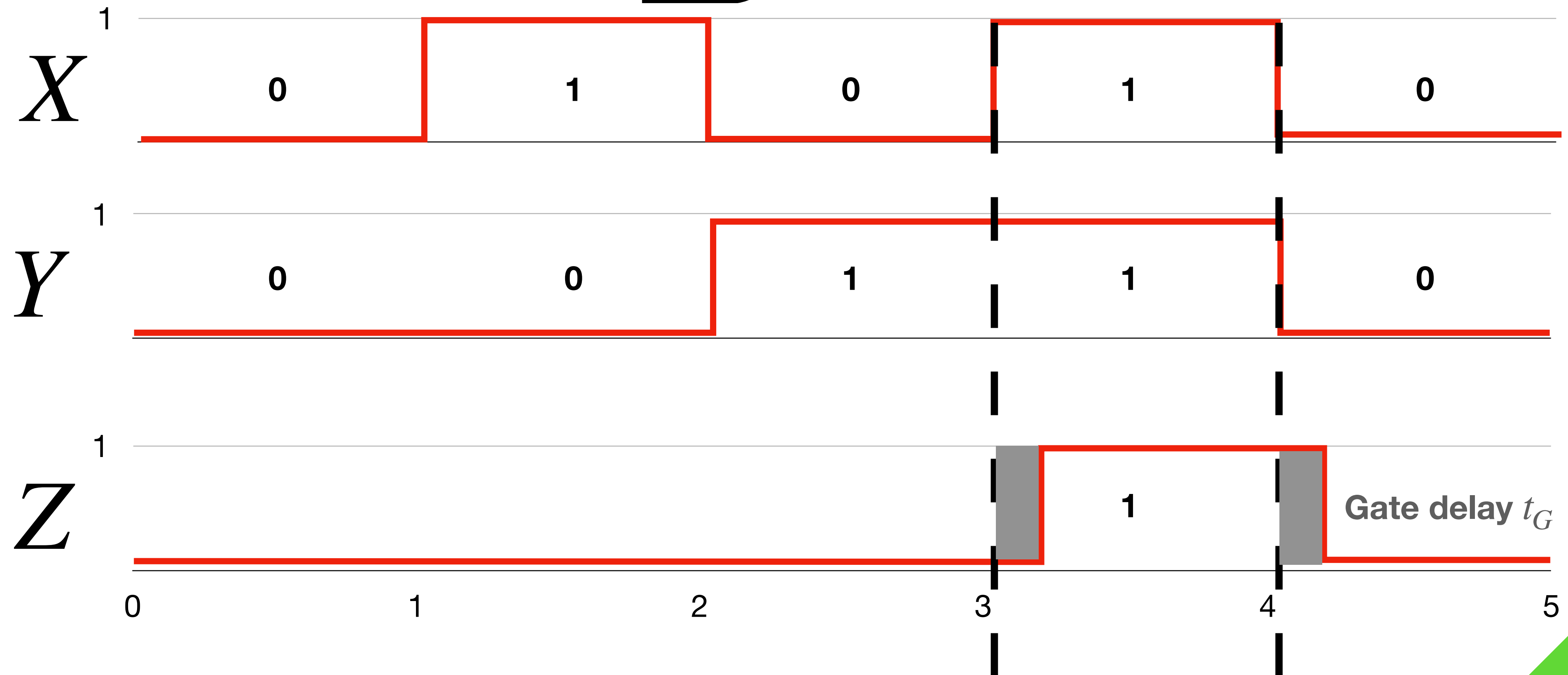
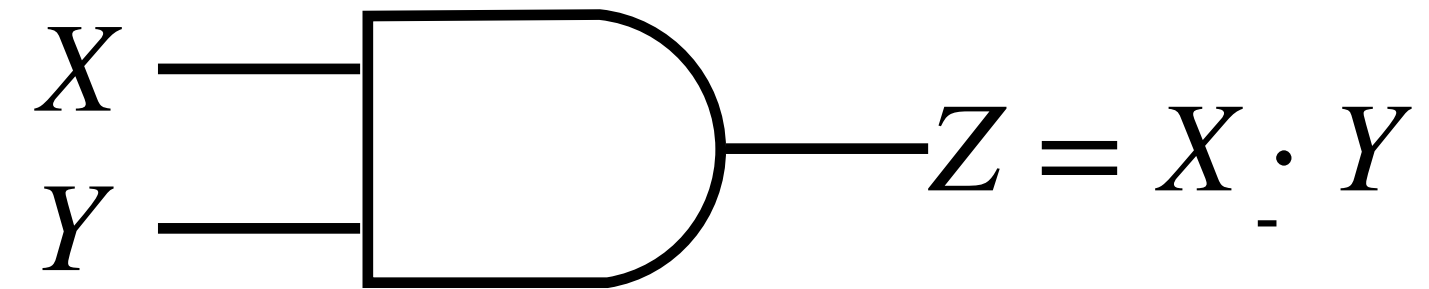
Timing Diagram (**REALITY**)





# Gate Delay

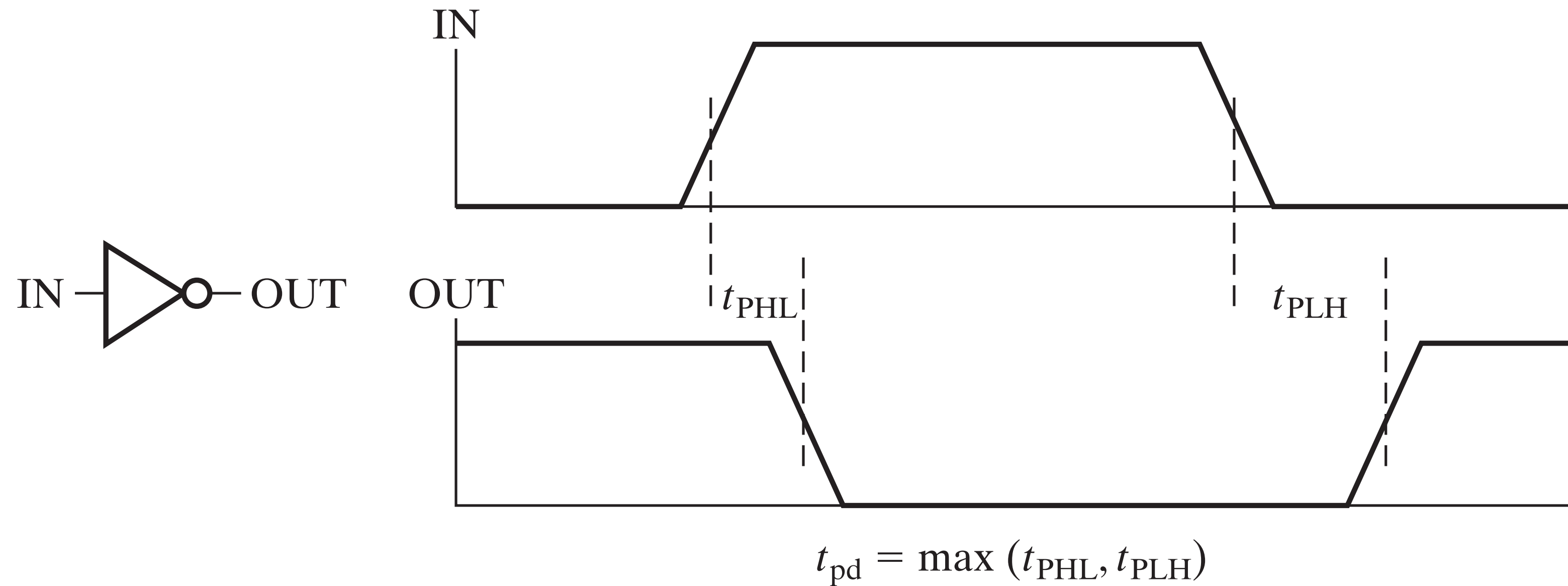
Timing Diagram (**REALITY**)



# Delays

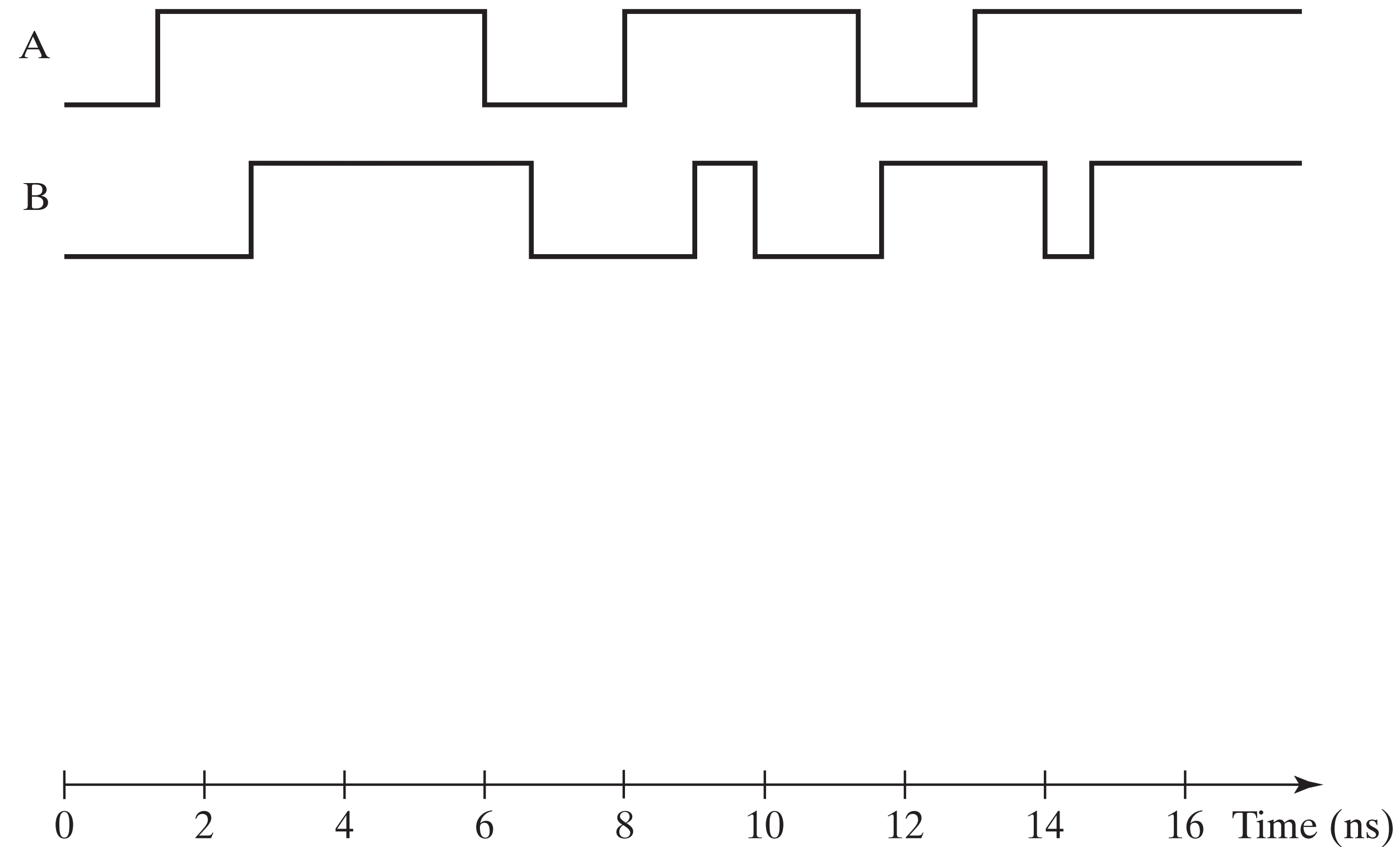
- Gate Delay
  - The time required for a change in value of a signal to propagate from input to output (of a Gate)
- Propagation Delay
  - The time required for a change in value of a signal to propagate from input to output (of a Circuit)

