30.01.20 12:40

CSCI 150
Introduction to Digital and Computer System Design
Lecture 2: Combinational Logical Circuits IV

Jetic Gū

2020 Winter Semester (S1)

Overview

- Focus: Boolean Algebra
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch2 2.4, 2.5; v5: Ch2 2.4, 2.5
- Core Ideas:
    1. Boolean Algebra III: K-Map

P0
Review

Boolean Algebra I&II

- AND, OR, NOT Operators and Gates
  - Simple digital circuit implementation
  - Algebraic manipulation using Binary Identities
- Standard Forms
  - Minterm & Maxterm
  - Sum of Products & Product of Sums

Review

P1
Optimisation

Boolean Algebra III:
K-Map

Cost Criteria;
Map and Map Manipulation

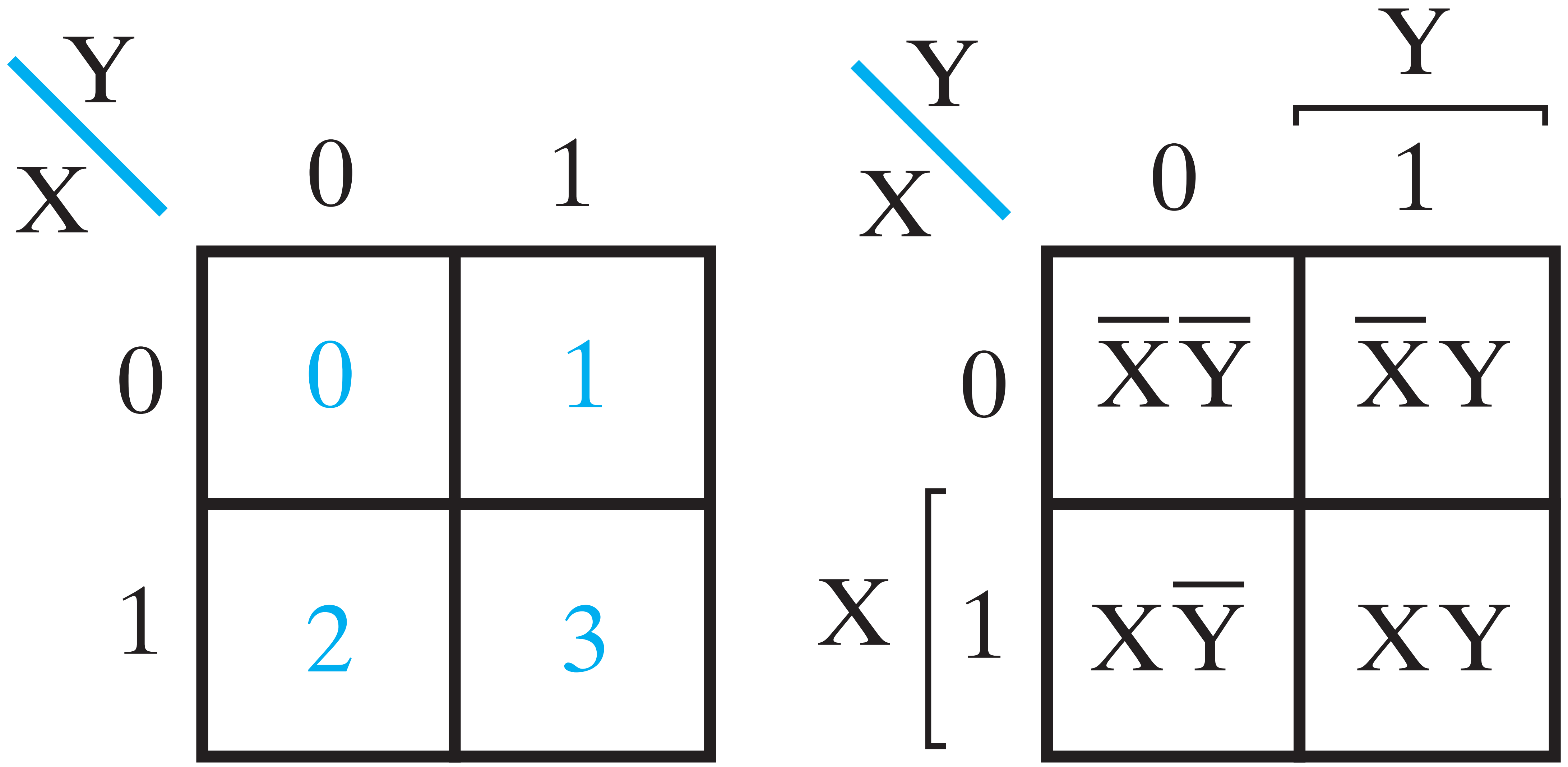
Summary

K-Map

- Karnaugh Map, or just K-Map
  - For optimising 2-4 variable boolean expressions
    - Skip: 5,6 variable K-Maps can also be drawn but are not very intuitive to use

P1
Optimisation

Two Variable Maps

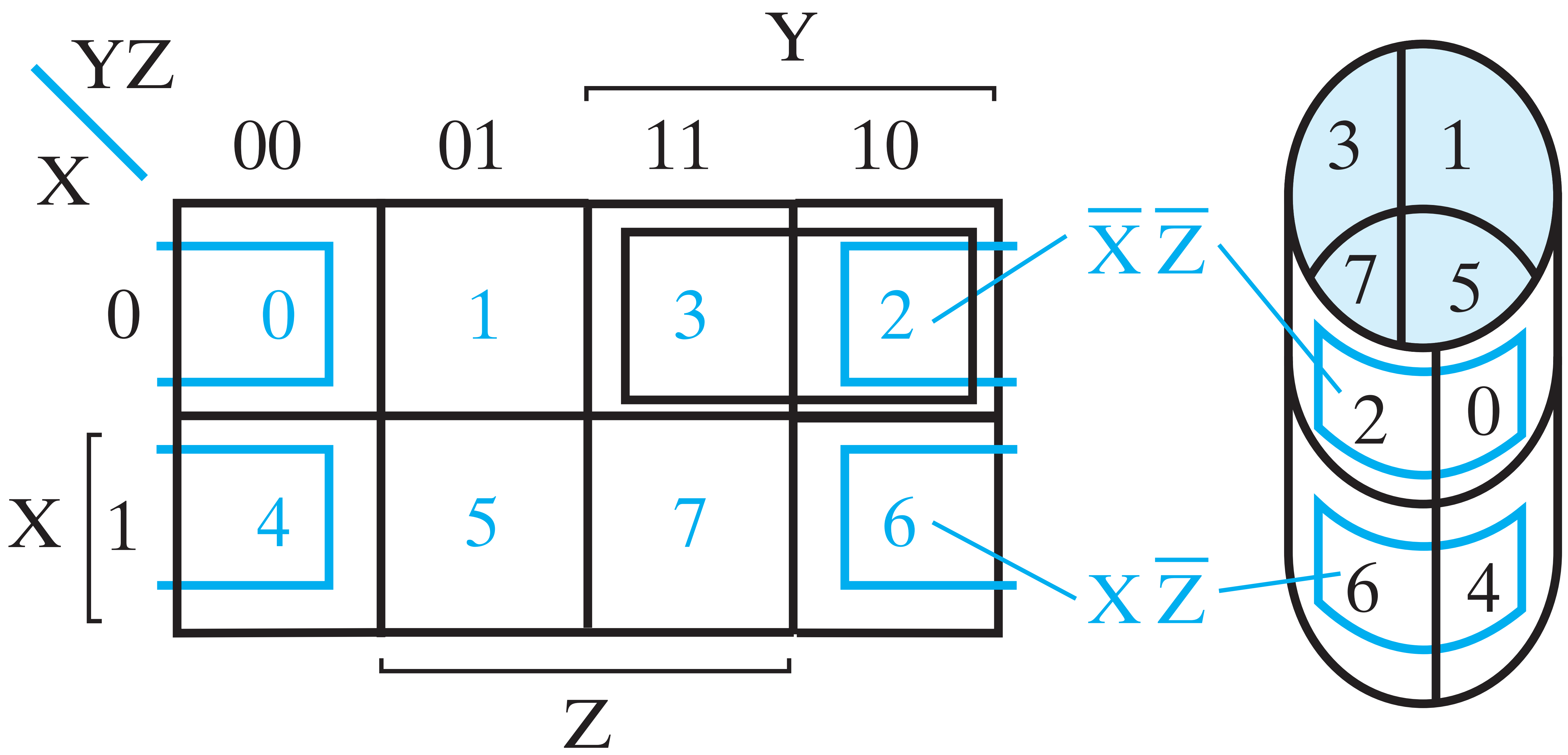


- Number of squares in each map is equal to the number of minterms for the same number of variables, light blue digit above is the index (of minterm)
- Two squares are adjacent if they only differ in one variable
- Binary value inside at each position indicates the truth table value for that term

Concept

P1
Optimisation

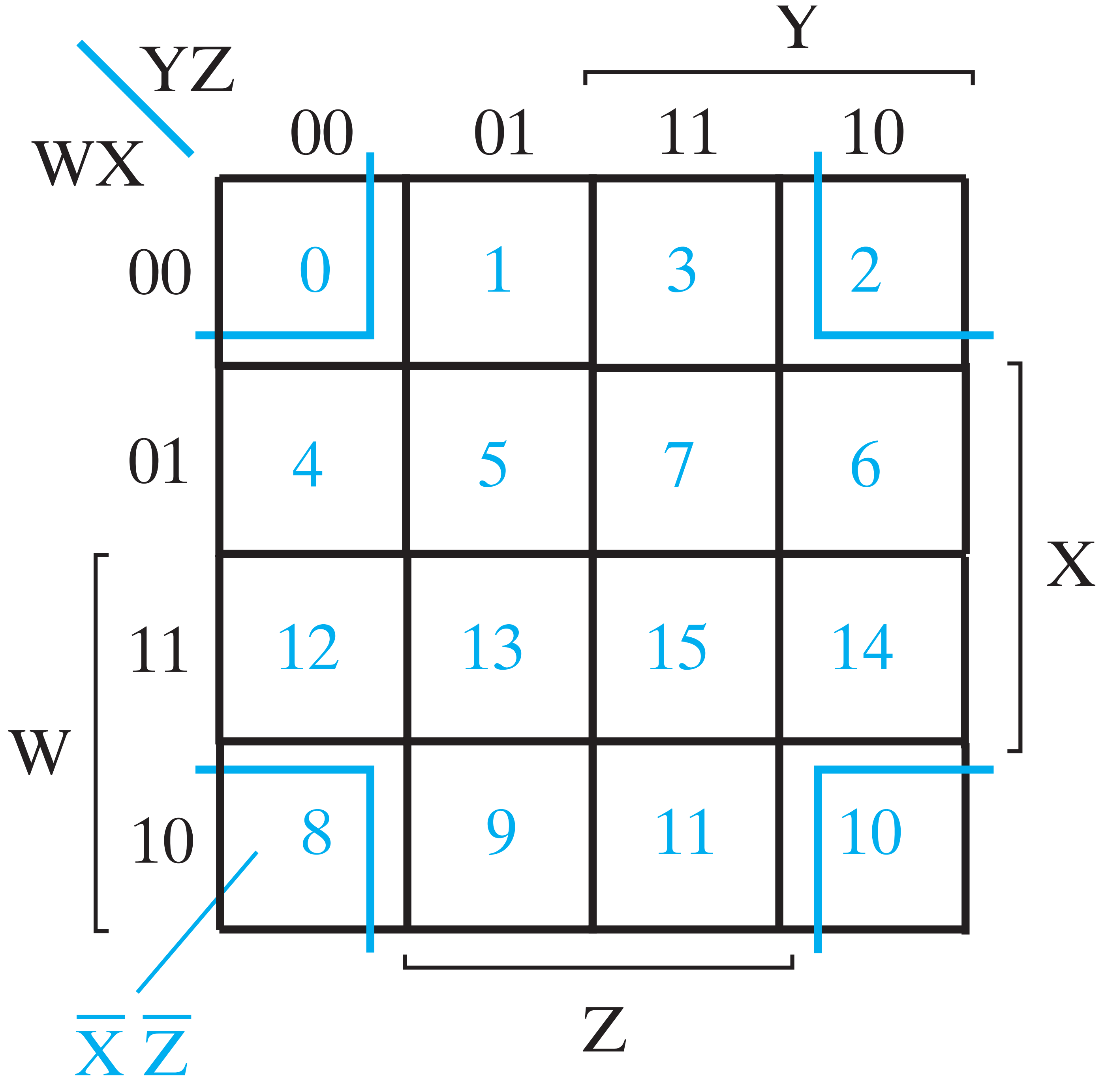
Three Variable Maps

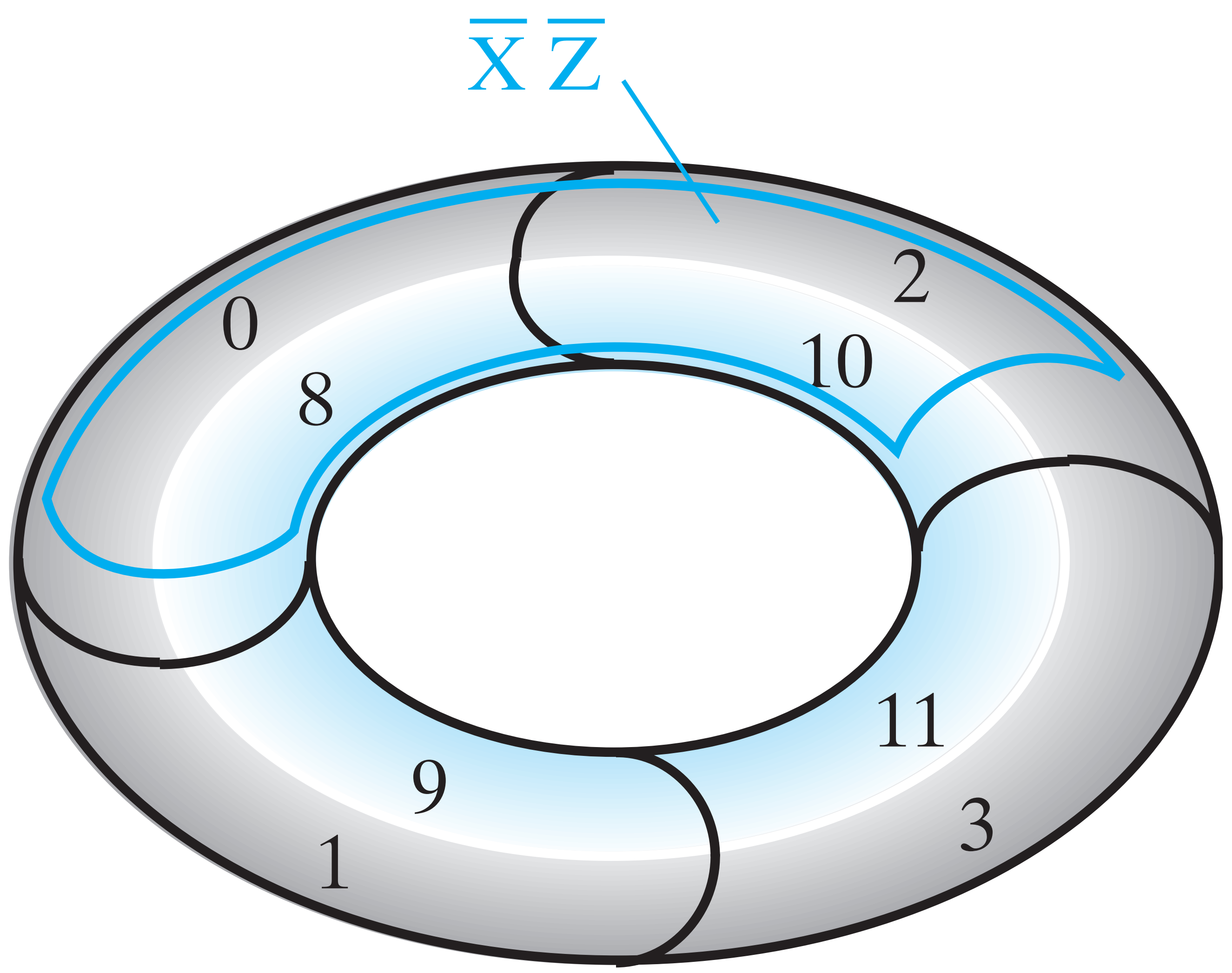


- Number of squares in each map is equal to the number of minterms for the same number of variables, light blue digit above is the index (of minterm)
- Two squares are adjacent if they only differ in one variable
- Binary value inside at each position indicates the truth table value for that term

Concept

Four Variable Maps

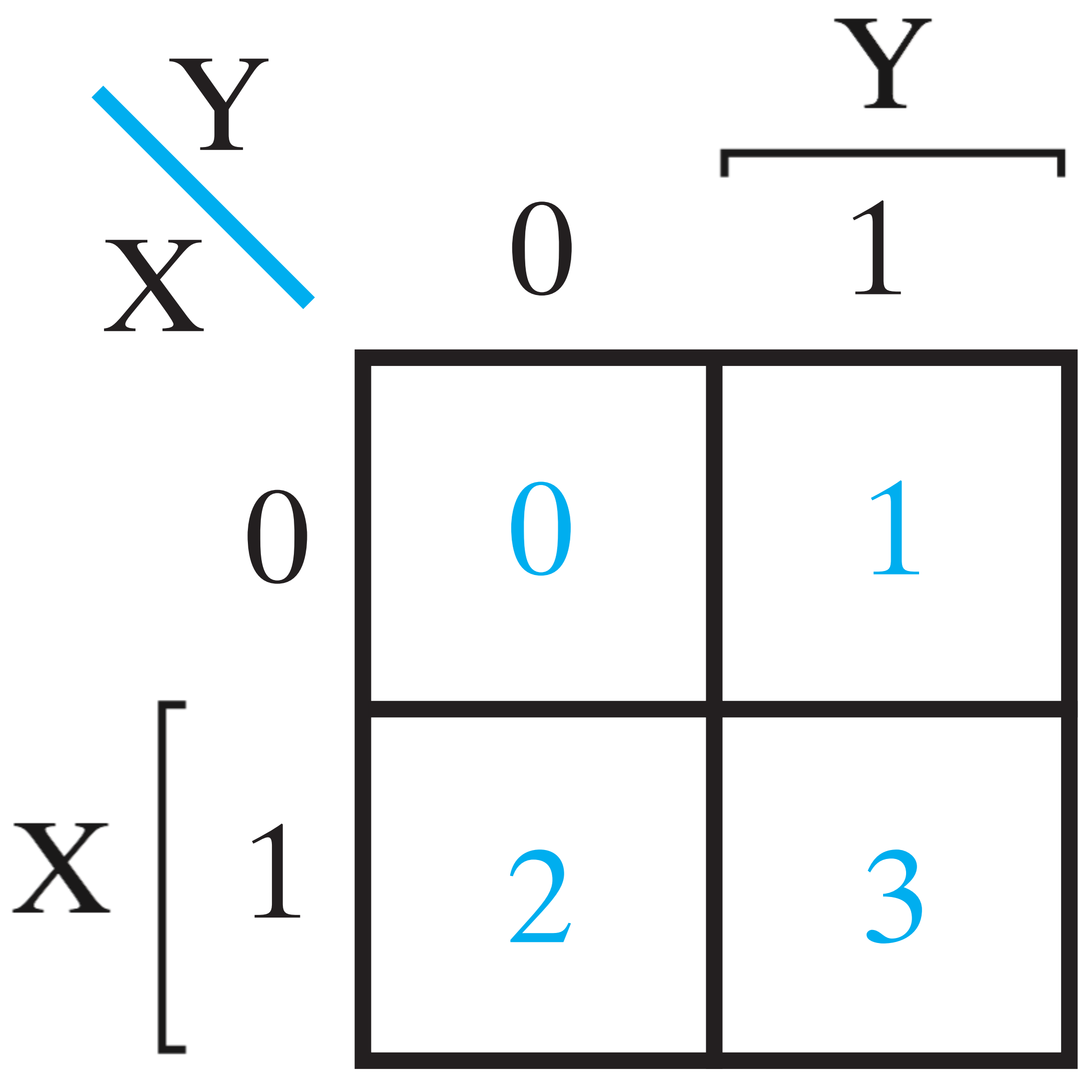




- Number of squares in each map is equal to the number of minterms for the same number of variables, light blue digit above is the index (of minterm)
- Two squares are adjacent if they only differ in one variable
- Binary value inside at each position indicates the truth table value for that term

Concept

Two Variable Maps
Optimisation



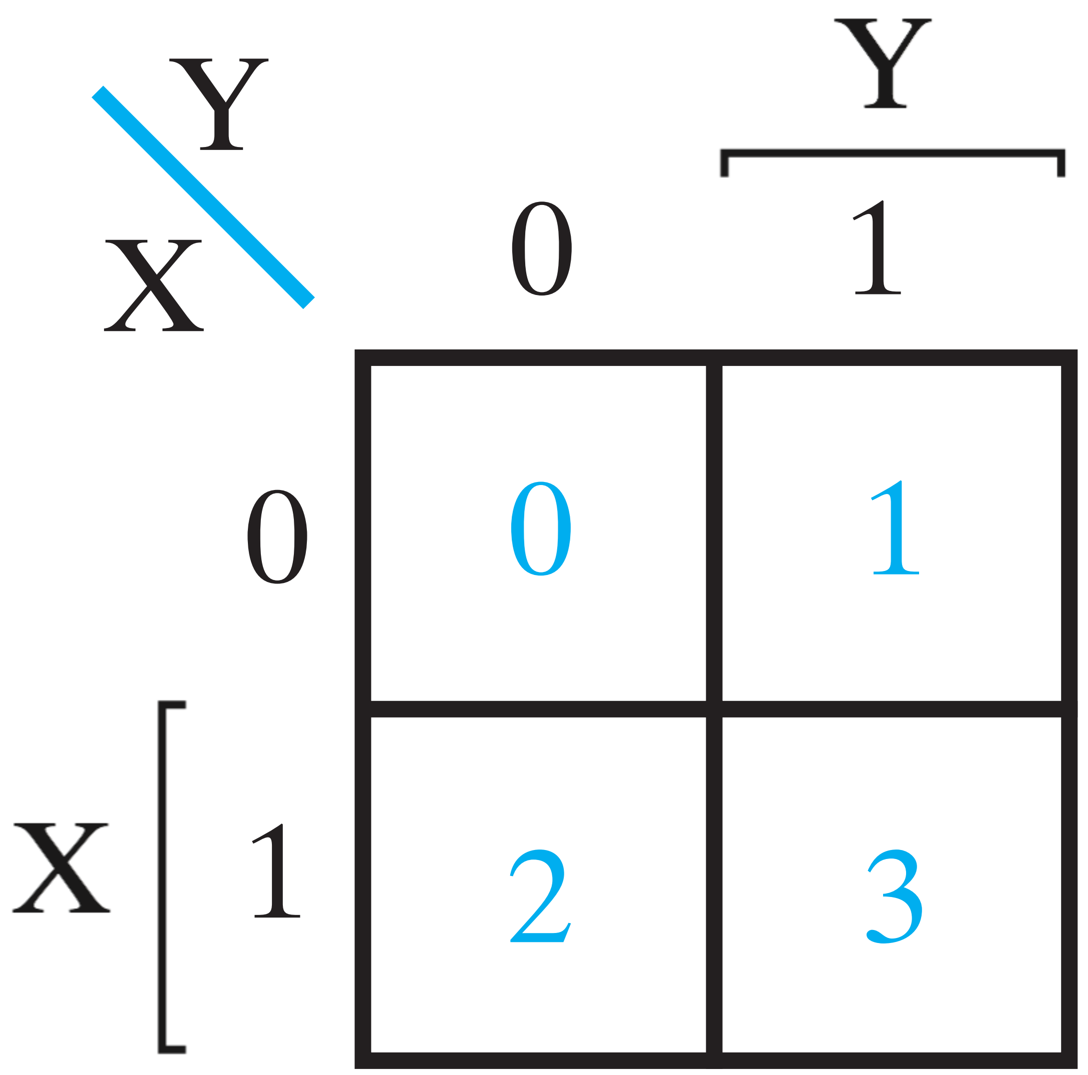
Truth Table

X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

Concept

P1 Optimisation

Two Variable Maps Optimisation



Truth Table

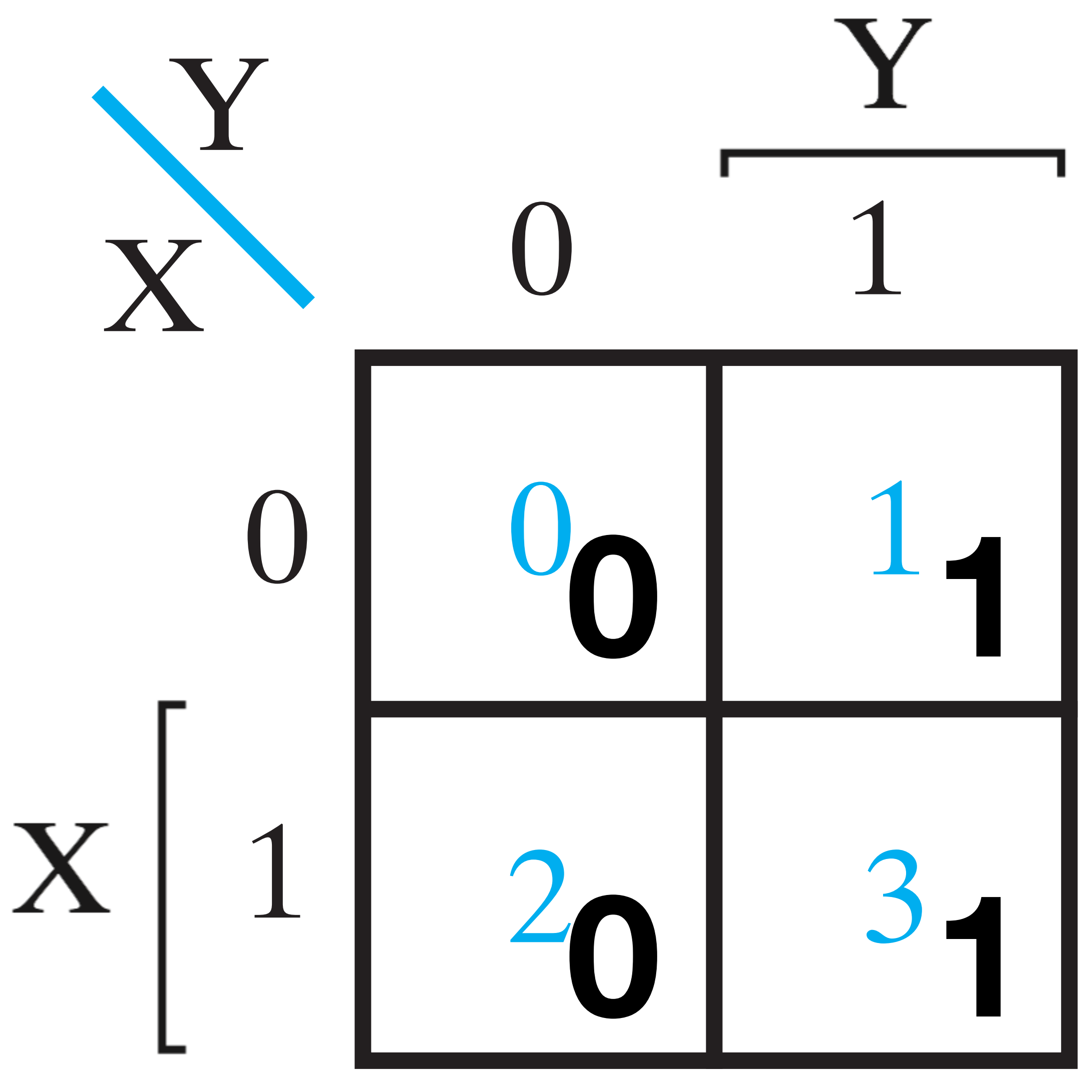
X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values