# Delays

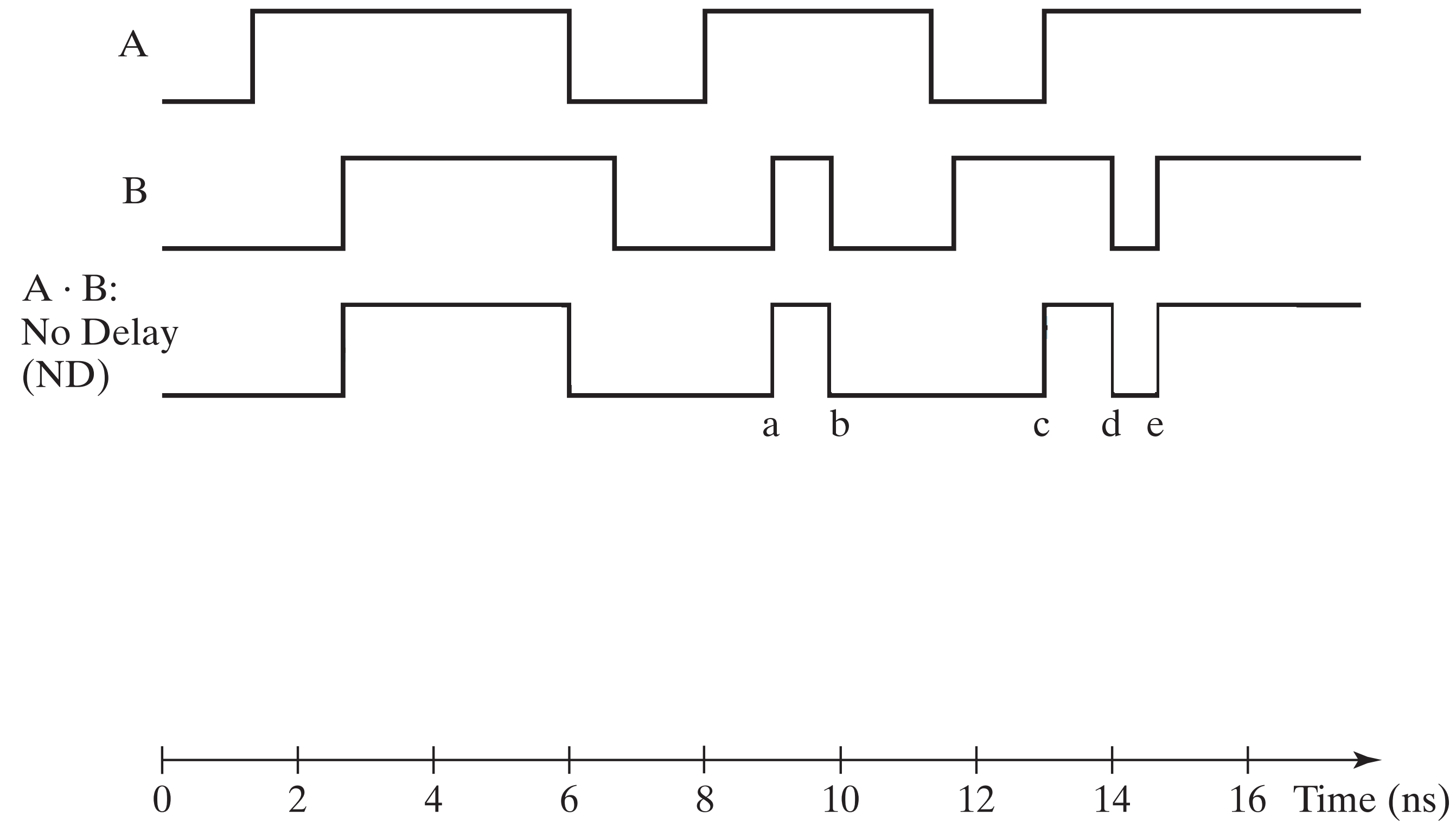


- $t_{pd}$ : Propagation Delay: longest time for propagating from input to output
- $t_{PHL}$ : Hight2Low propagation time;  $t_{PLH}$ : Low2High propagation time;