Concept

P1
Optimisation

Two Variable Maps
Optimisation



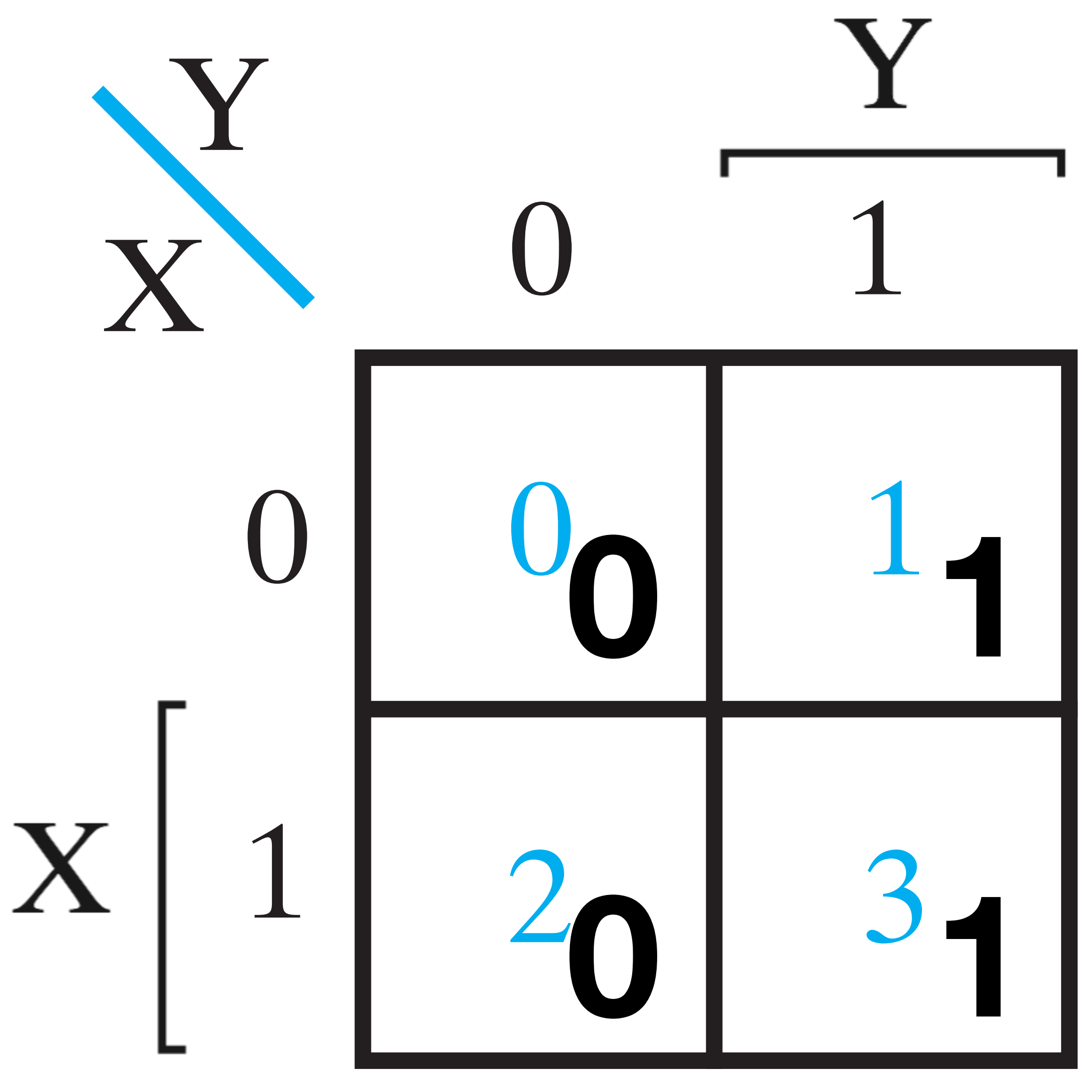
Truth Table

X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values

Concept

Two Variable Maps Optimisation



Truth Table

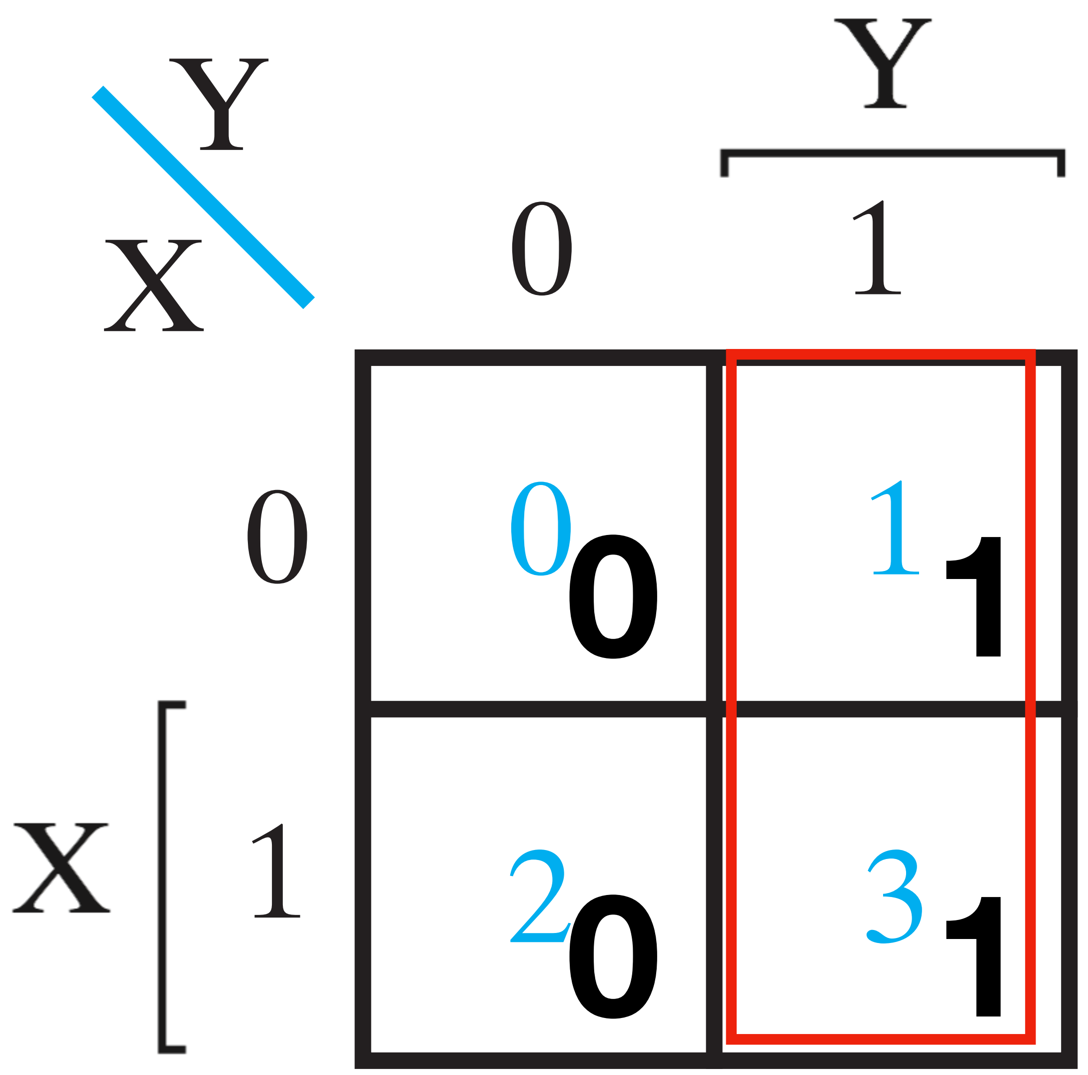
X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s

Concept

P1
Optimisation

Two Variable Maps Optimisation



Truth Table

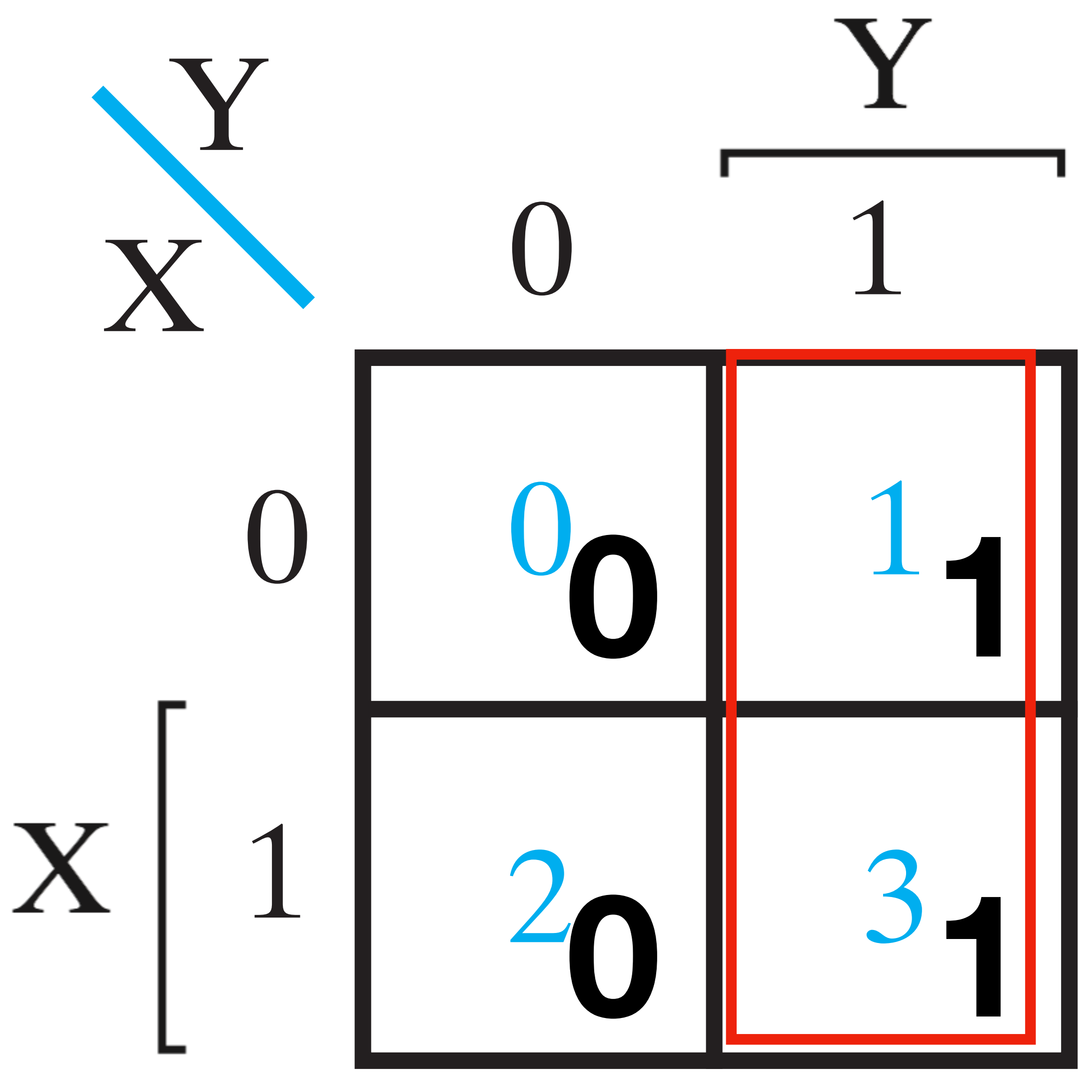
X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s

Concept

P1
Optimisation

Two Variable Maps Optimisation



Truth Table

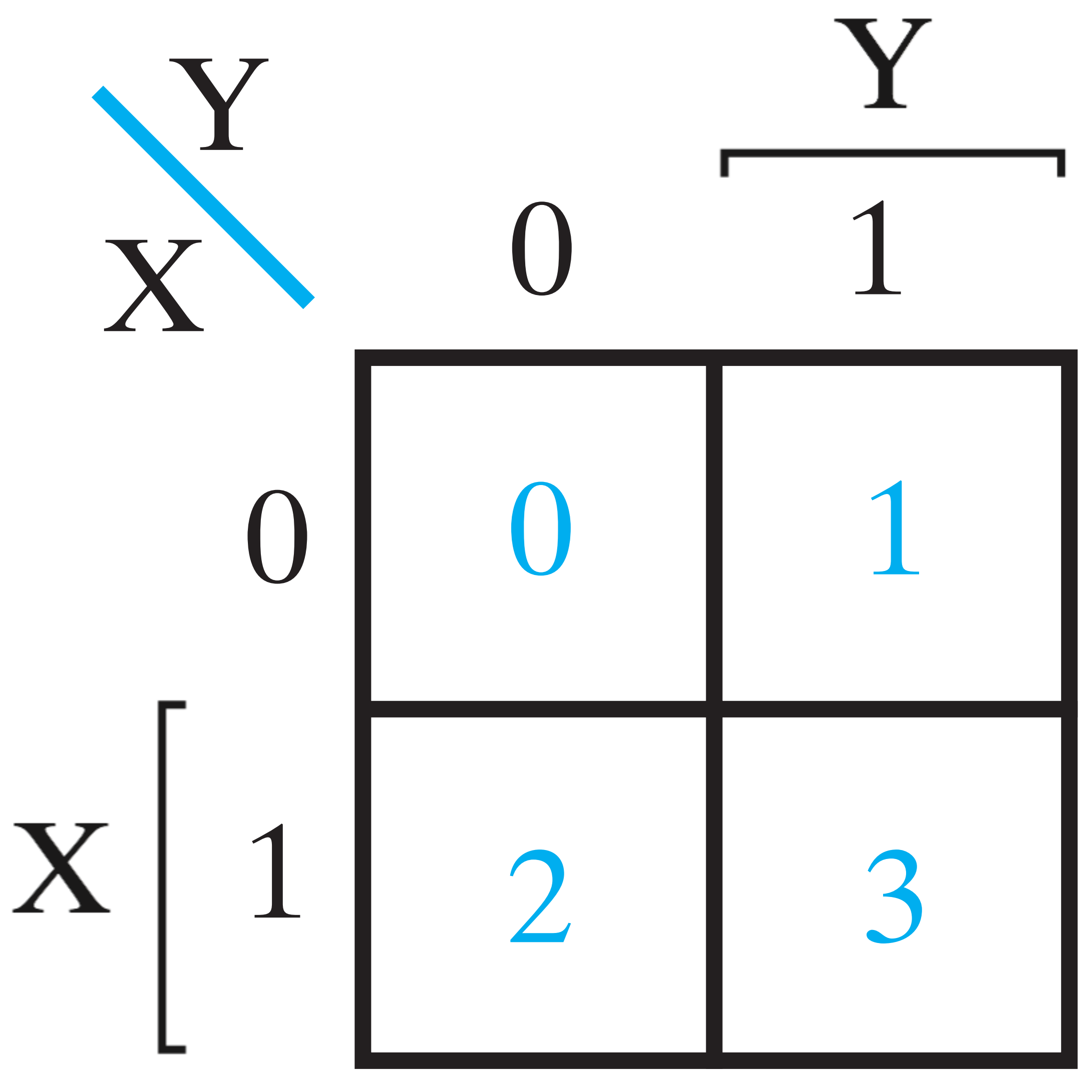
X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Concept