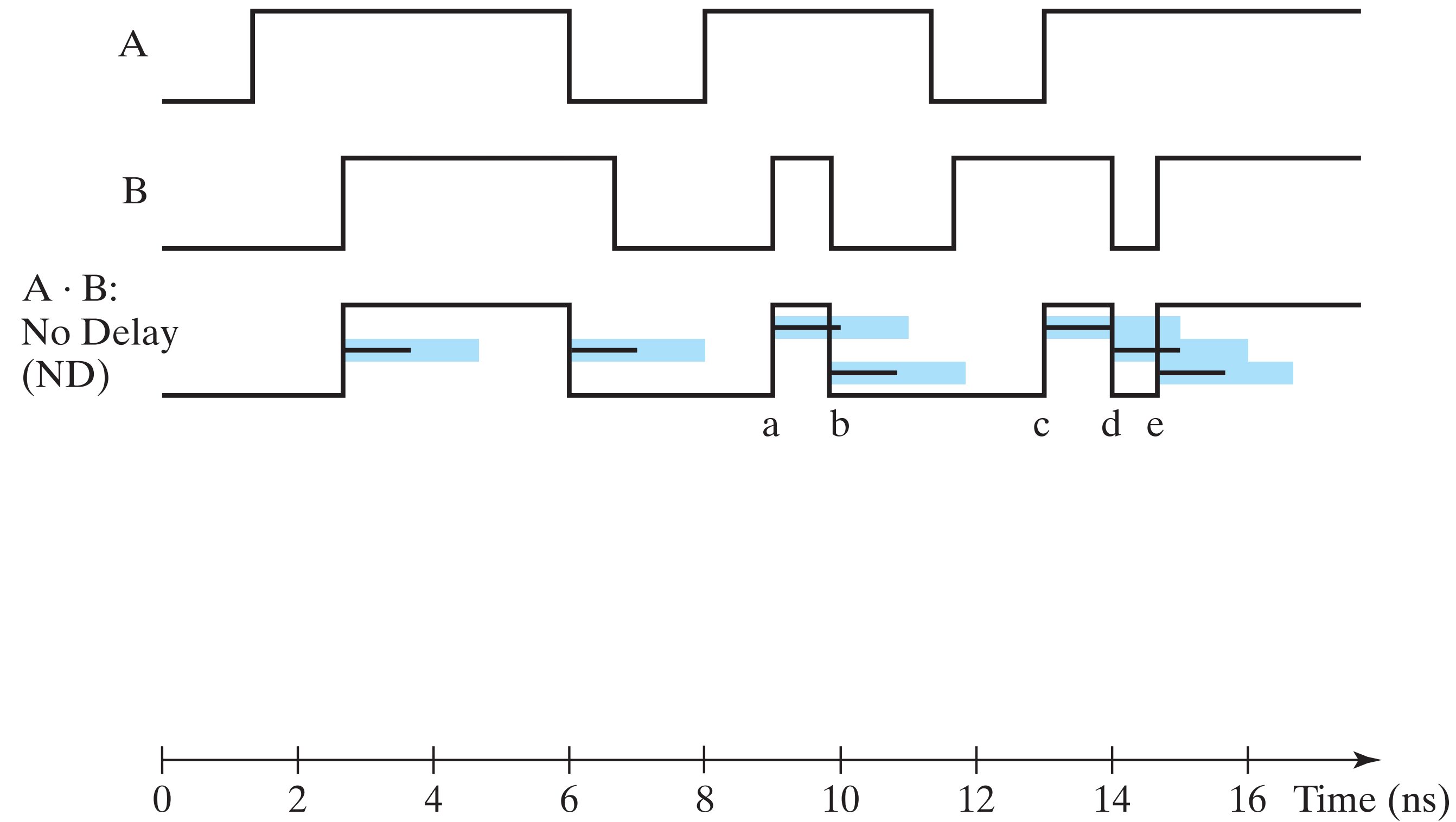
# Transport and Inertial Delays



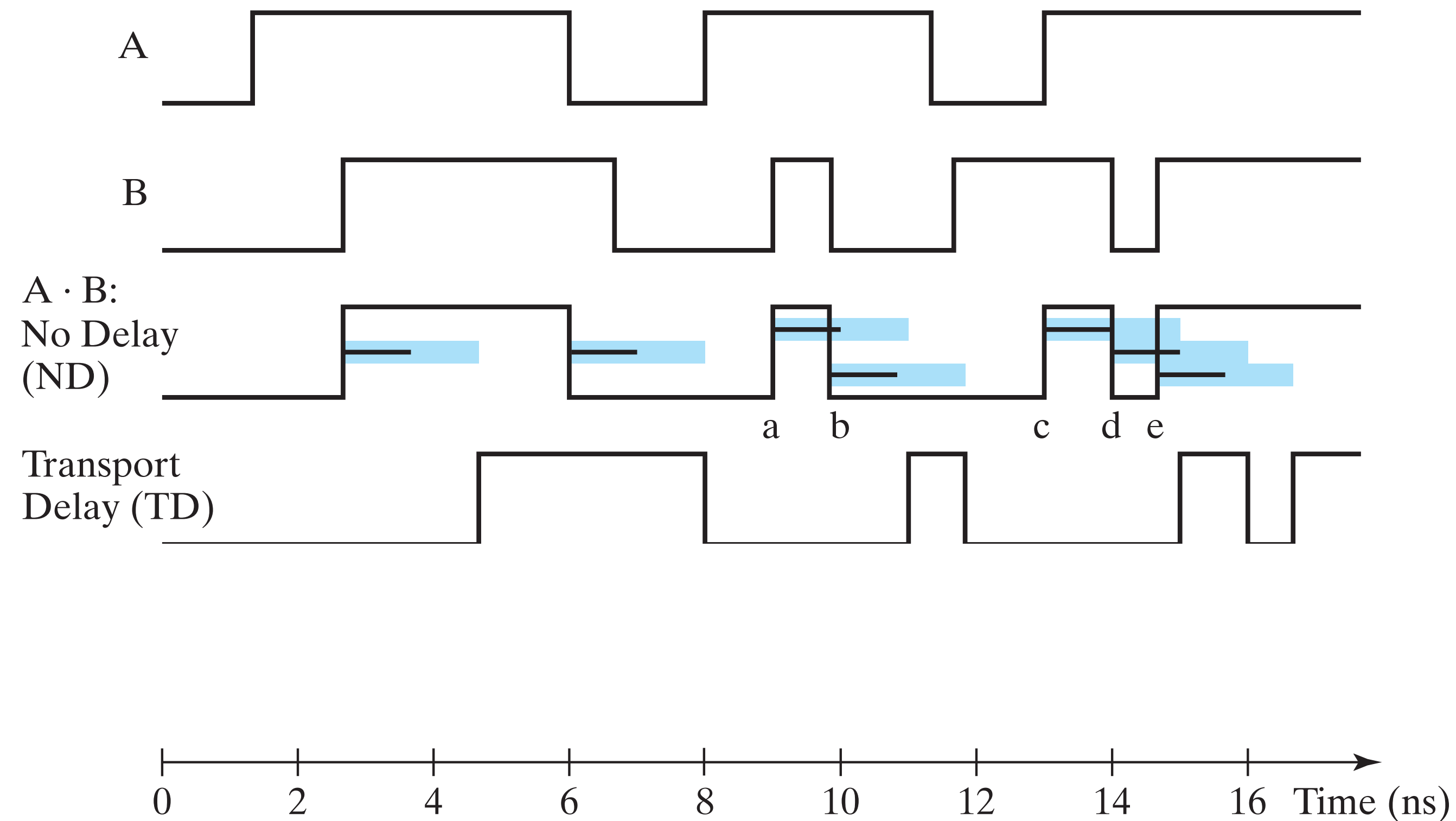
# Transport and Inertial Delays



# Transport and Inertial Delays

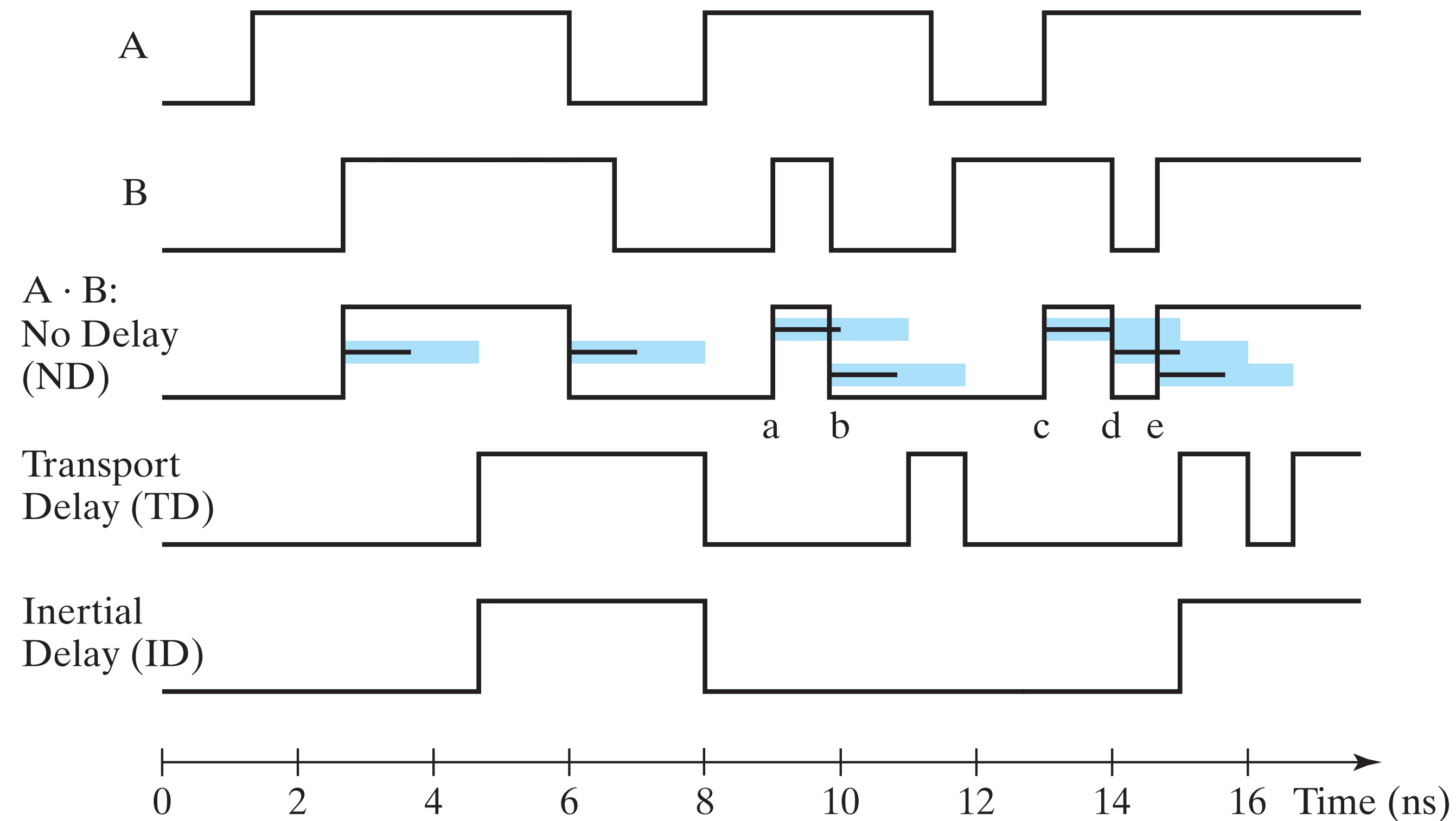


# Transport and Inertial Delays



- **Transport Delay:** output in response to input changes after propagation delay

# Transport and Inertial Delays



- **Transport Delay:** output in response to input changes after propagation delay
- **Inertial Delay:** if a value changes twice in a short time (**rejection time**), ignore



# Parameters of Gate Delay

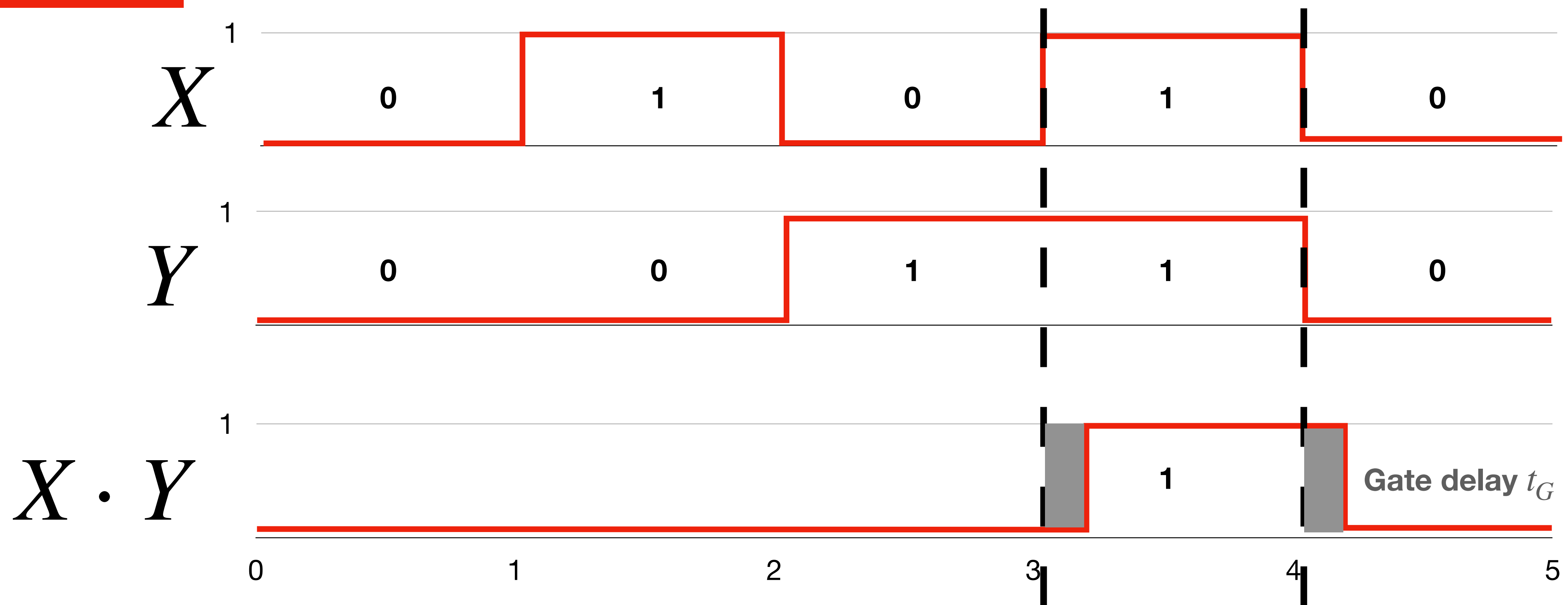
# Parameters of Gate Delay

- Fan-in  
number of inputs of a logic gate

# Parameters of Gate Delay

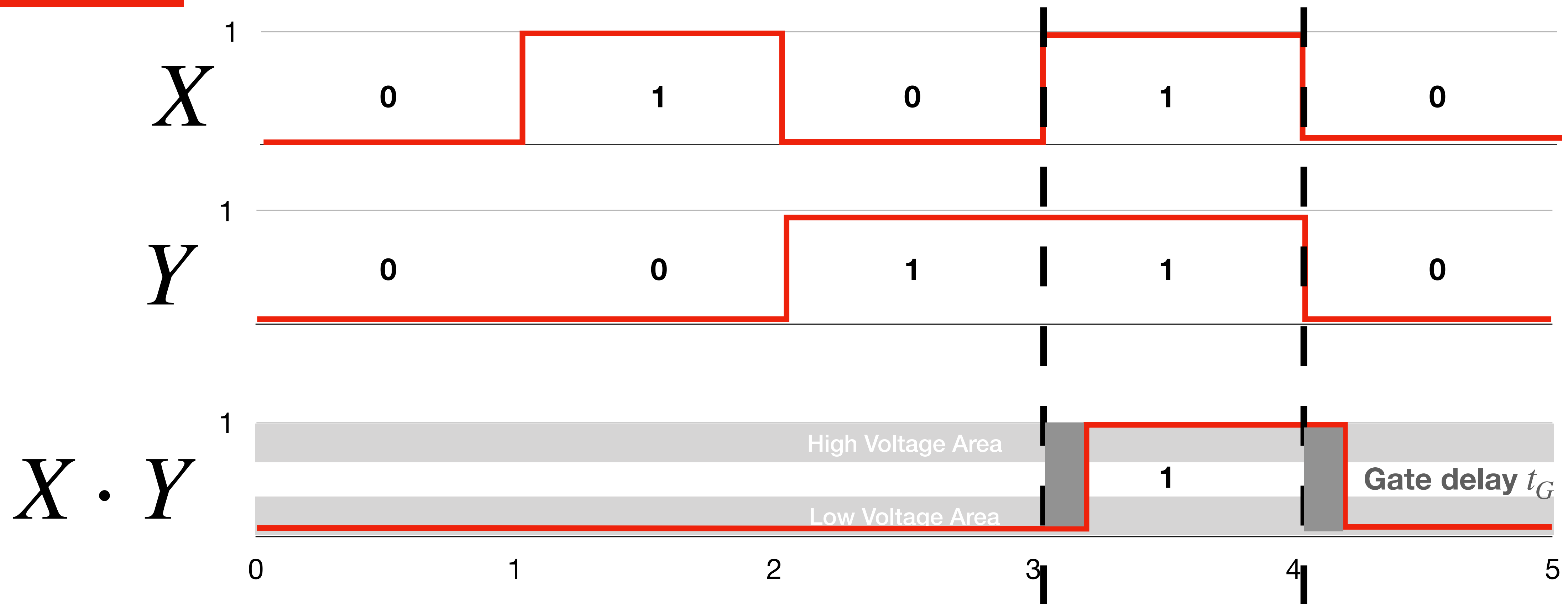
- Fan-in  
number of inputs of a logic gate
- Fan-out (standard load)  
number of gates that each logic gate can drive while providing voltage levels in the guaranteed range is called the standard load or fan-out

# Standard Load



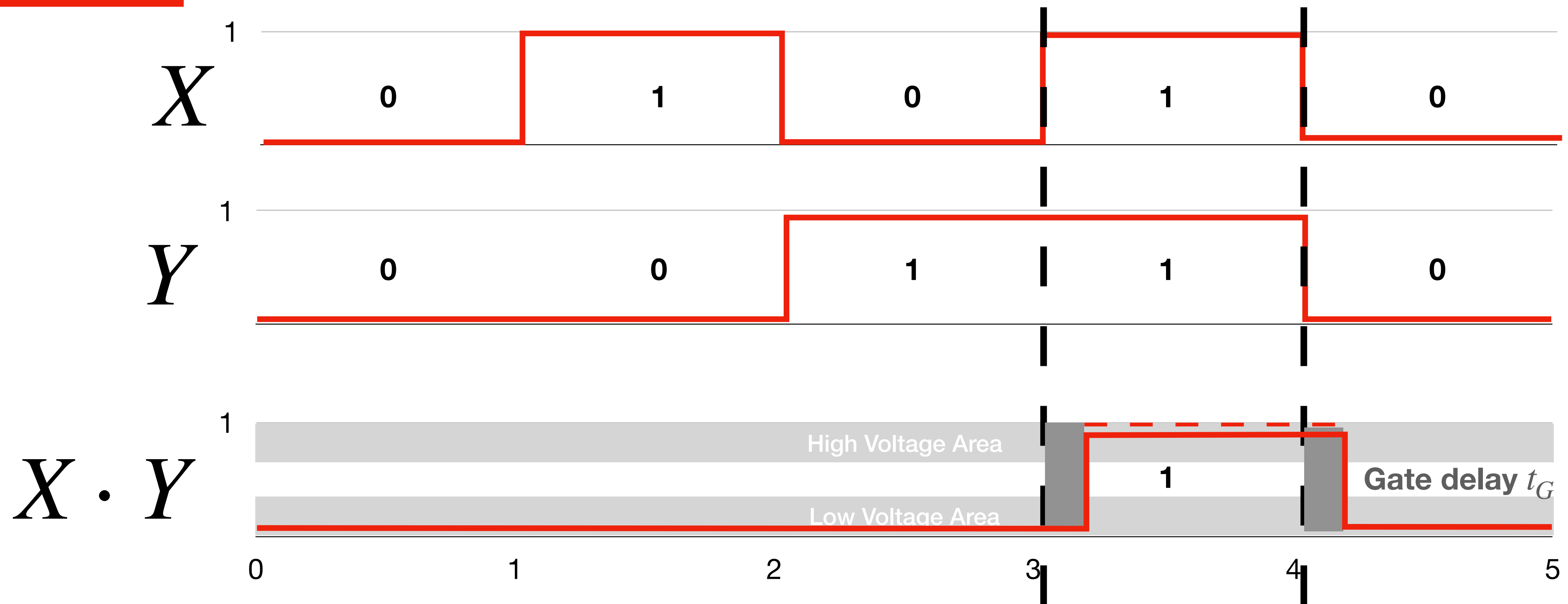
- Digital circuits are driven by power, each component also takes power

# Standard Load



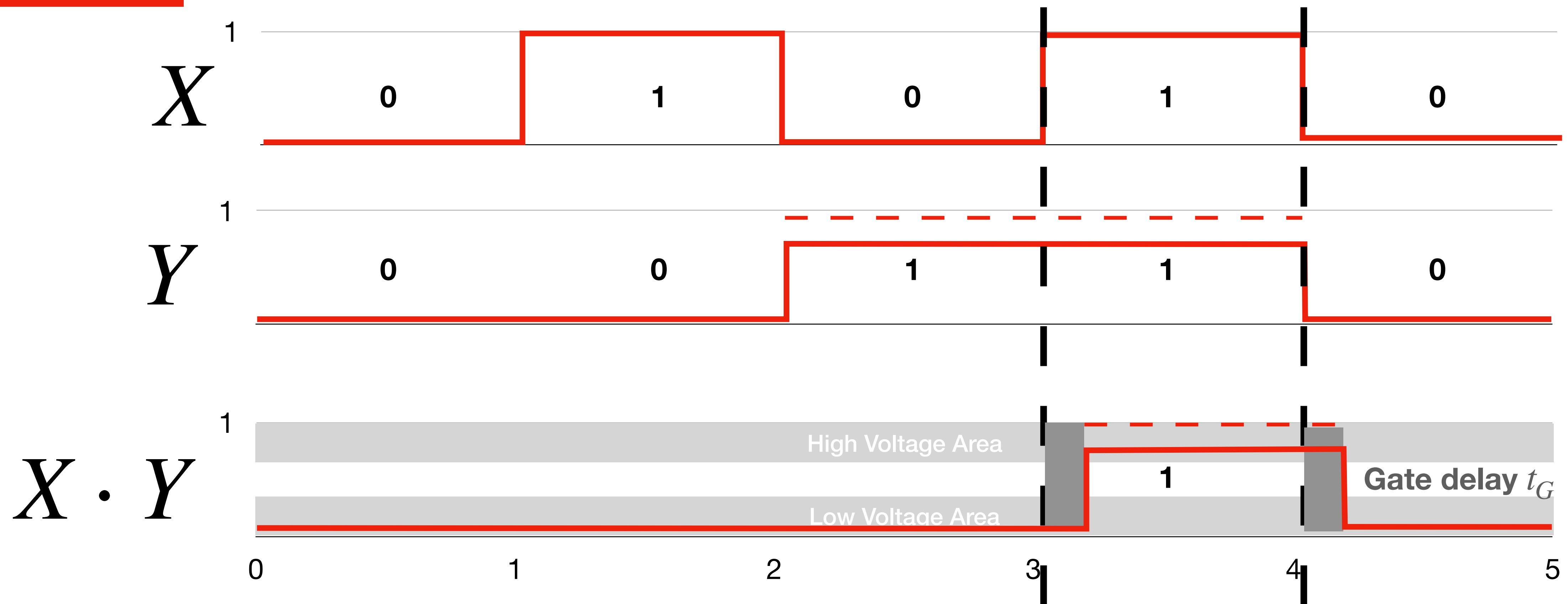
- Digital circuits are driven by power, each component also takes power

# Standard Load



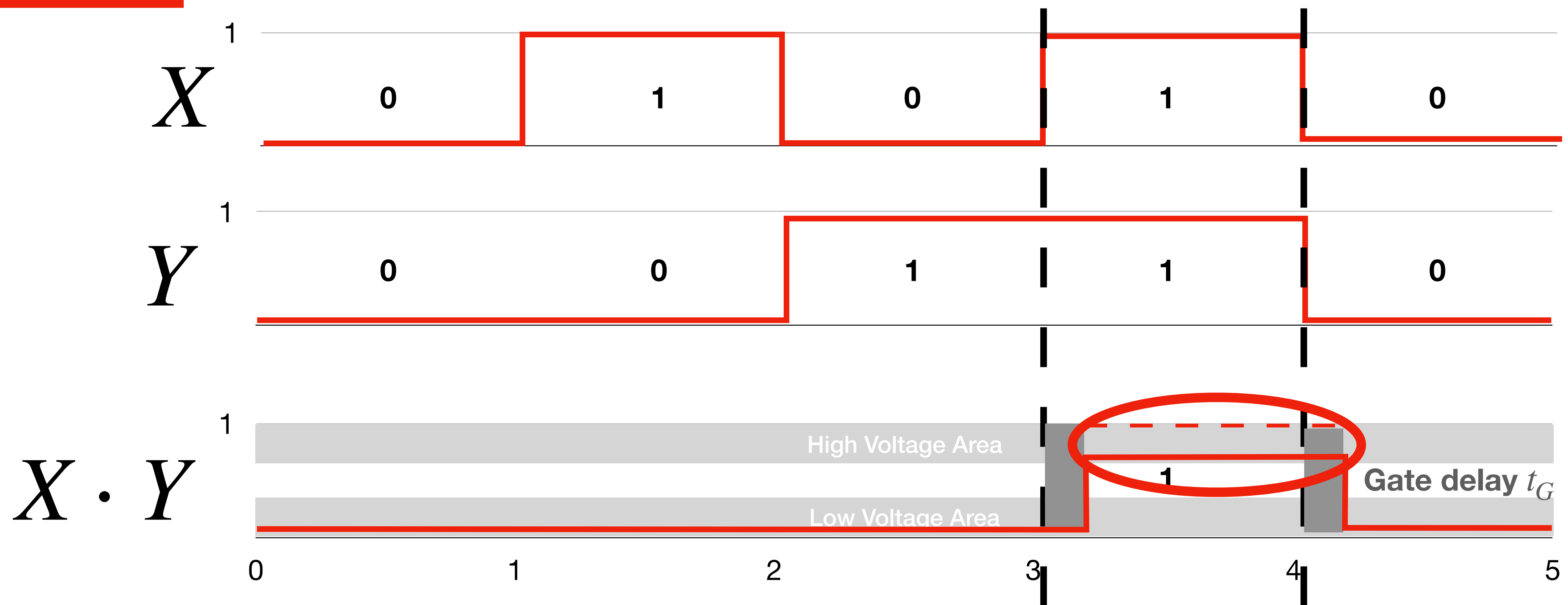
- Digital circuits are driven by power, each component also takes power

# Standard Load



- Changes in input voltage can also affect output voltage

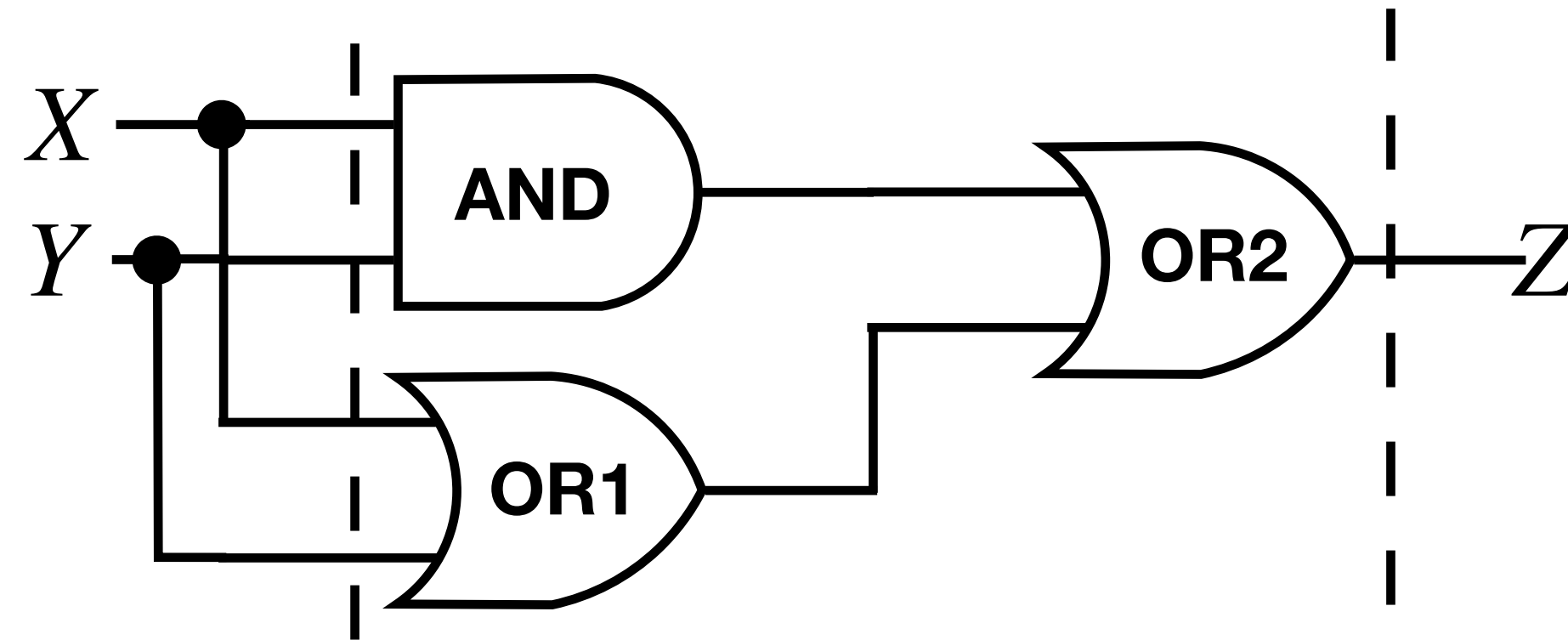
# Standard Load



- If it takes too much power, it will not be able to guarantee output voltage
- Standard load is defining that power consumption, which also affect delay