P1
Optimisation

Two Variable Maps
Optimisation



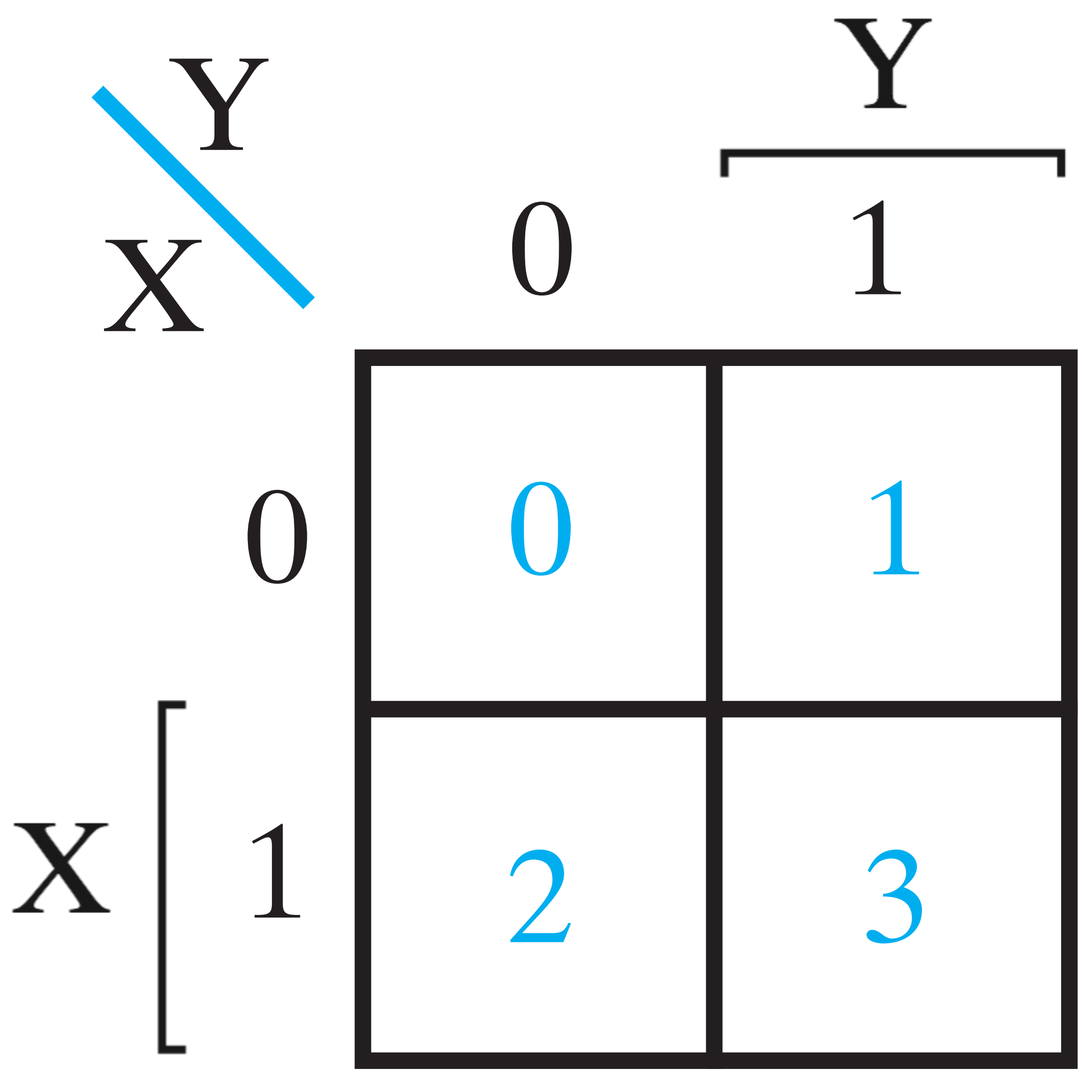
Truth Table

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	1

Demo

P1
Optimisation

Two Variable Maps Optimisation



Truth Table

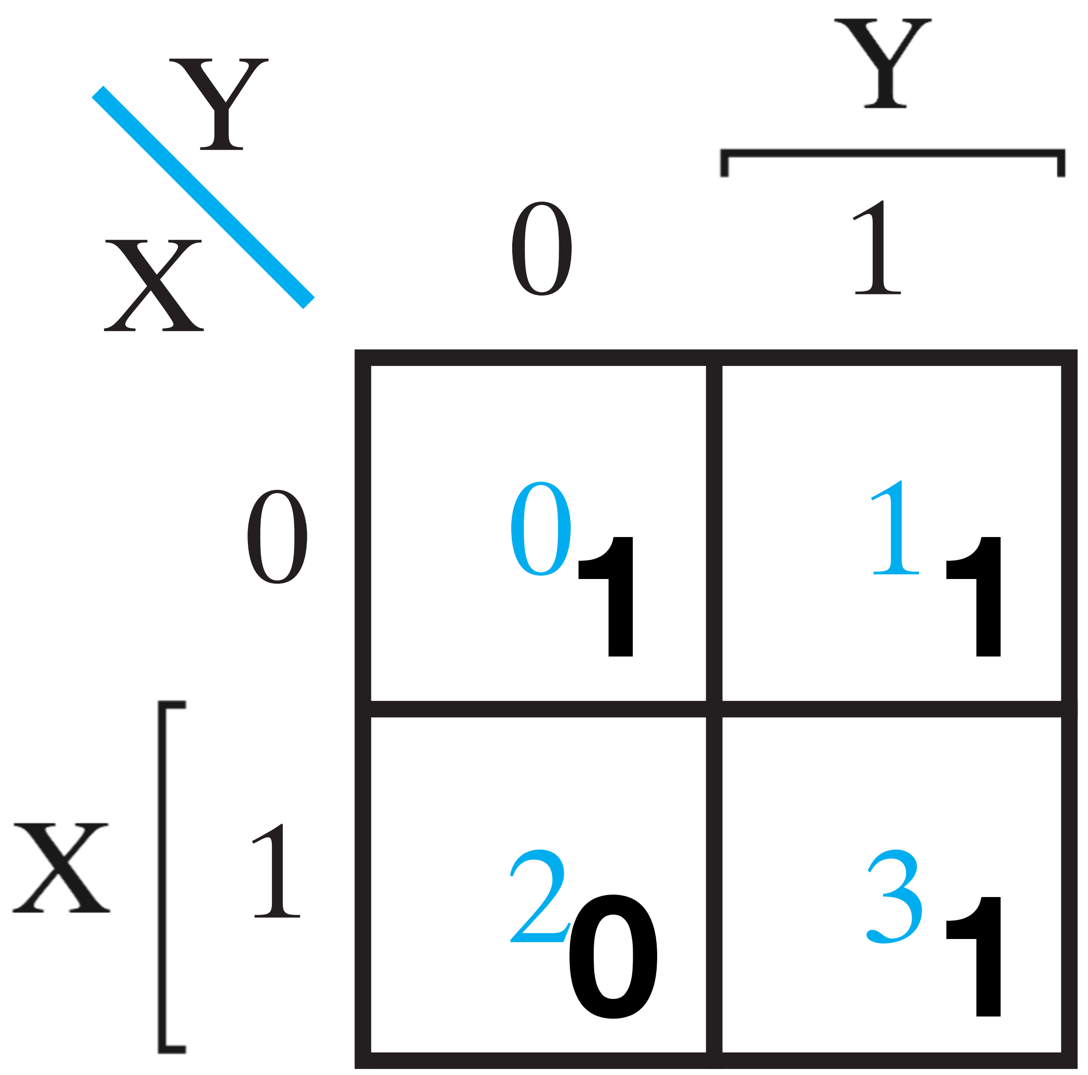
X	Y	F
0	0	1
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values

Demo

P1
Optimisation

Two Variable Maps Optimisation



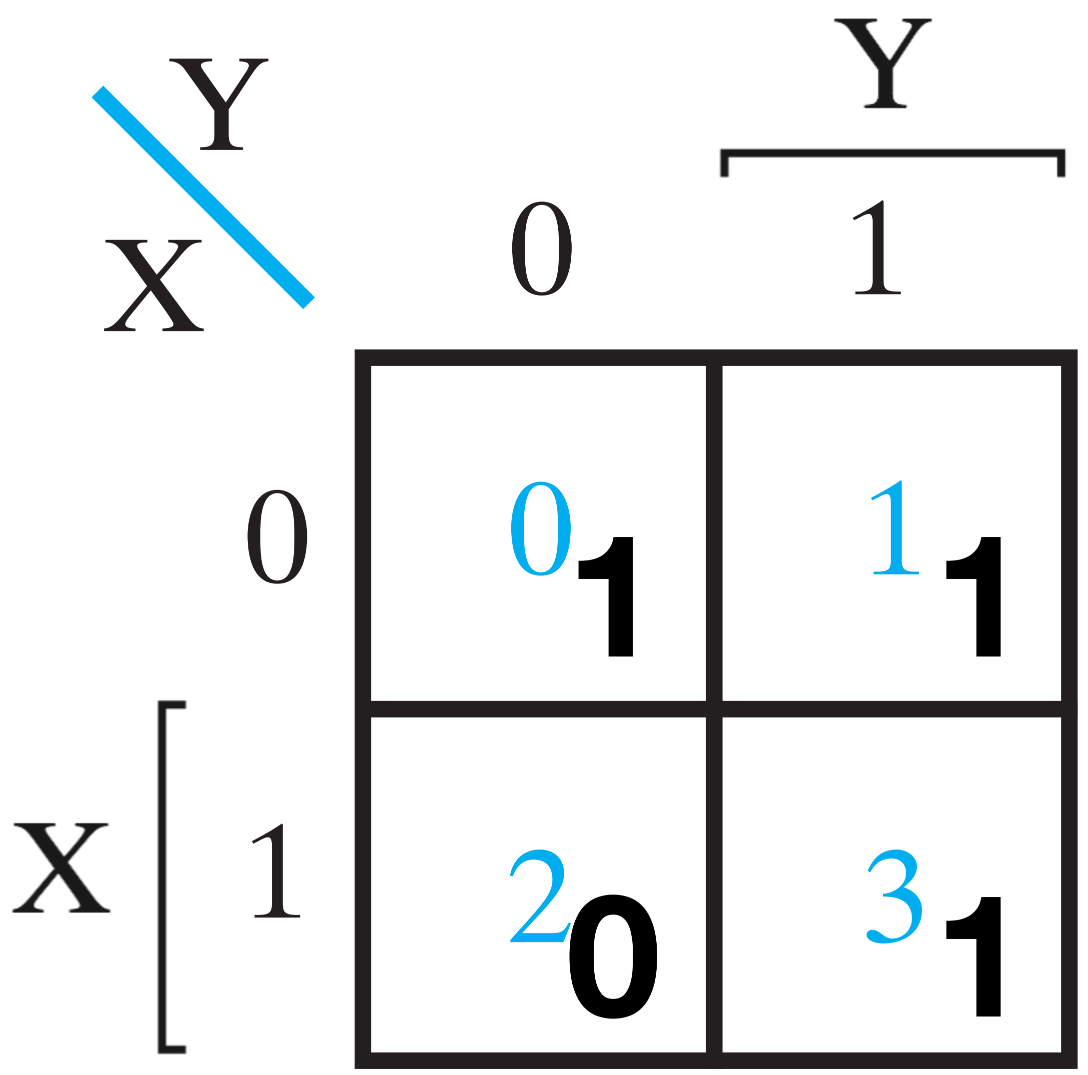
- Step 1: Enter the values

Truth Table

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	1

Demo

Two Variable Maps Optimisation



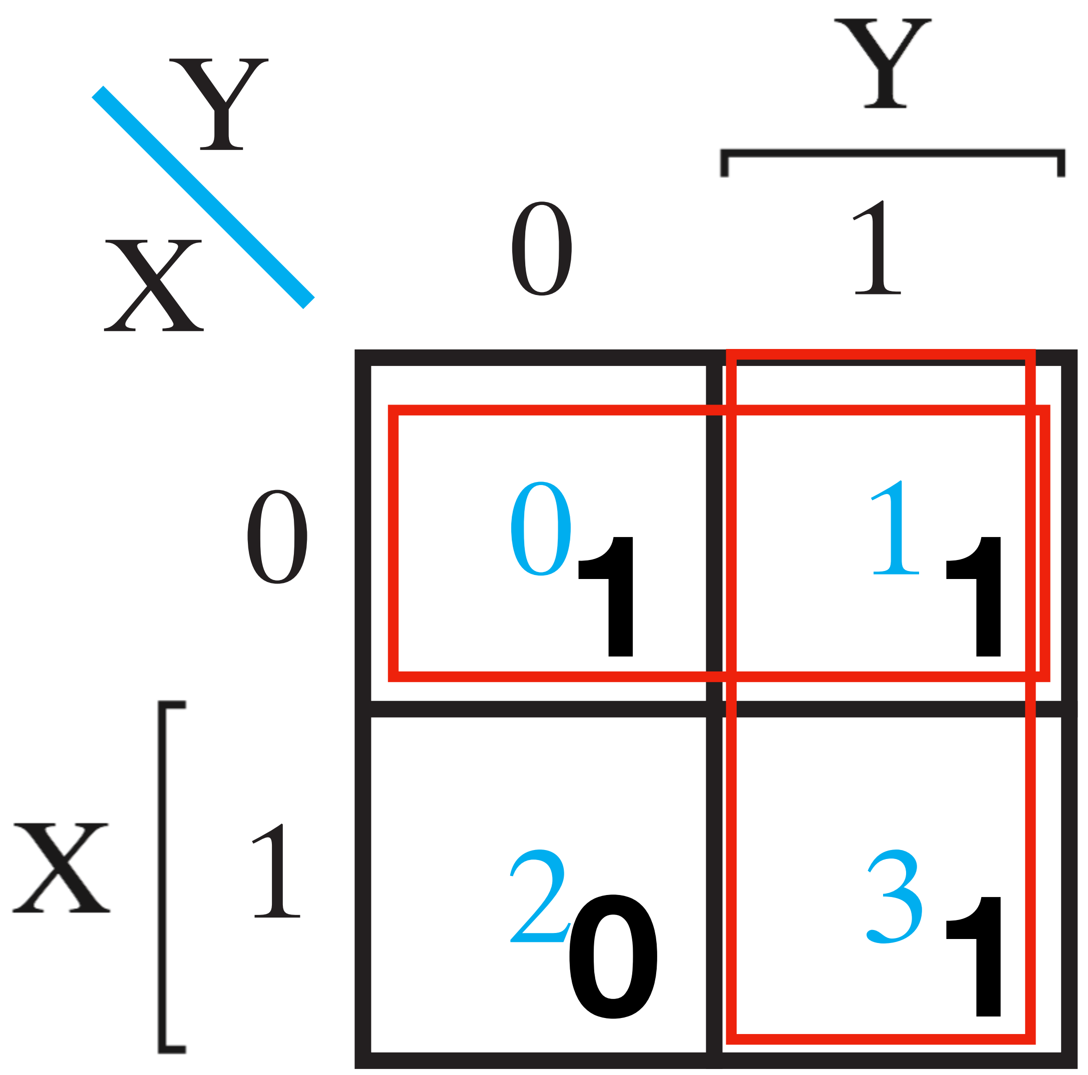
Truth Table

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s

Demo

Two Variable Maps Optimisation

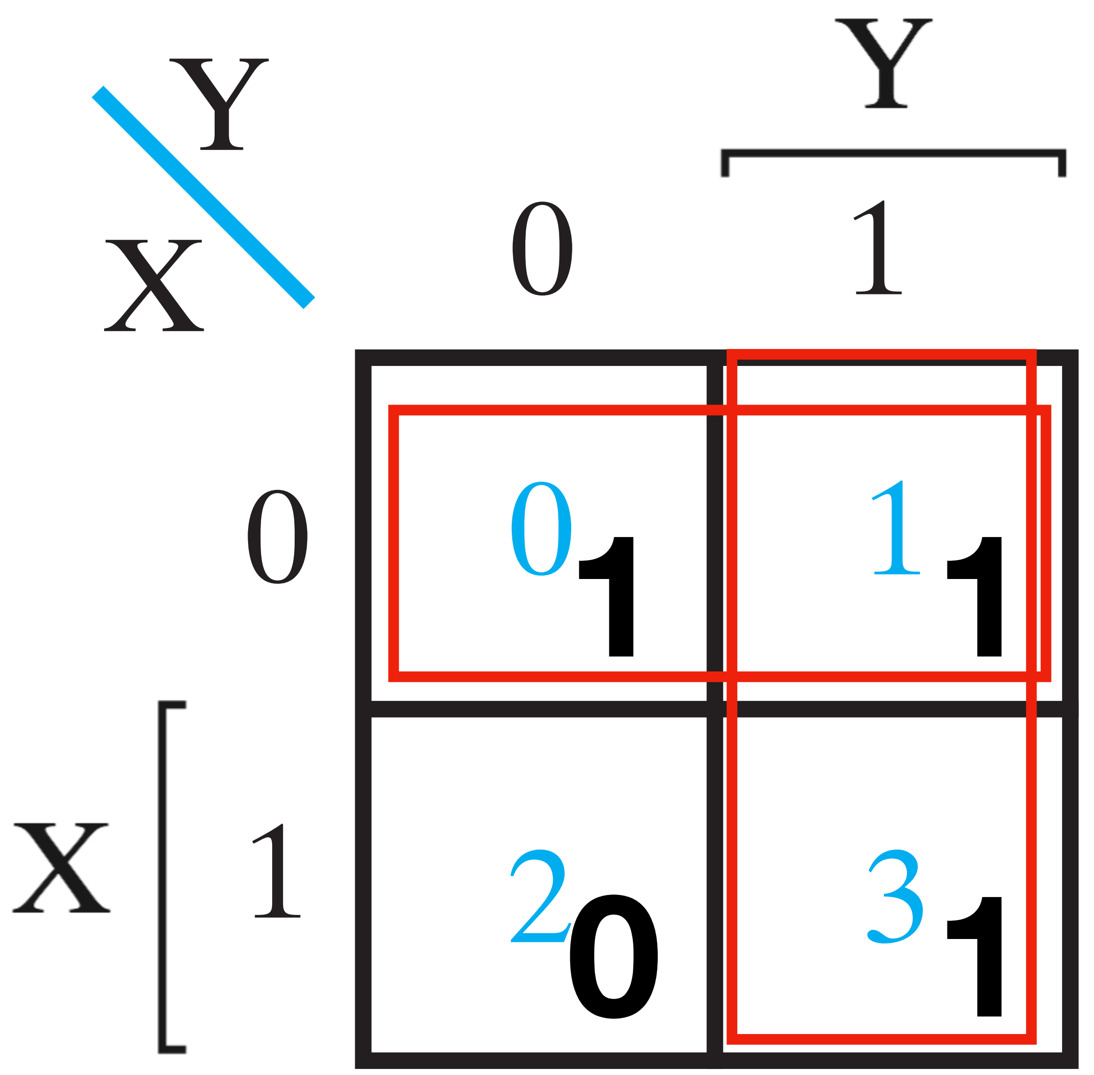


Truth Table

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s

Two Variable Maps
Optimisation



Truth Table

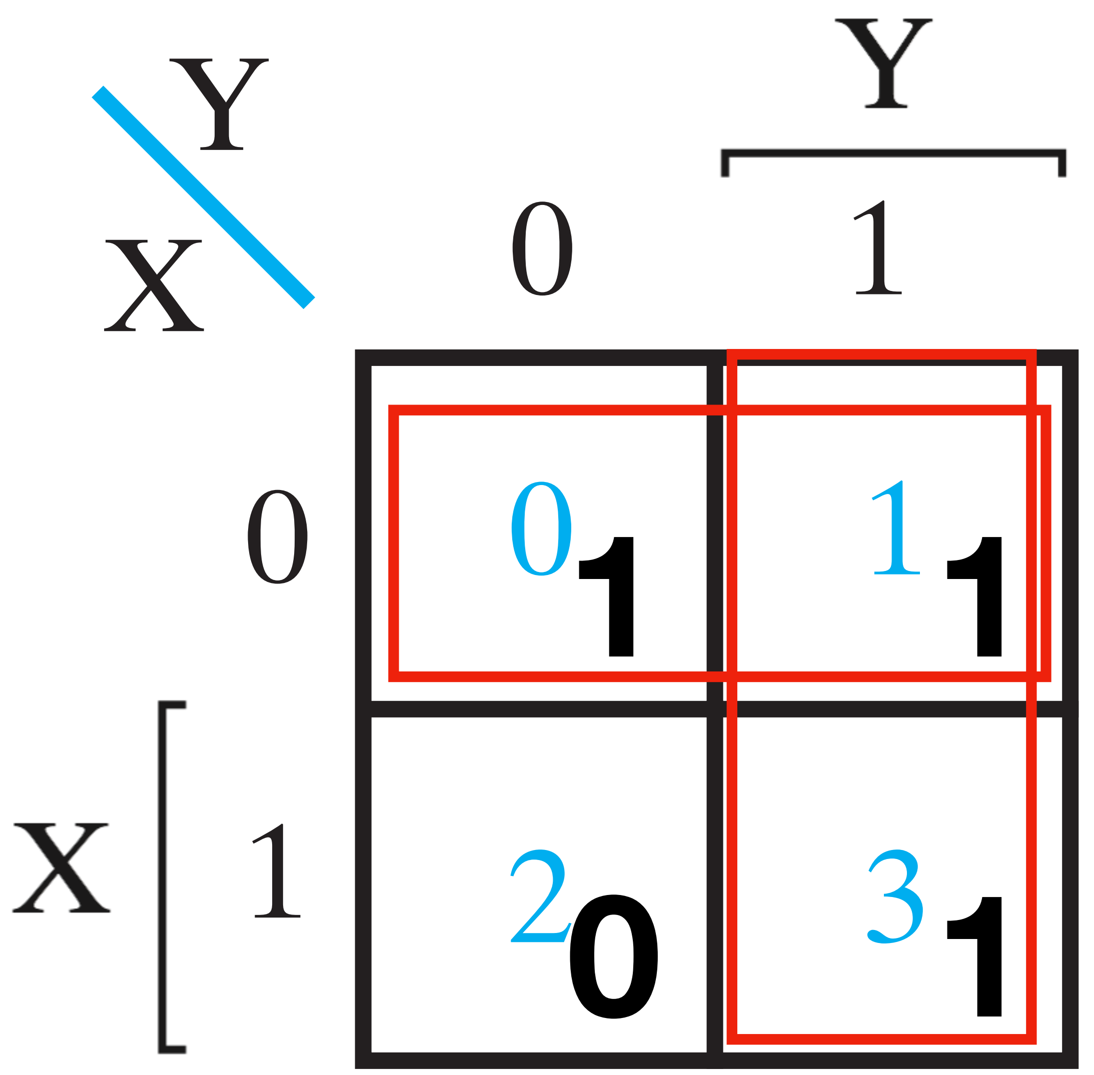
X	Y	F
0	0	1
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Demo