# Standard Load



- OR Gate: 0.8 SL; AND Gate: 1.00 SL;

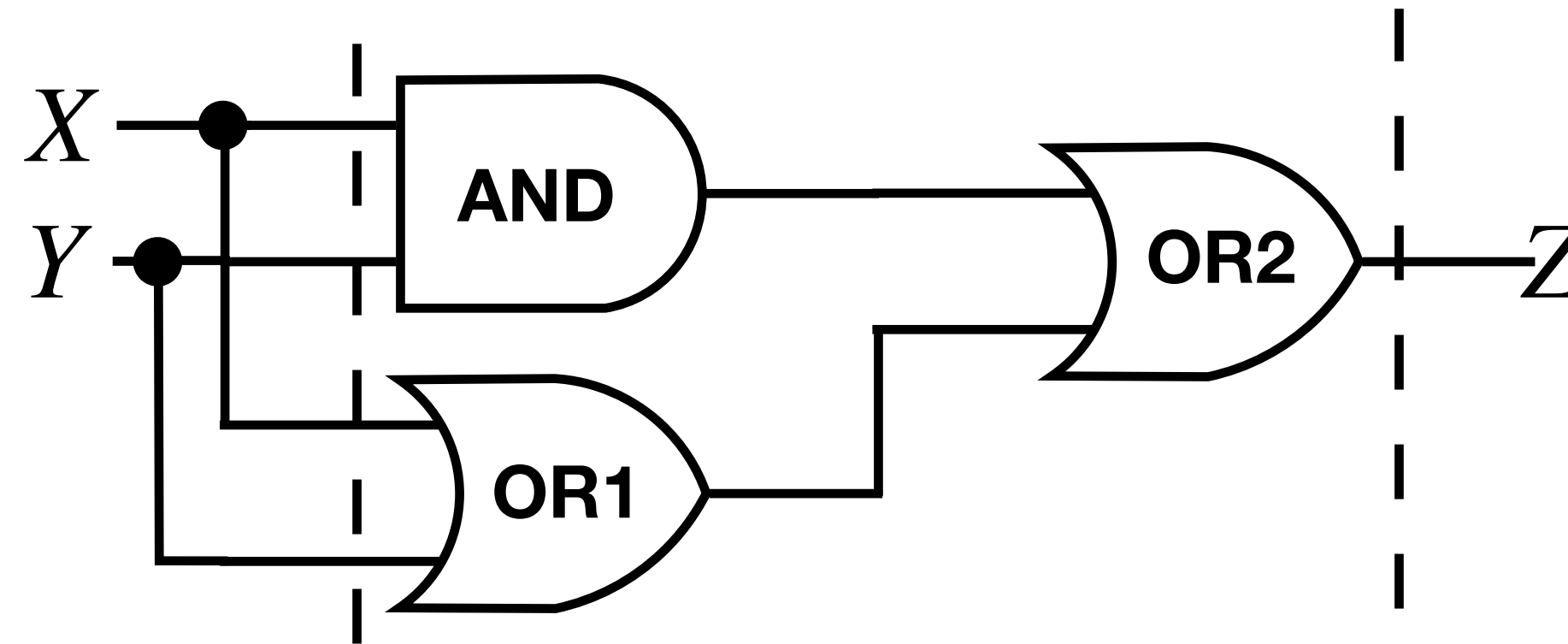
- Delay for AND:

$$t_{pd} = 0.07 + 0.021 \times \sum InputSL$$

- Delay for OR:

$$t_{pd} = 0.05 + 0.02 \times \sum InputSL$$

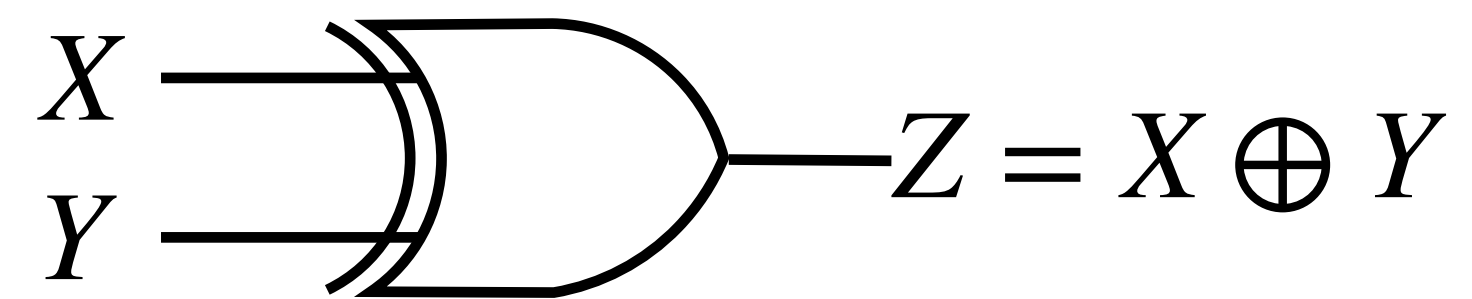
# Standard Load



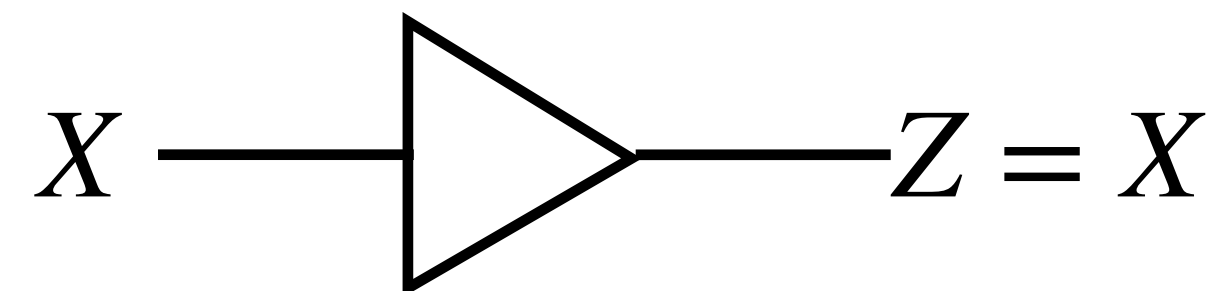
- OR Gate: 0.8 SL; AND Gate: 1.00 SL;
- Estimation for OR2
$$t_{pd} = 0.05 + 0.02 \times (0.8 + 1.00) = 0.086\text{ns}$$
- Delay for AND:
$$t_{pd} = 0.07 + 0.021 \times \sum \text{InputSL}$$
- Precise estimation is very hard, since we lack a lot of these information
- Delay for OR:
$$t_{pd} = 0.05 + 0.02 \times \sum \text{InputSL}$$
- We can ignore this computation, but you should know roughly

# Other Gates

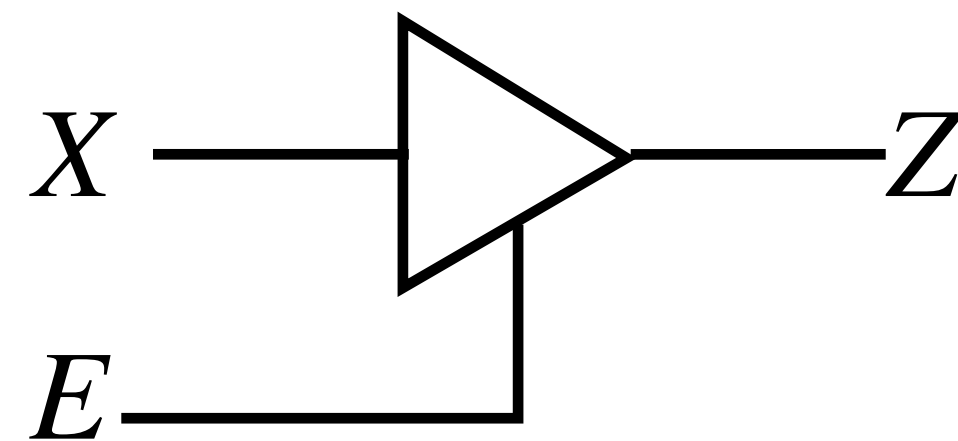
**XOR Gate**  
**Exclusive-OR**



**Buffer**

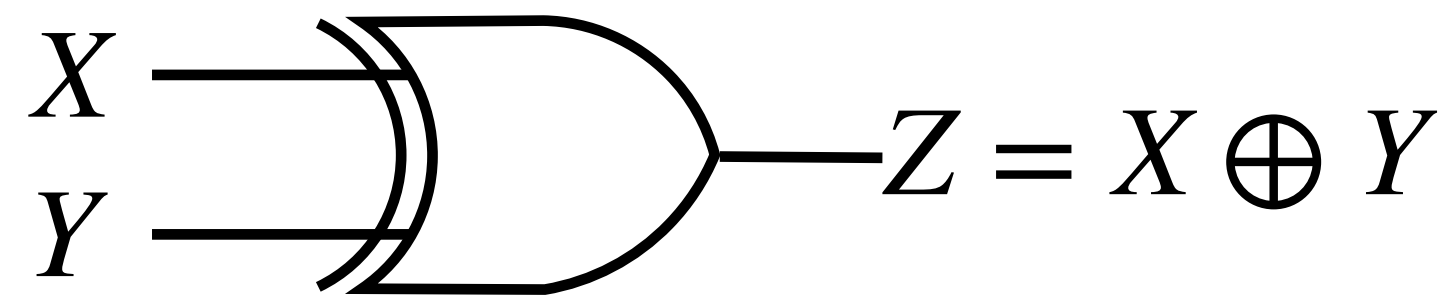


**3-State Buffer**

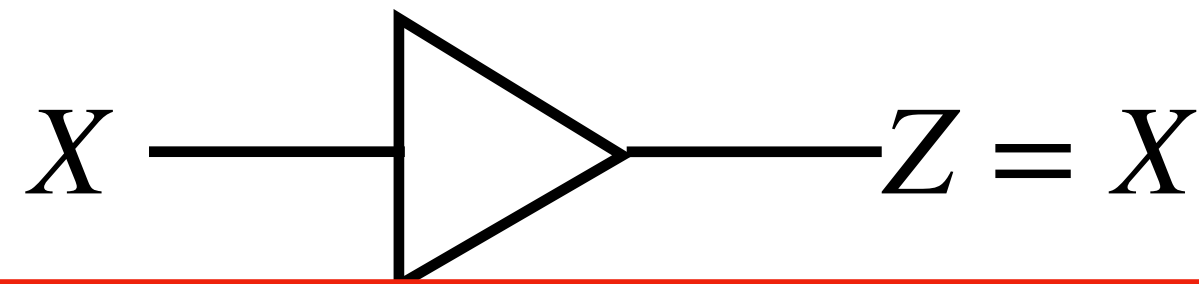


# Buffer

**XOR Gate**  
**Exclusive-OR**

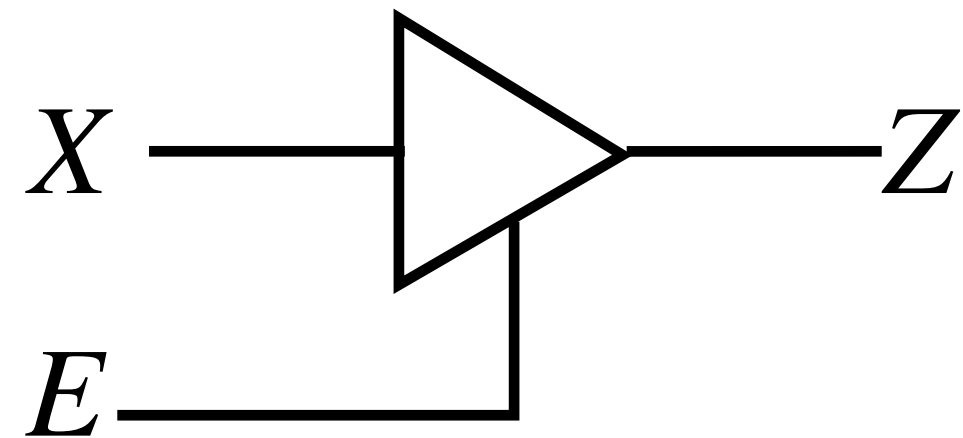


**Buffer**



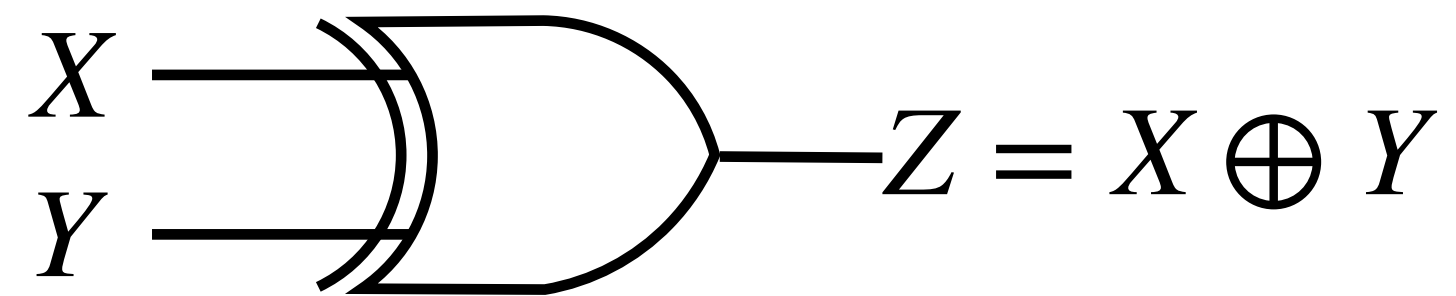
- Has delay
- Increases output Voltage  
e.g.: input 0.8 V, output 1.0 V

**3-State Buffer**

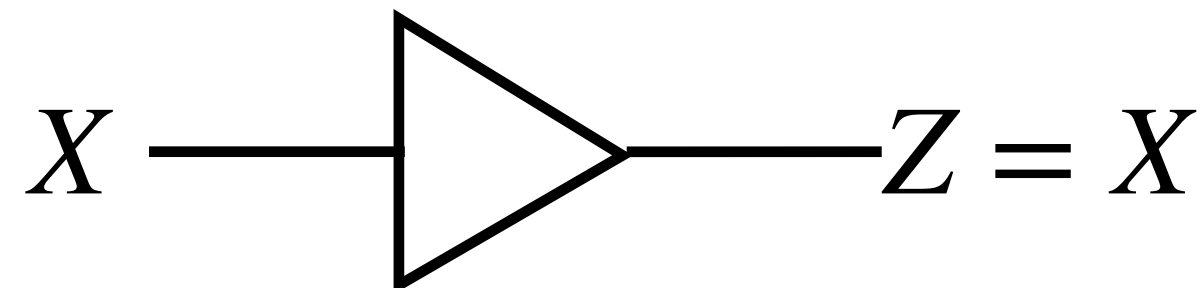


# 3-State Buffer

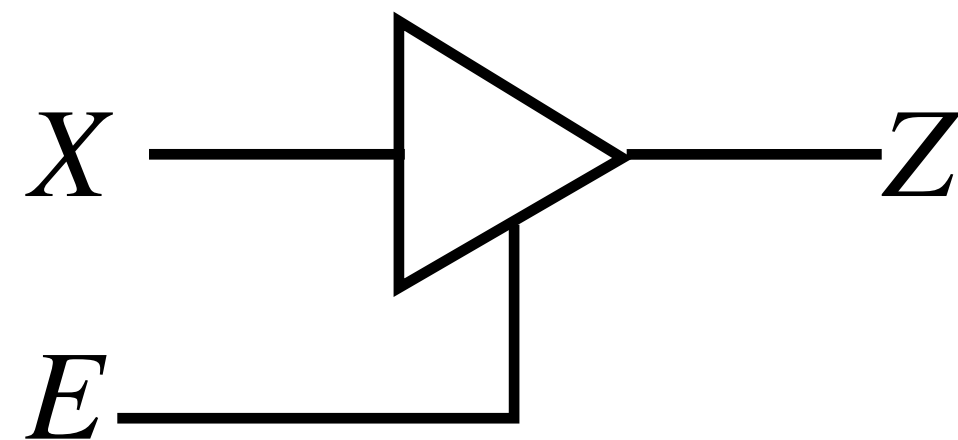
**XOR Gate**  
**Exclusive-OR**



**Buffer**



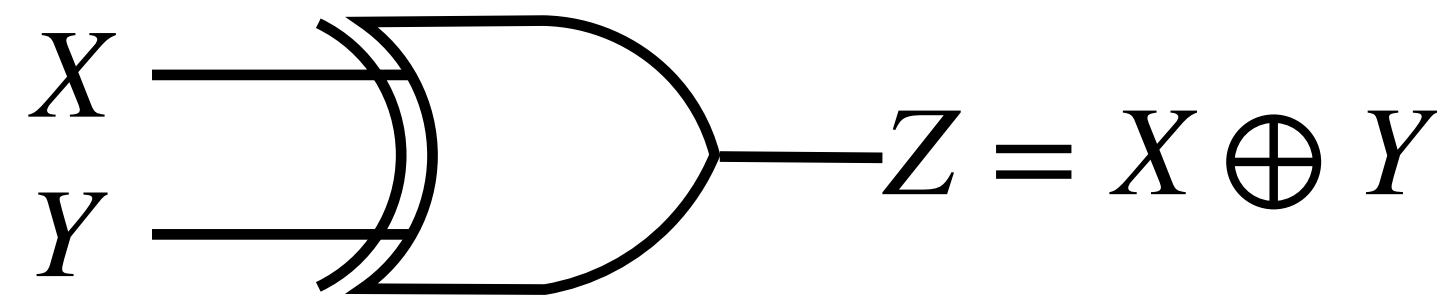
**3-State Buffer**



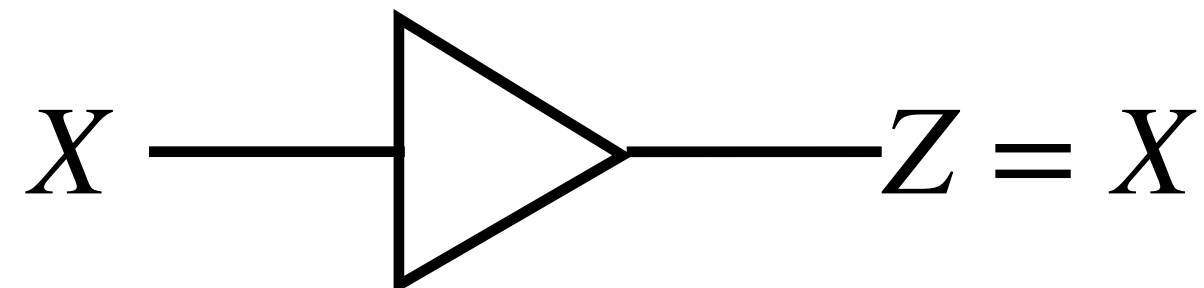
- Has delay
- Increases output Voltage  
e.g.: input 0.8 V, output 1.0 V
- When  $E = 0$ , outputs no electricity  
(Open circuit/Hi-Z/High Impedance)

# 3-State Buffer

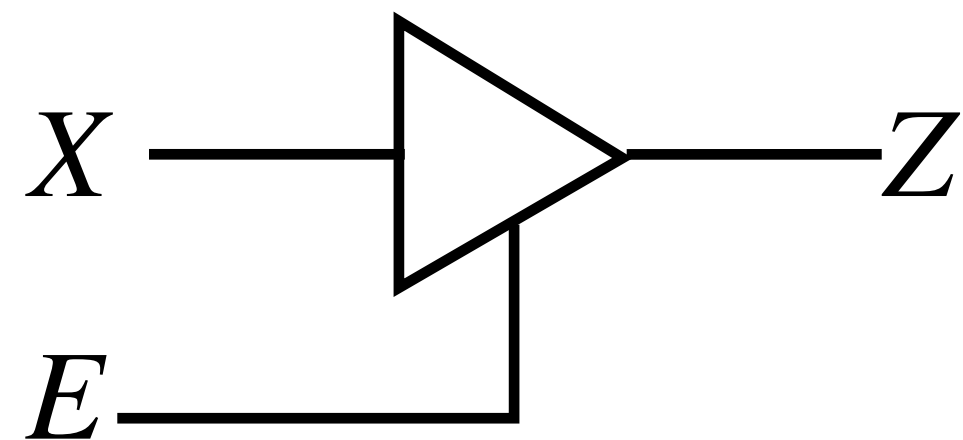
**XOR Gate**  
**Exclusive-OR**



**Buffer**



**3-State Buffer**

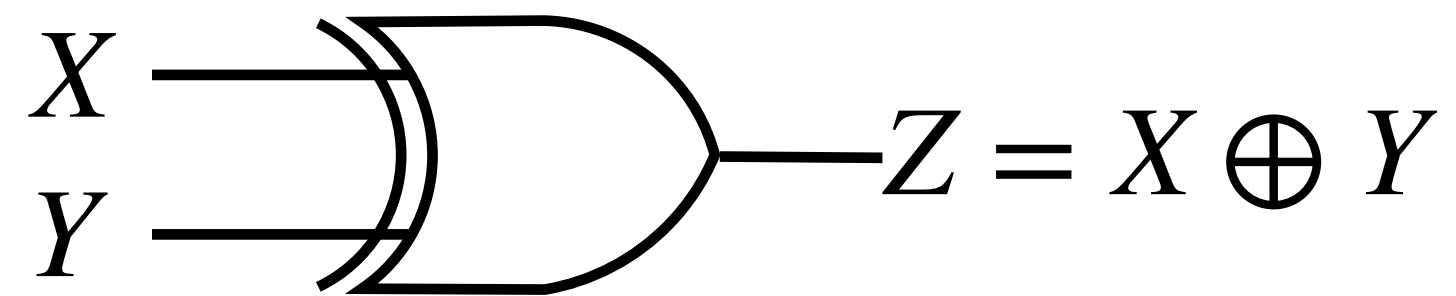


- Has delay
- Increases output Voltage  
e.g.: input 0.8 V, output 1.0 V
- When  $E = 0$ , outputs no electricity  
(Open circuit/Hi-Z/High Impedance)