P1
Optimisation

Two Variable Maps Optimisation



$$\overline{X} + Y$$

Truth Table

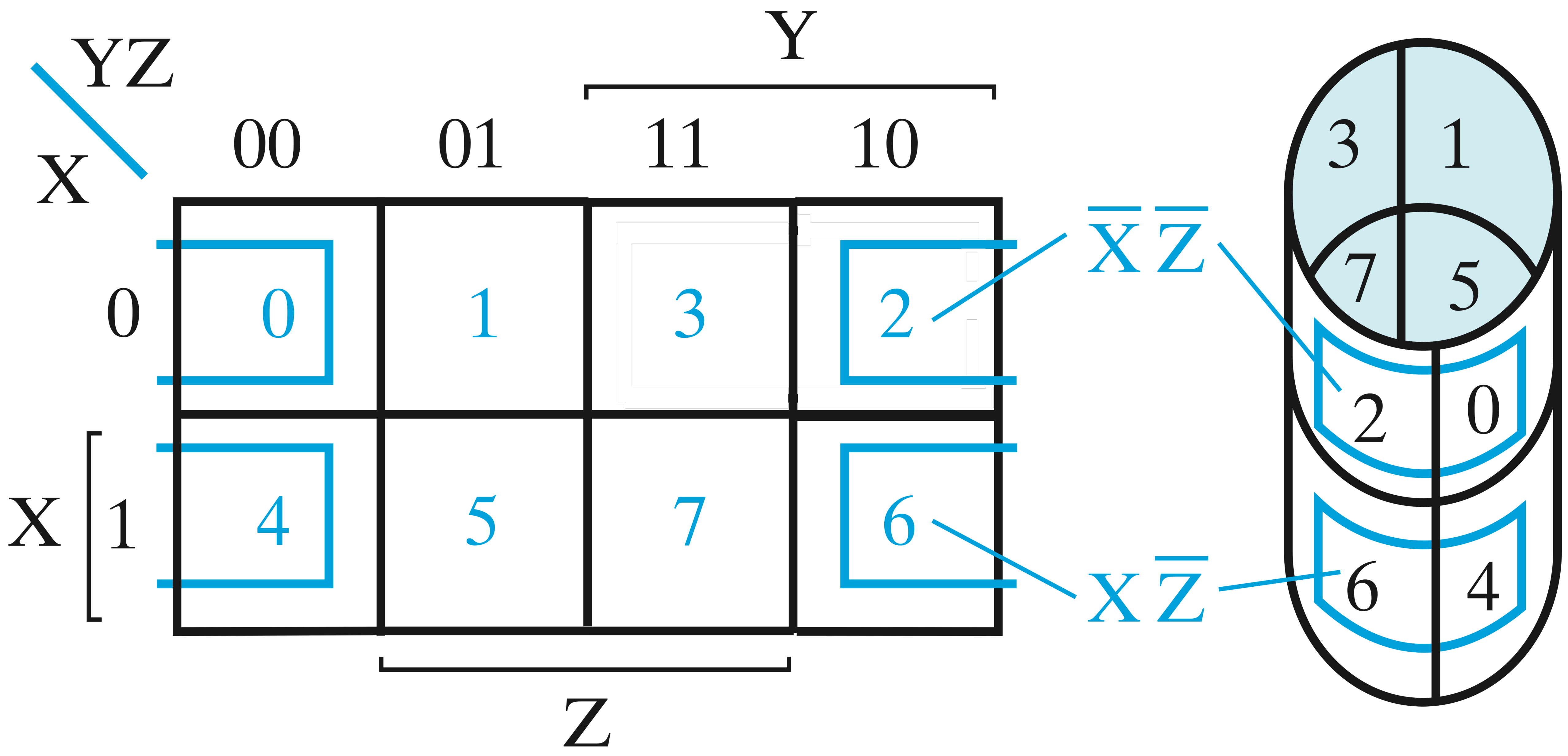
X	Y	F
0	0	1
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Demo