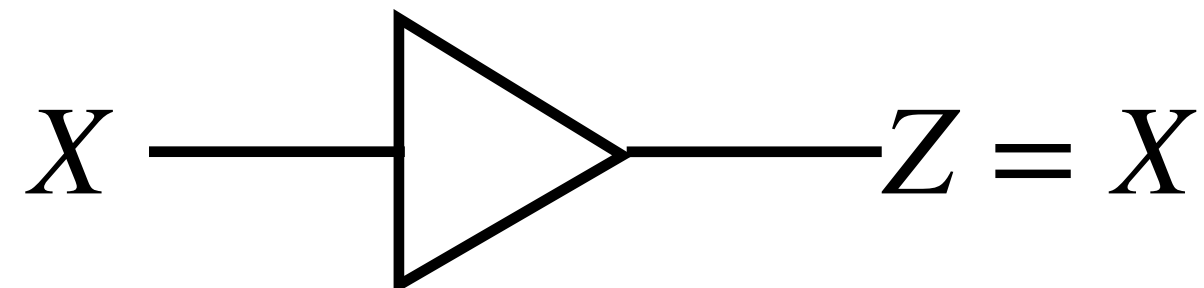


# 3-State Buffer

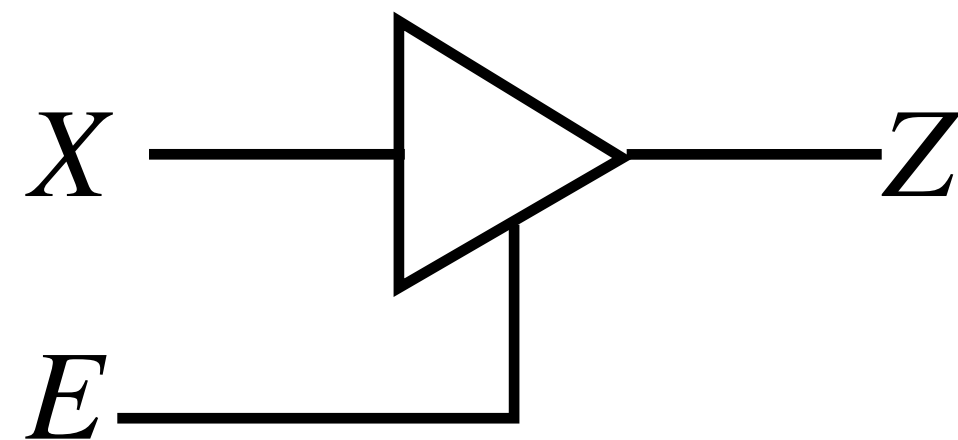
**XOR Gate**  
**Exclusive-OR**



**Buffer**



**3-State Buffer**

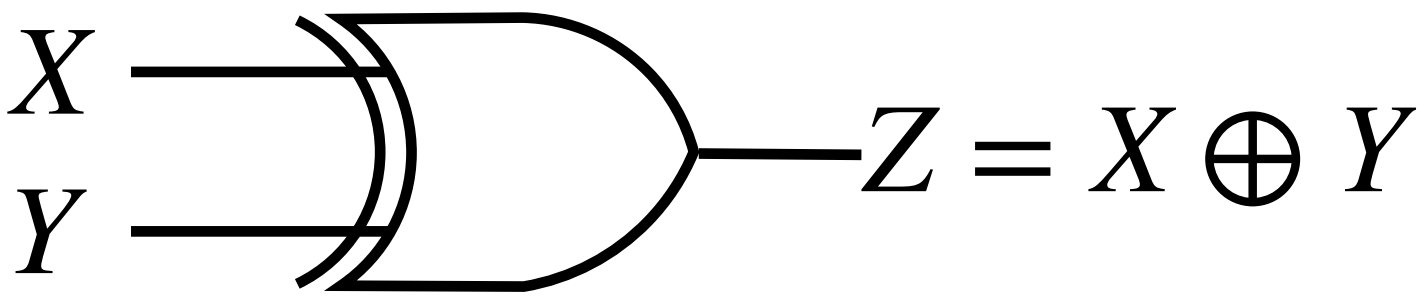


- Has delay
- Increases output Voltage  
e.g.: input 0.8 V, output 1.0 V
- When  $E = 0$ , outputs no electricity  
(Open circuit/Hi-Z/High Impedance)

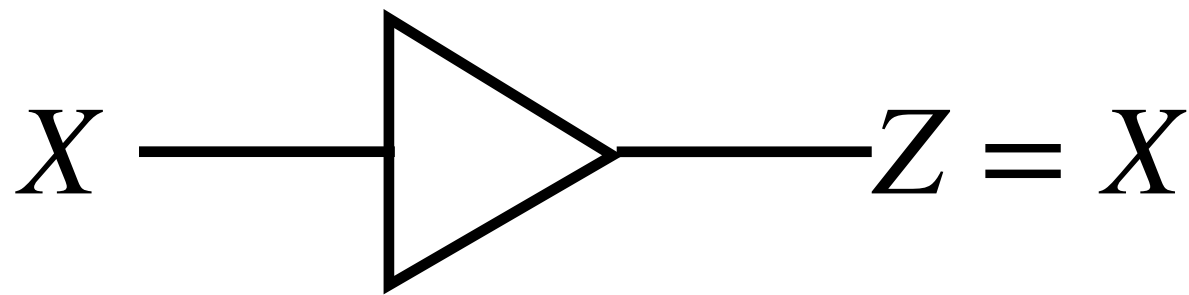


# 3-State Buffer

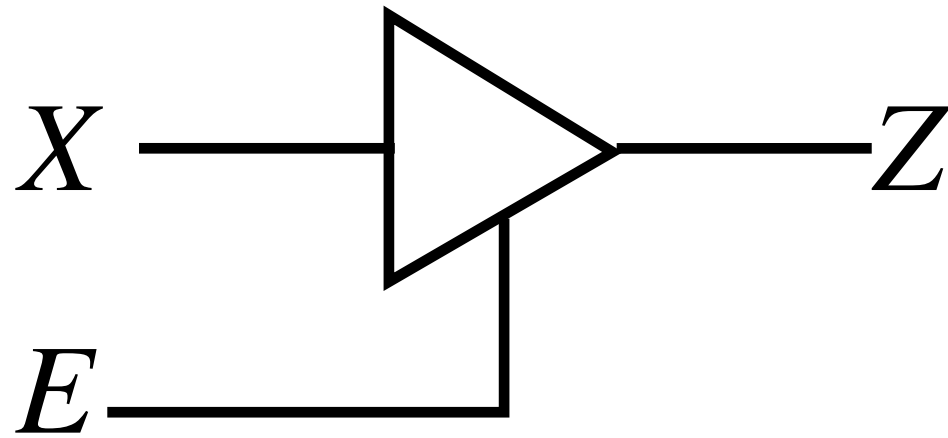
XOR Gate  
Exclusive-OR



Buffer



3-State Buffer



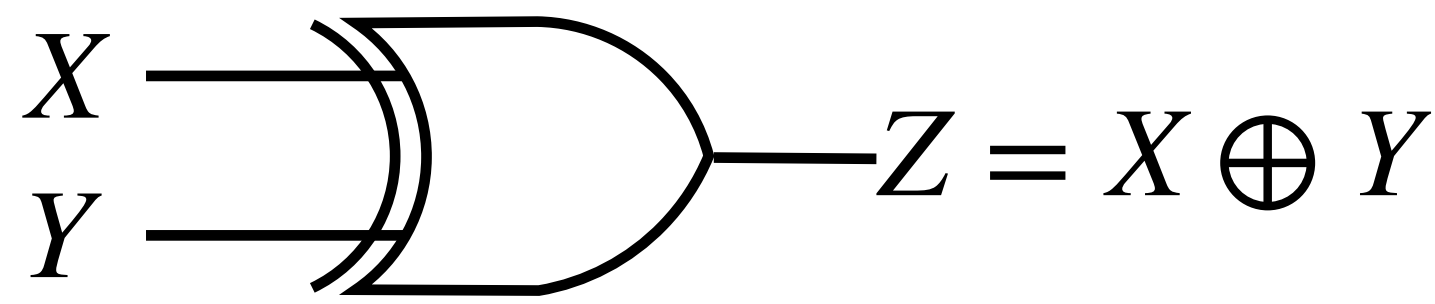
3-State Buffer Truth Table

$E$	$X$	$Z$
0	0	Hi-Z
0	1	Hi-Z
1	0	0
1	1	1



# XOR Gate

XOR Gate  
Exclusive-OR

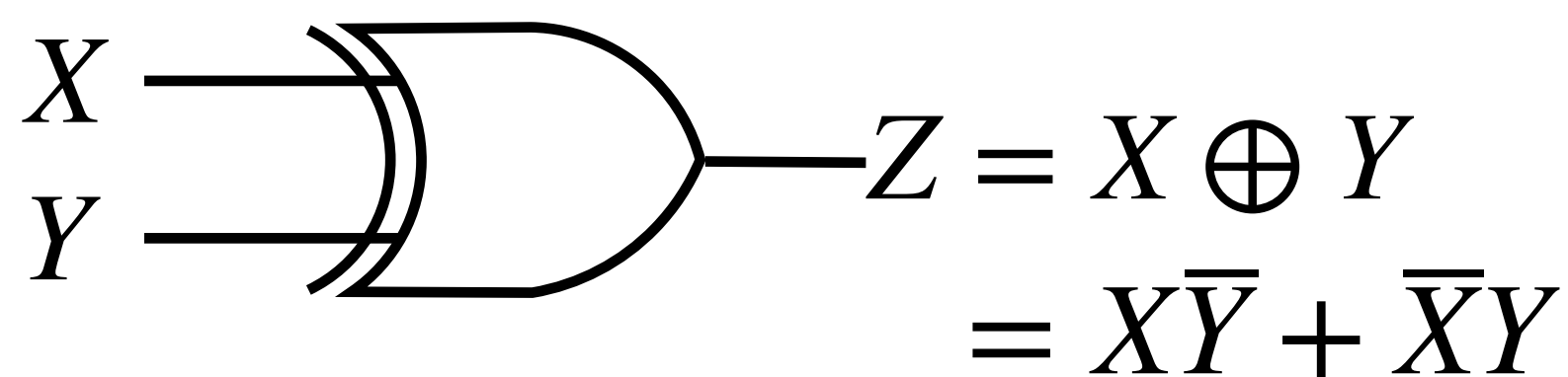


XOR Truth Table

$X$	$Y$	$Z = X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

# XOR Gate

XOR Gate  
Exclusive-OR



XOR Truth Table

$X$	$Y$	$Z = X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

# XOR Gate

- $X \oplus 0 = X$

- $X \oplus X = 0$

- $X \oplus \bar{Y} = \overline{X \oplus Y}$

- $X \oplus 1 = \bar{X}$

- $X \oplus \bar{X} = 1$

- $\bar{X} \oplus Y = \overline{X \oplus Y}$

# Odd and Even Functions

- Odd Functions
  - Outputs 1 if the **number of 1s** in the input is an **Odd** number
  - 2 variables: XOR
- Even Functions
  - Outputs 1 if the **number of 1s** in the input is an **Even** number
  - 2 variables: XNOR

# Odd Function K-Map

		Y			
		00	01	11	10
X	0		1		1
	1	1		1	

Z

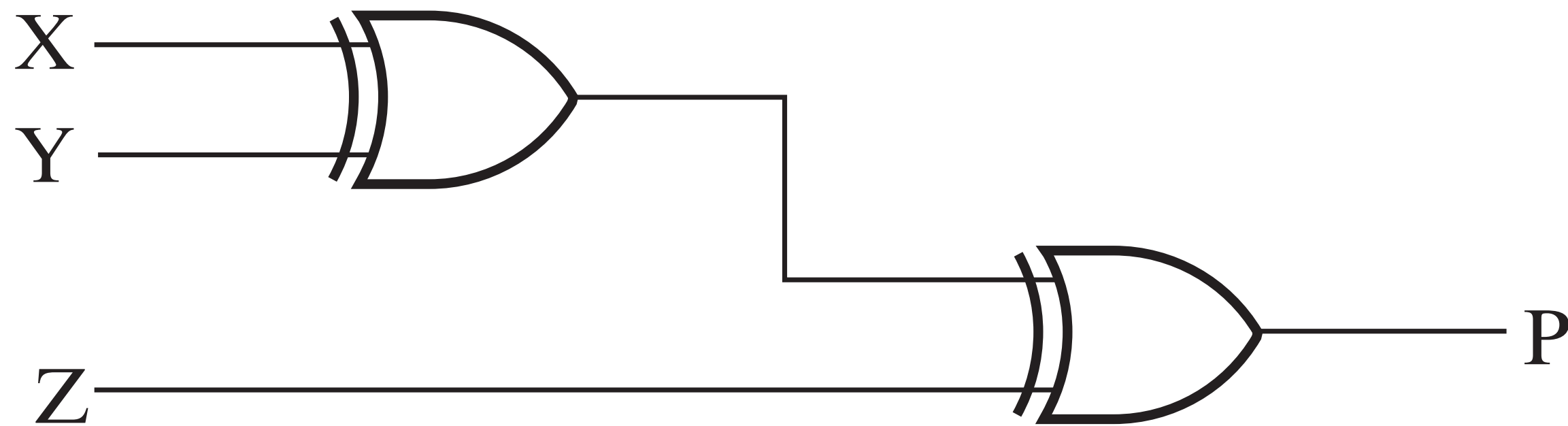
(a)  $X \oplus Y \oplus Z$

		C			
		00	01	11	10
A	00		1		1
	01	1		1	
	11		1		1
	10	1		1	

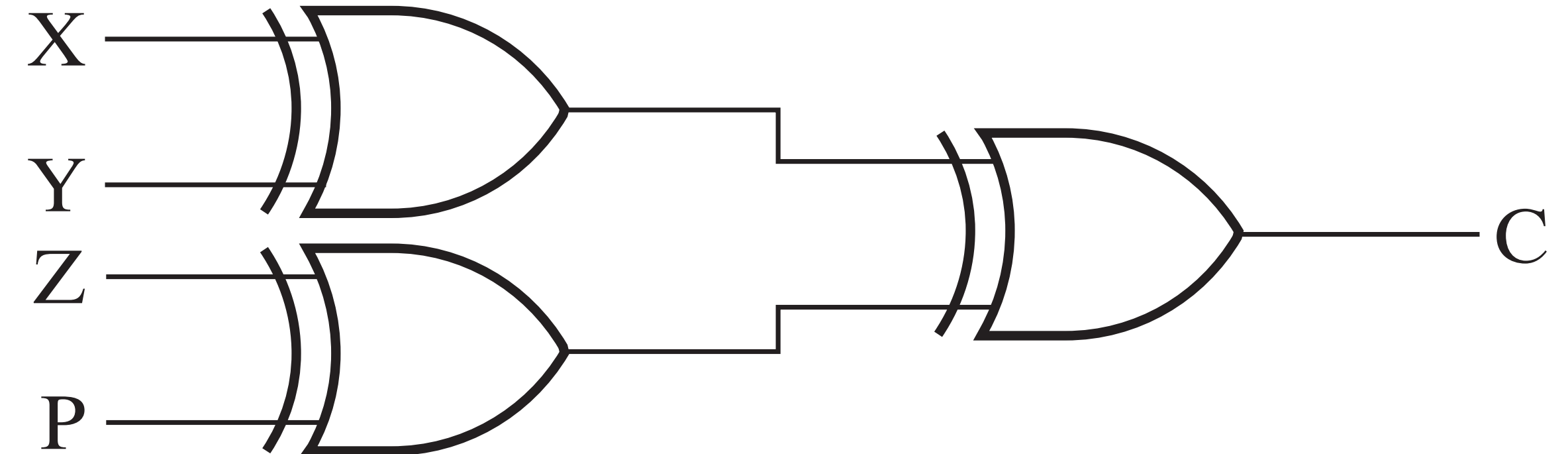
D

(b)  $A \oplus B \oplus C \oplus D$

# Odd Function Boolean Expressions



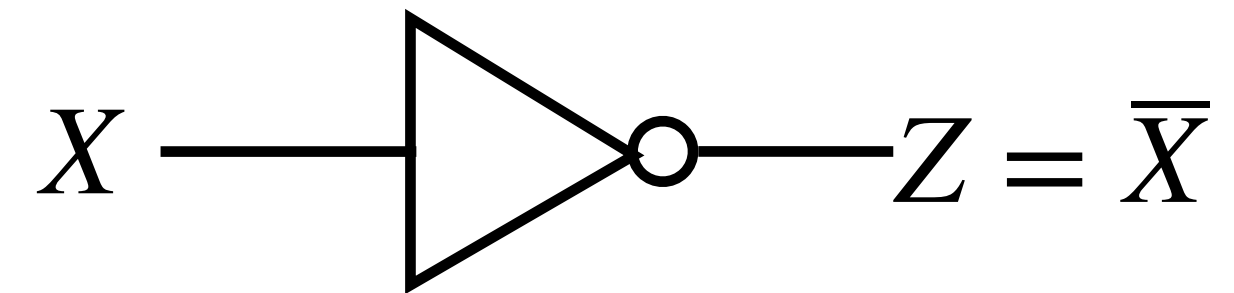
(a)  $P = X \oplus Y \oplus Z$



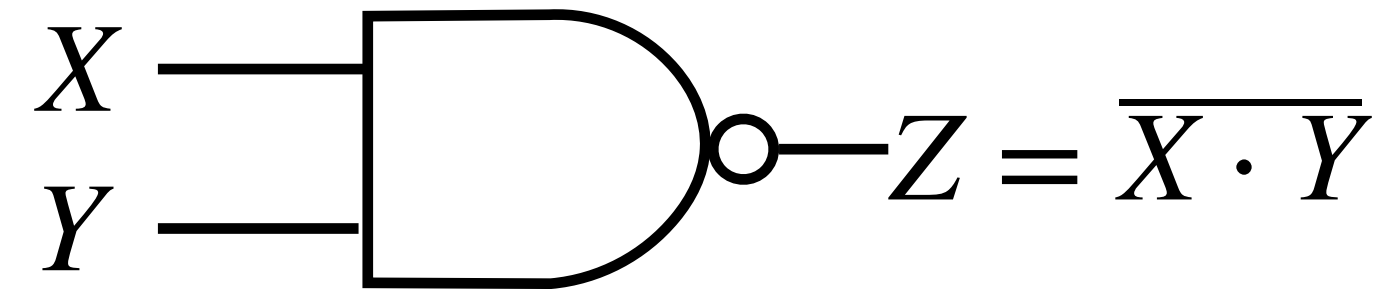
(b)  $C = X \oplus Y \oplus Z \oplus P$

# N-Gates

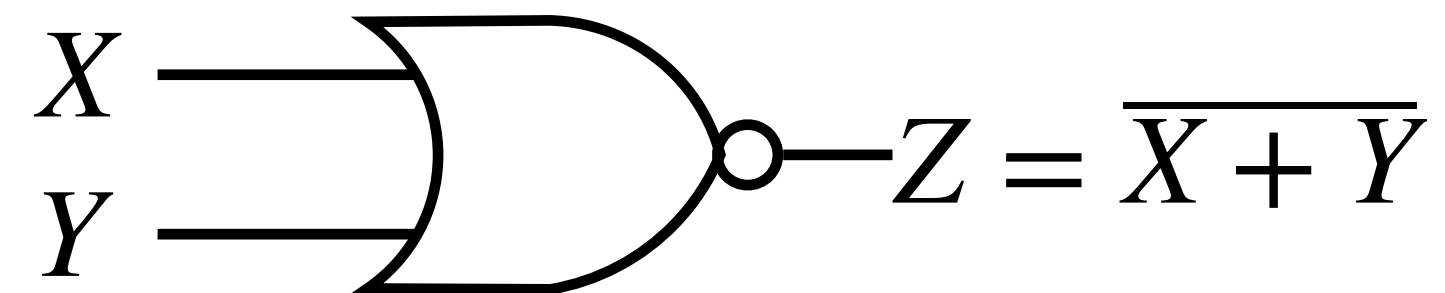
NOT Gate



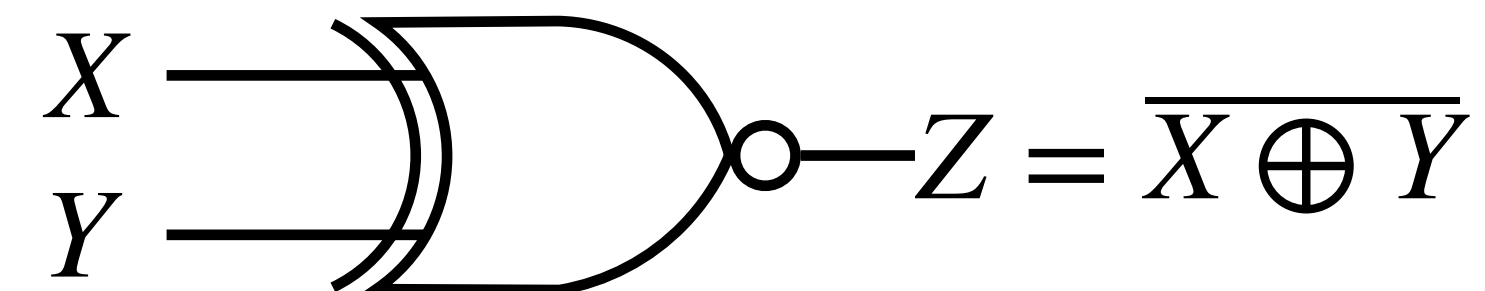
NAND Gate



NOR Gate



XNOR Gate



# Summary



# Summary

- Delay: Remember the Definitions!

# Summary

- Delay: Remember the Definitions!
- Gate Delay and Propagation Delay

# Summary

- Delay: Remember the Definitions!
- Gate Delay and Propagation Delay
- Transport and Inertial Delays

# Summary

- Delay: Remember the Definitions!
  - Gate Delay and Propagation Delay
  - Transport and Inertial Delays
  - Standard Load

# Summary

- Delay: Remember the Definitions!
  - Gate Delay and Propagation Delay
  - Transport and Inertial Delays
  - Standard Load
- Other Gates: Remember the Definitions!

# Summary

- Delay: Remember the Definitions!
  - Gate Delay and Propagation Delay
  - Transport and Inertial Delays
  - Standard Load
- Other Gates: Remember the Definitions!
  - XOR, Buffer, 3-State, NAND, NOR