P1 Optimisation

Three Variable Maps Optimisation

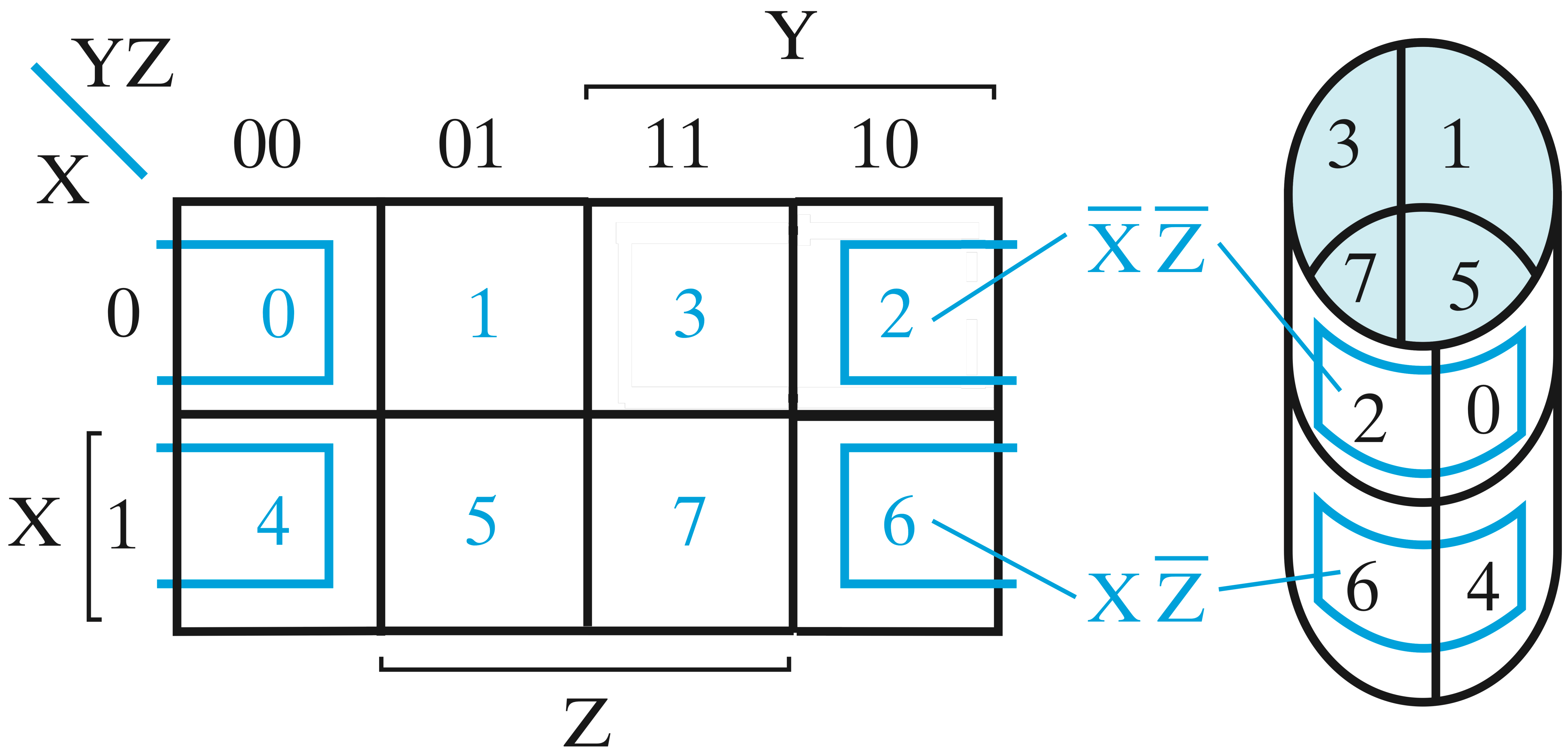


$$F(X, Y, Z) = \Sigma m(0,1,2,3,4,5)$$

Concept

P1
Optimisation

Three Variable Maps Optimisation

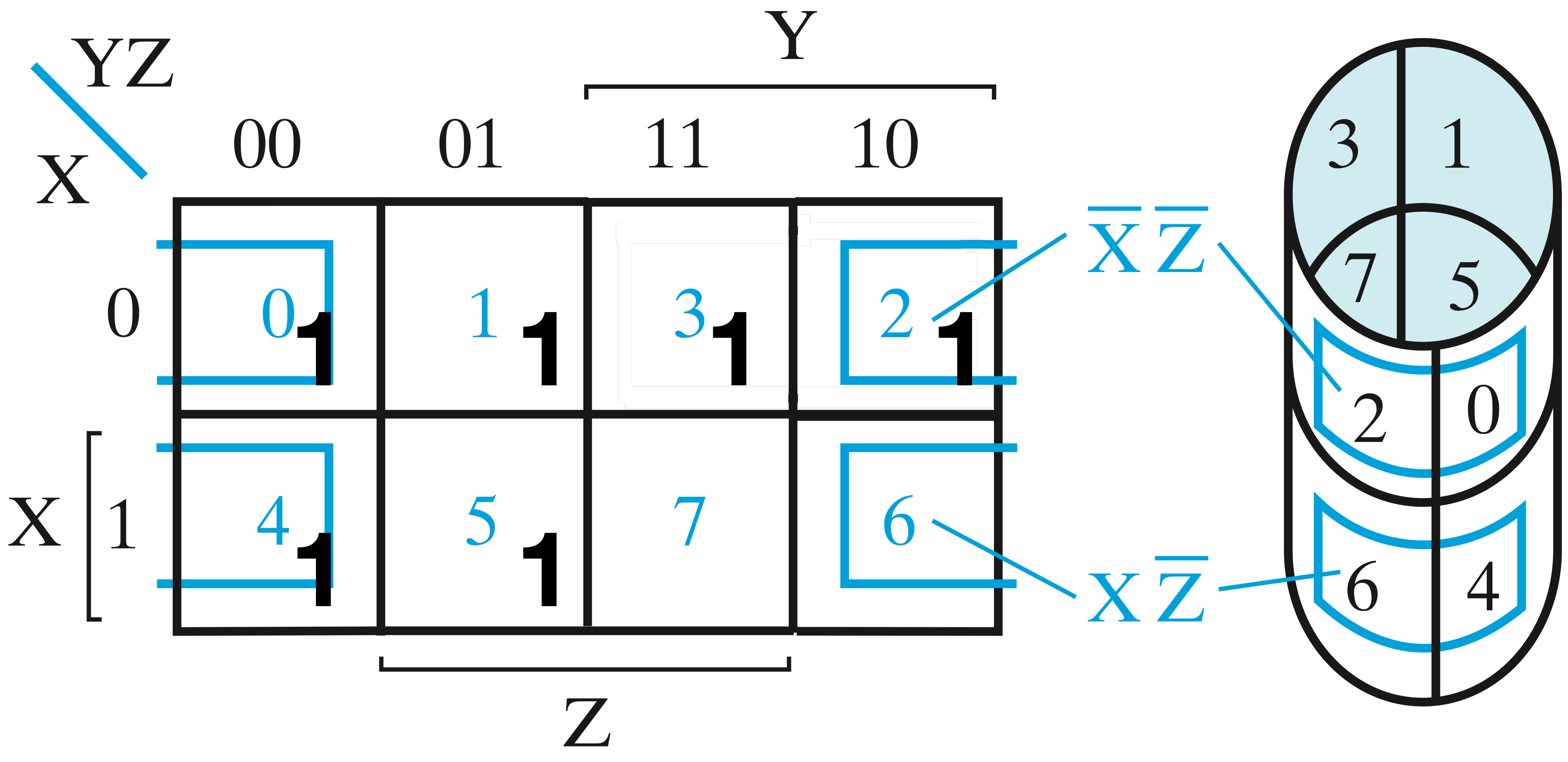


$$F(X,Y,Z) = \Sigma m(0,1,2,3,4,5)$$

- Step 1: Enter the values

Concept

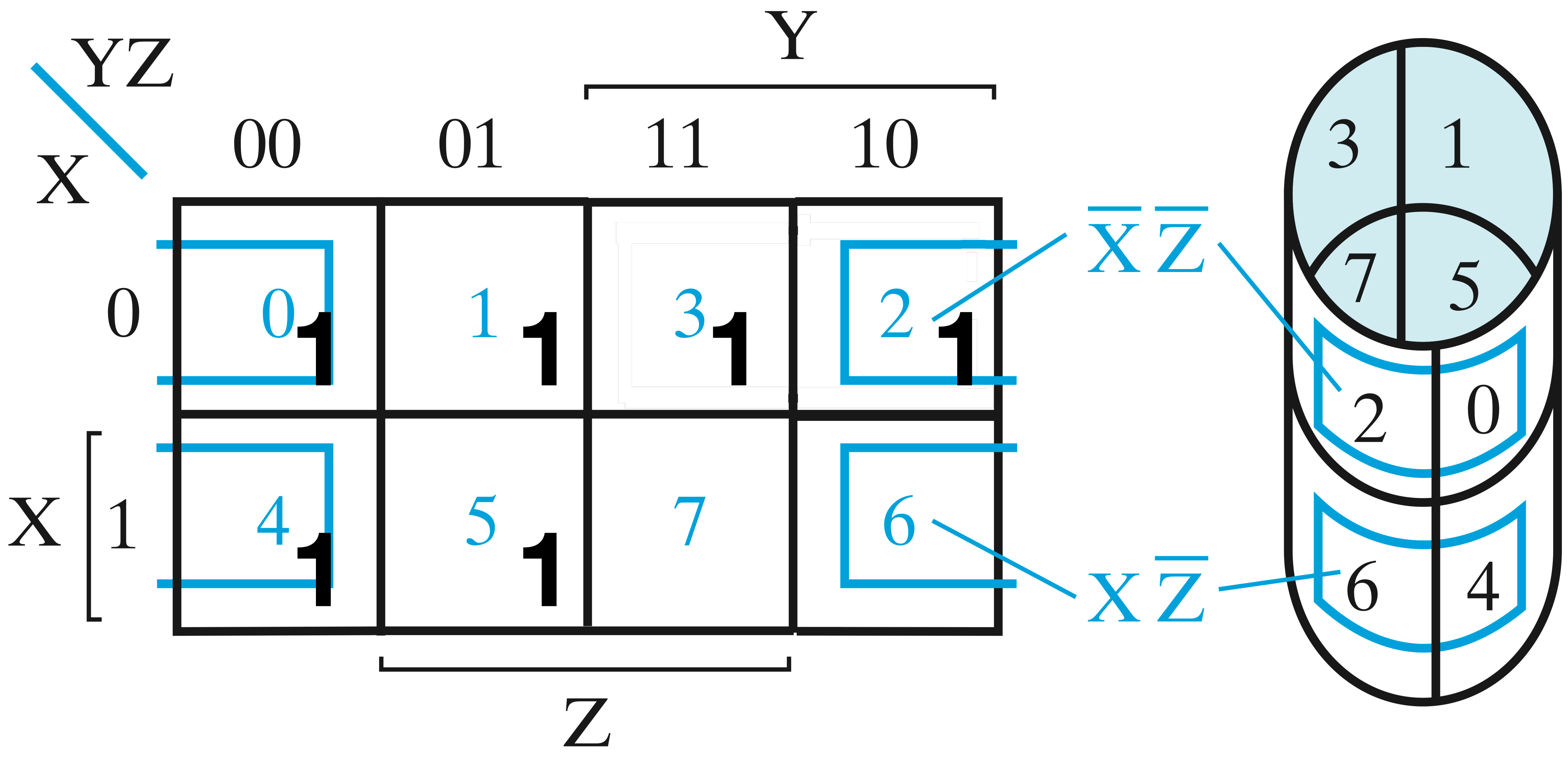
Three Variable Maps Optimisation



$$F(X, Y, Z) = \Sigma m(0,1,2,3,4,5)$$

- Step 1: Enter the values

Three Variable Maps Optimisation

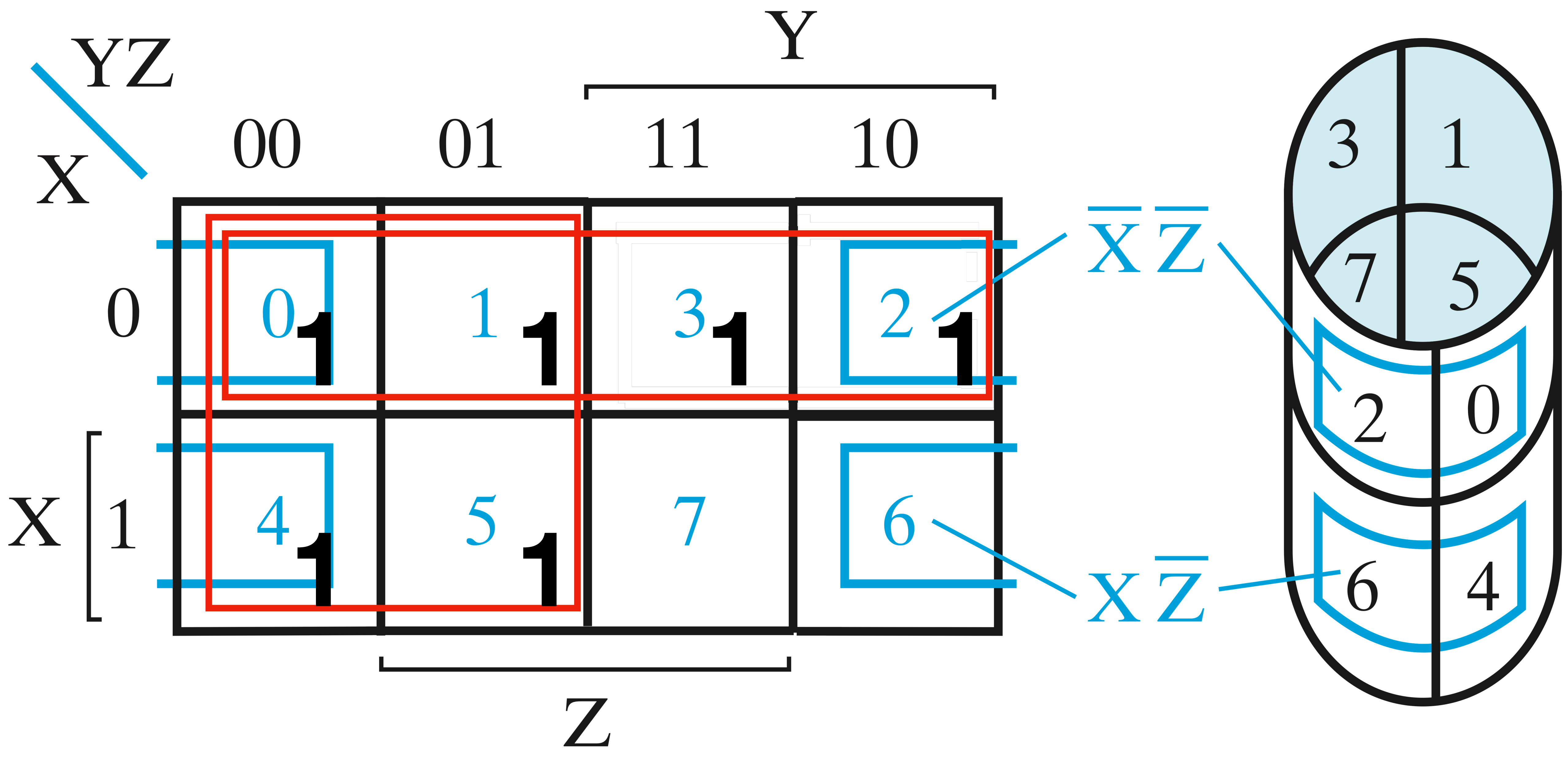


$F(X, Y, Z) = \Sigma m(0,1,2,3,4,5)$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s

P1 Optimisation

Three Variable Maps Optimisation

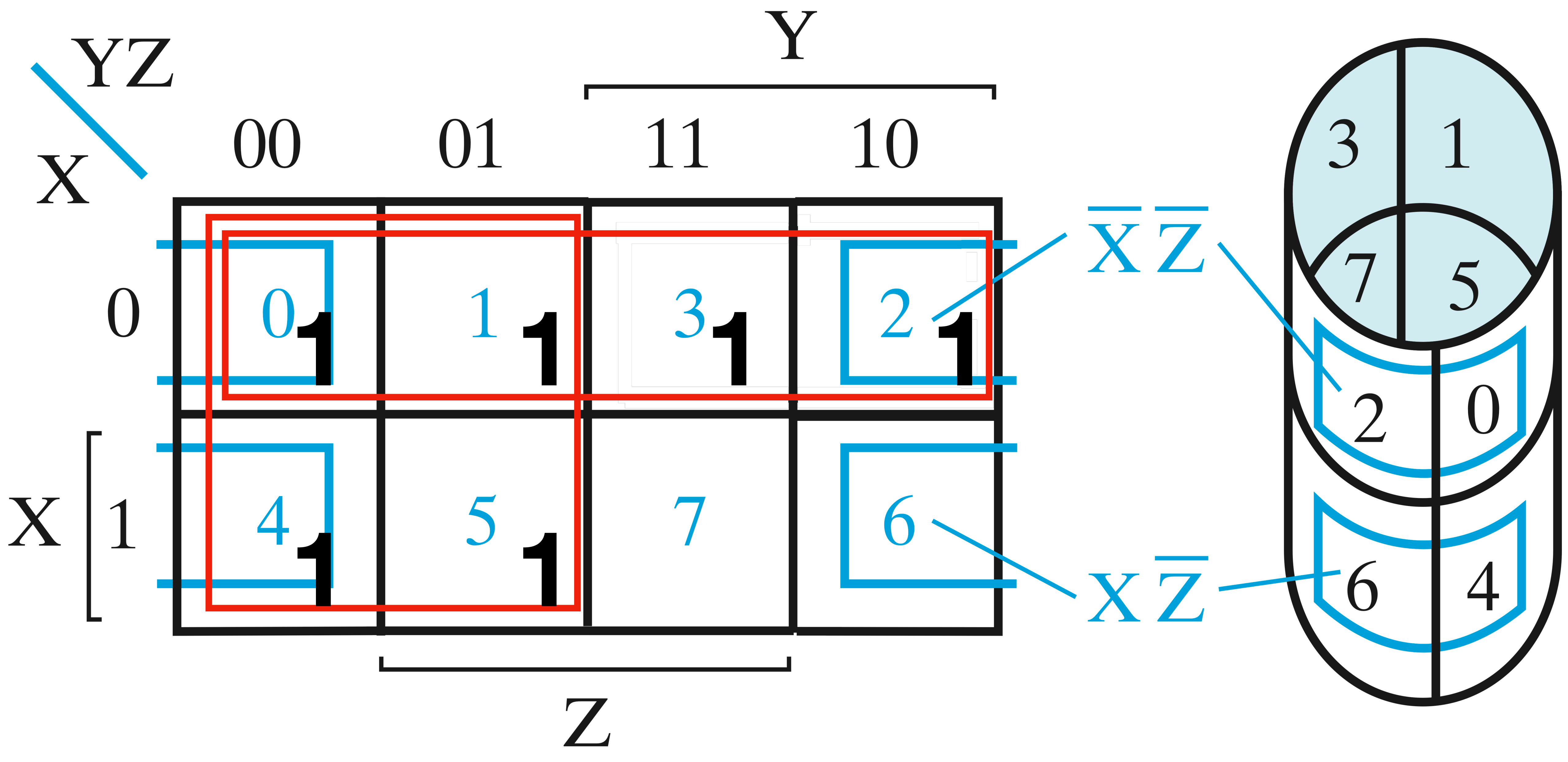


$$F(X, Y, Z) = \Sigma m(0,1,2,3,4,5)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s

Concept

Three Variable Maps Optimisation

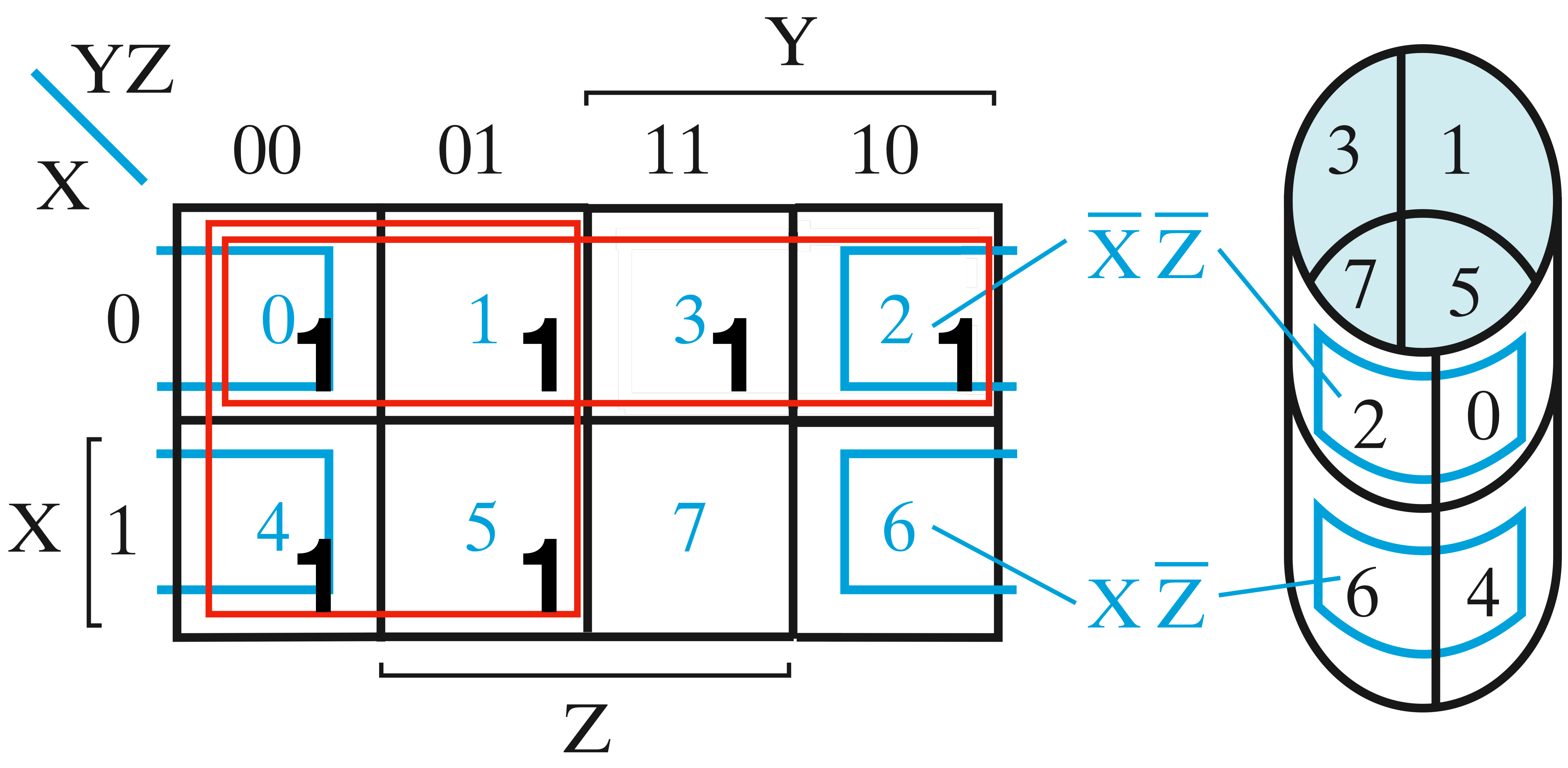


$$F(X,Y,Z) = \Sigma m(0,1,2,3,4,5)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Concept

Three Variable Maps Optimisation

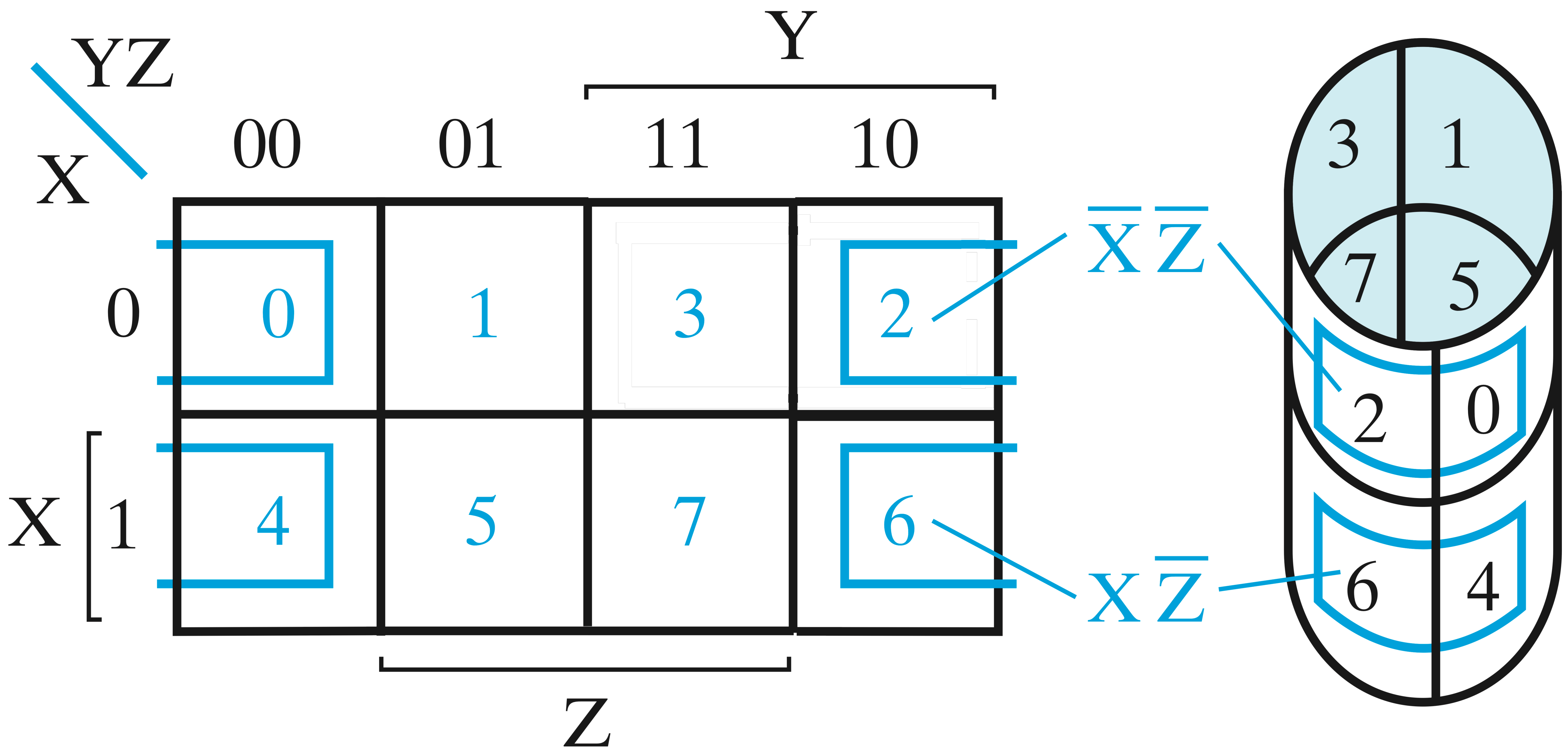


$$F(X,Y,Z) = \Sigma m(0,1,2,3,4,5)$$
$$= \overline{X} + \overline{Y}$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Concept

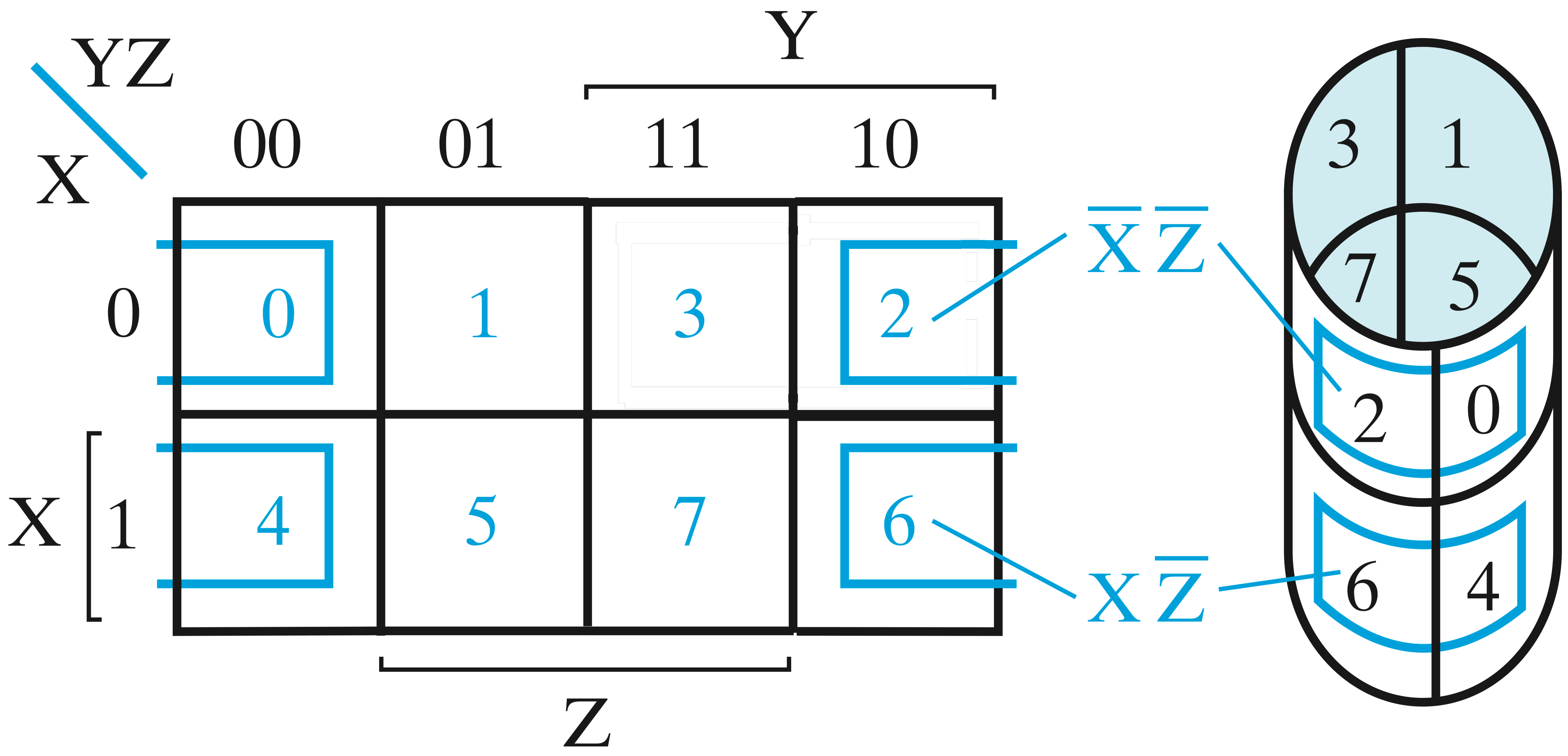
Three Variable Maps
Optimisation



$$F(X, Y, Z) = \Sigma m(0,2,4,5,6)$$

Demo

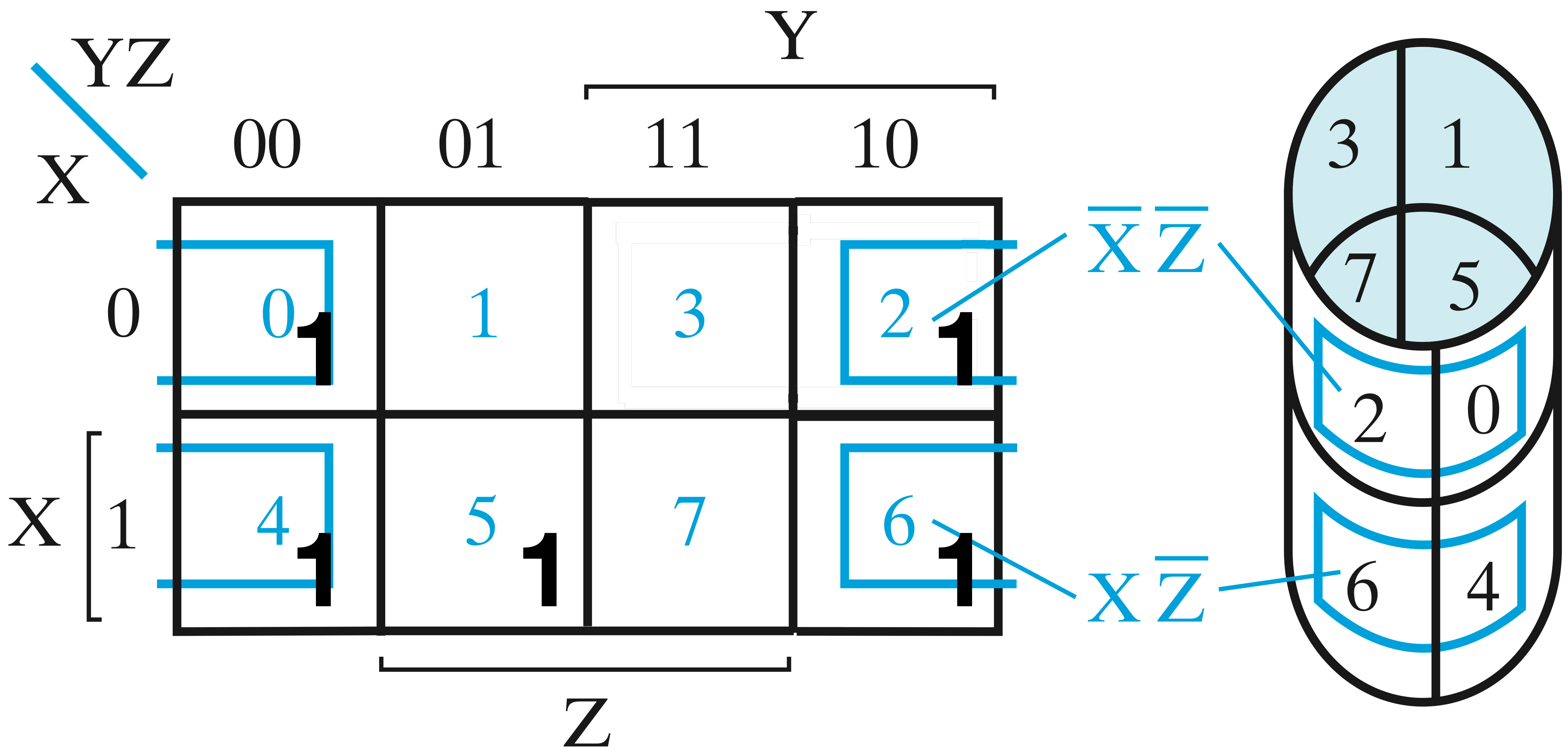
Three Variable Maps Optimisation



$$F(X,Y,Z) = \Sigma m(0,2,4,5,6)$$

- Step 1: Enter the values

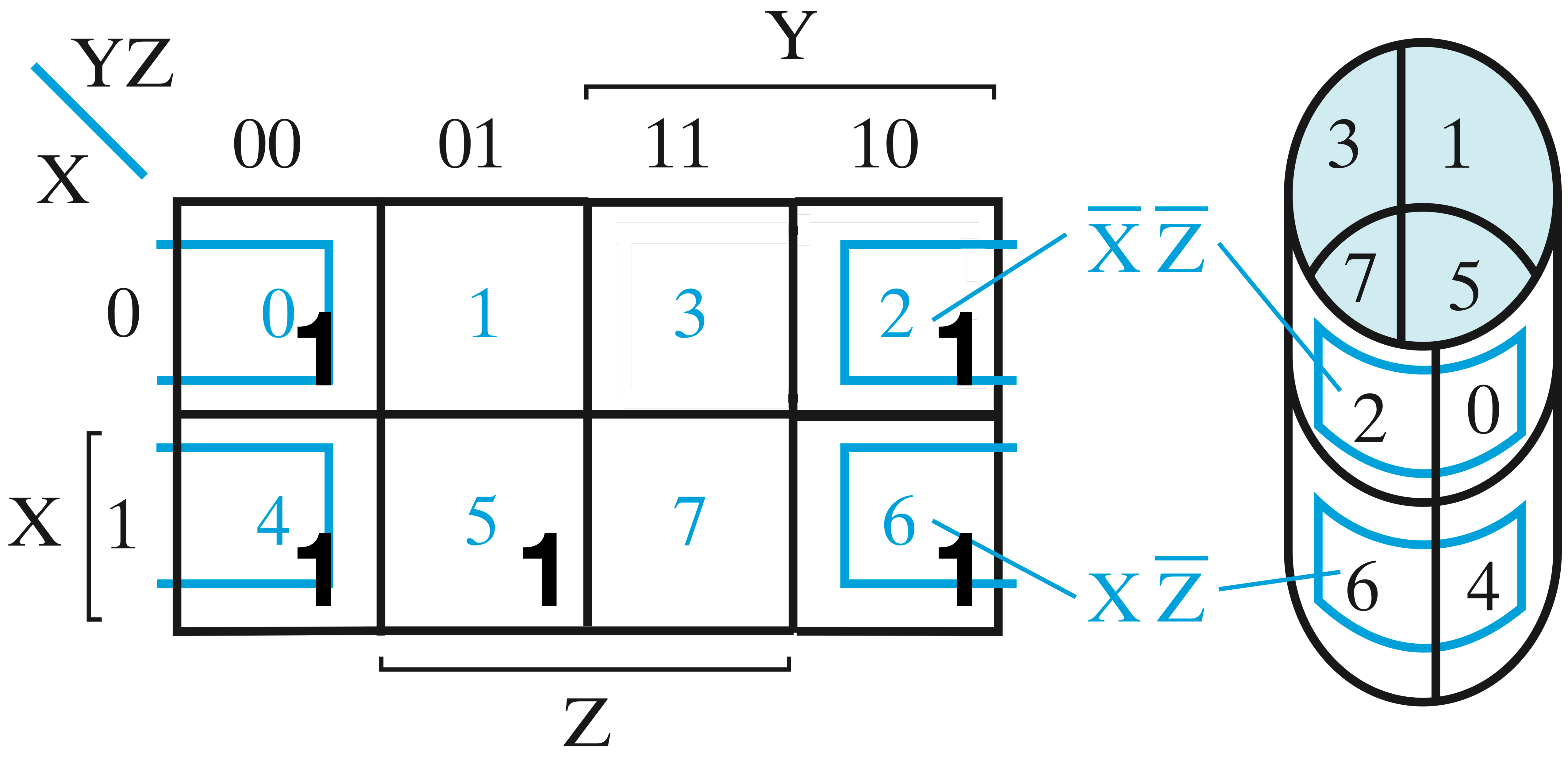
Three Variable Maps Optimisation



$$F(X, Y, Z) = \Sigma m(0,2,4,5,6)$$

- Step 1: Enter the values

Three Variable Maps Optimisation



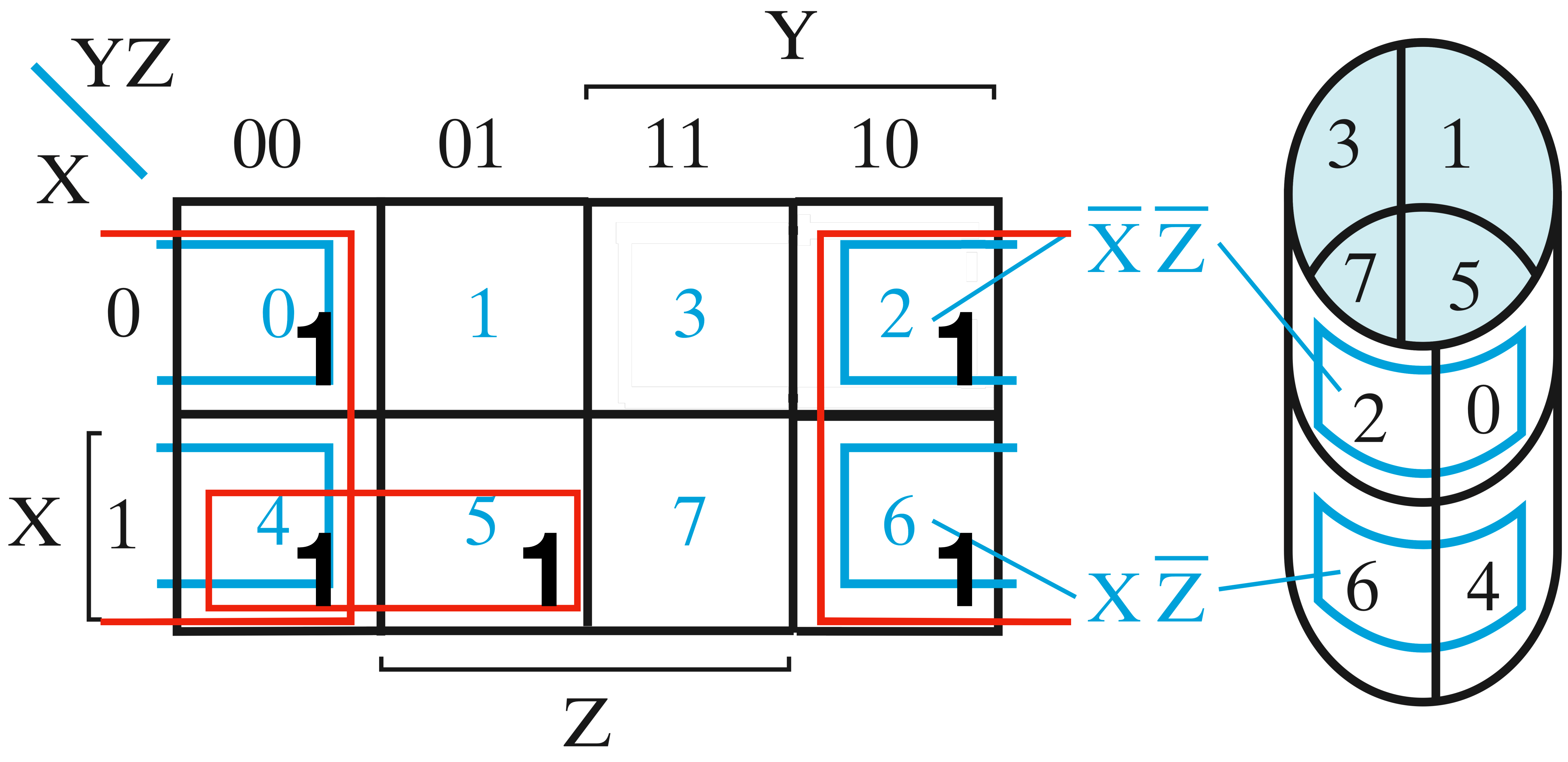
$$F(X, Y, Z) = \Sigma m(0,2,4,5,6)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s

Demo

P1
Optimisation

Three Variable Maps Optimisation

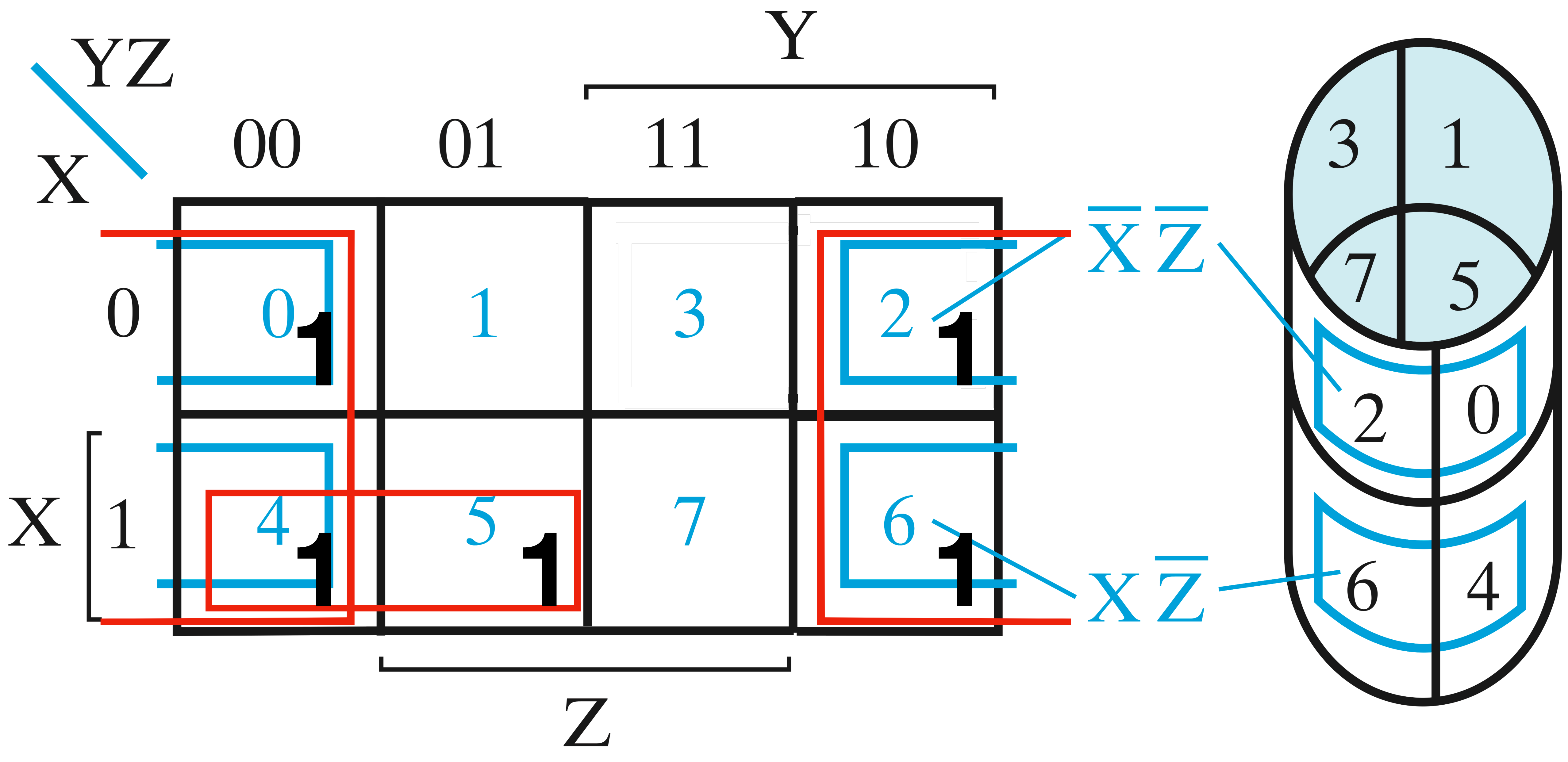


$$F(X,Y,Z) = \Sigma m(0,2,4,5,6)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s

Demo

Three Variable Maps Optimisation



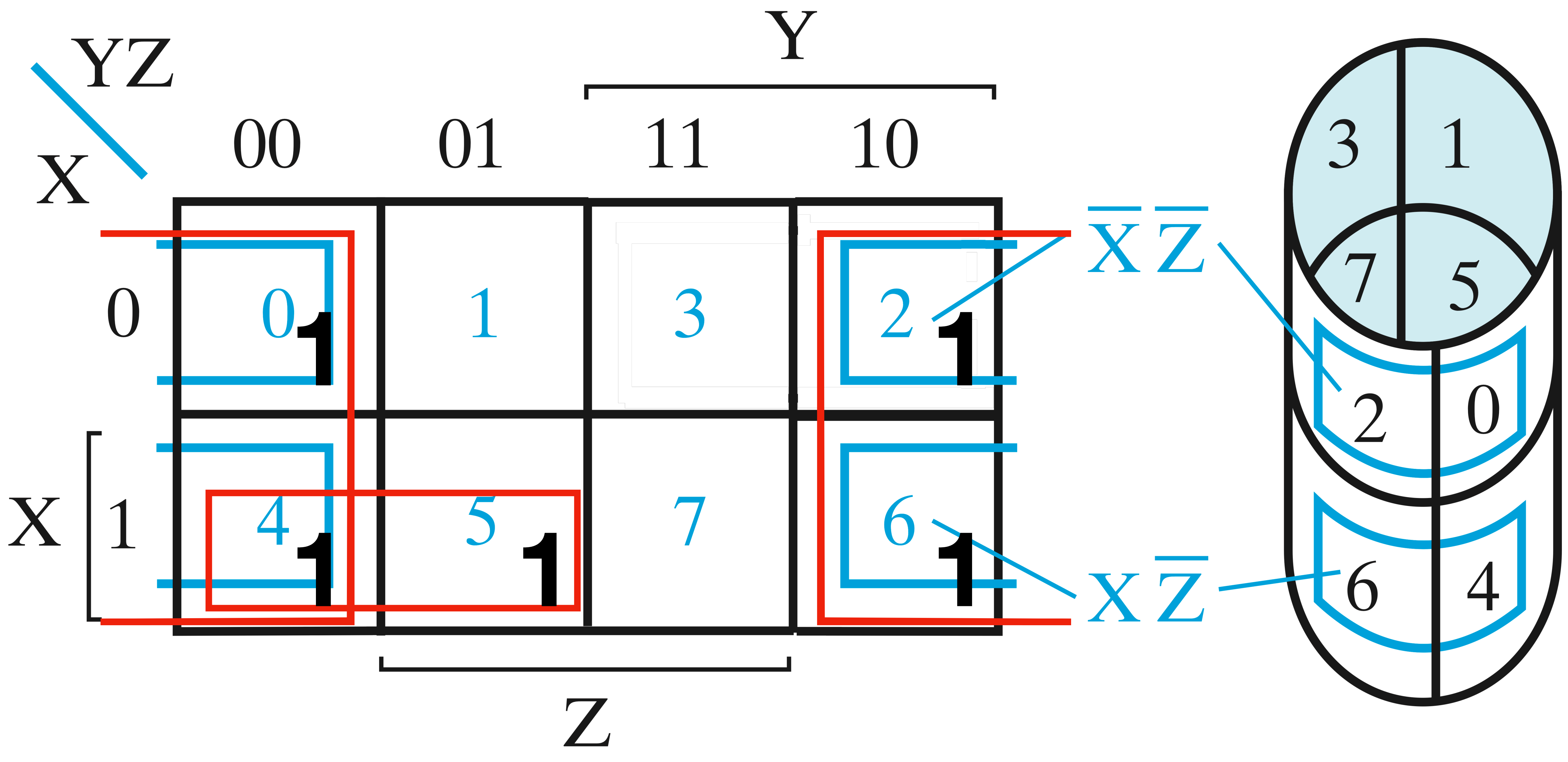
$$F(X, Y, Z) = \Sigma m(0,2,4,5,6)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Demo

P1
Optimisation

Three Variable Maps Optimisation

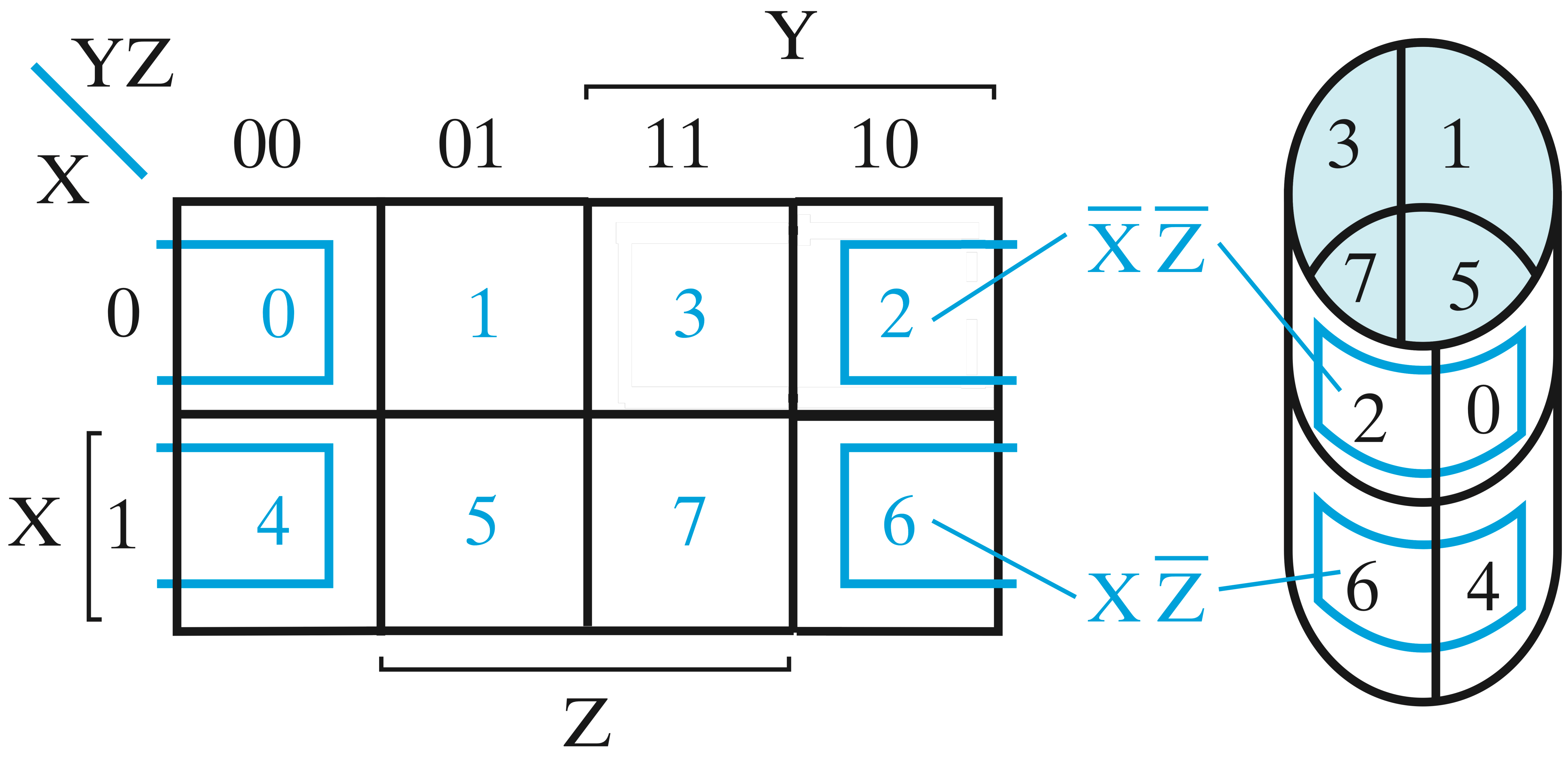


$$F(X,Y,Z) = \Sigma m(0,2,4,5,6)$$
$$= X\overline{Y} + \overline{Z}$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Demo

Three Variable Maps Optimisation

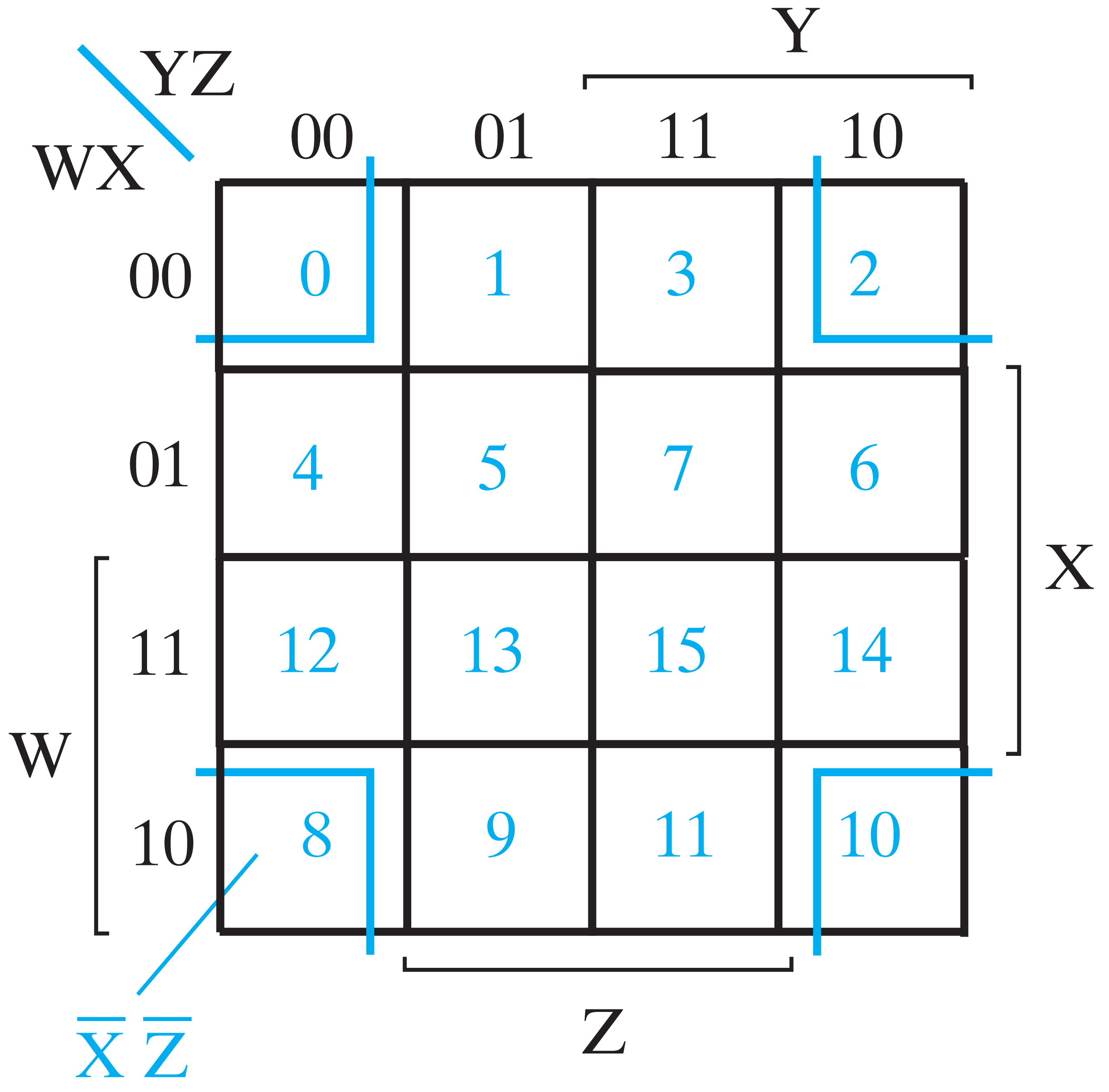


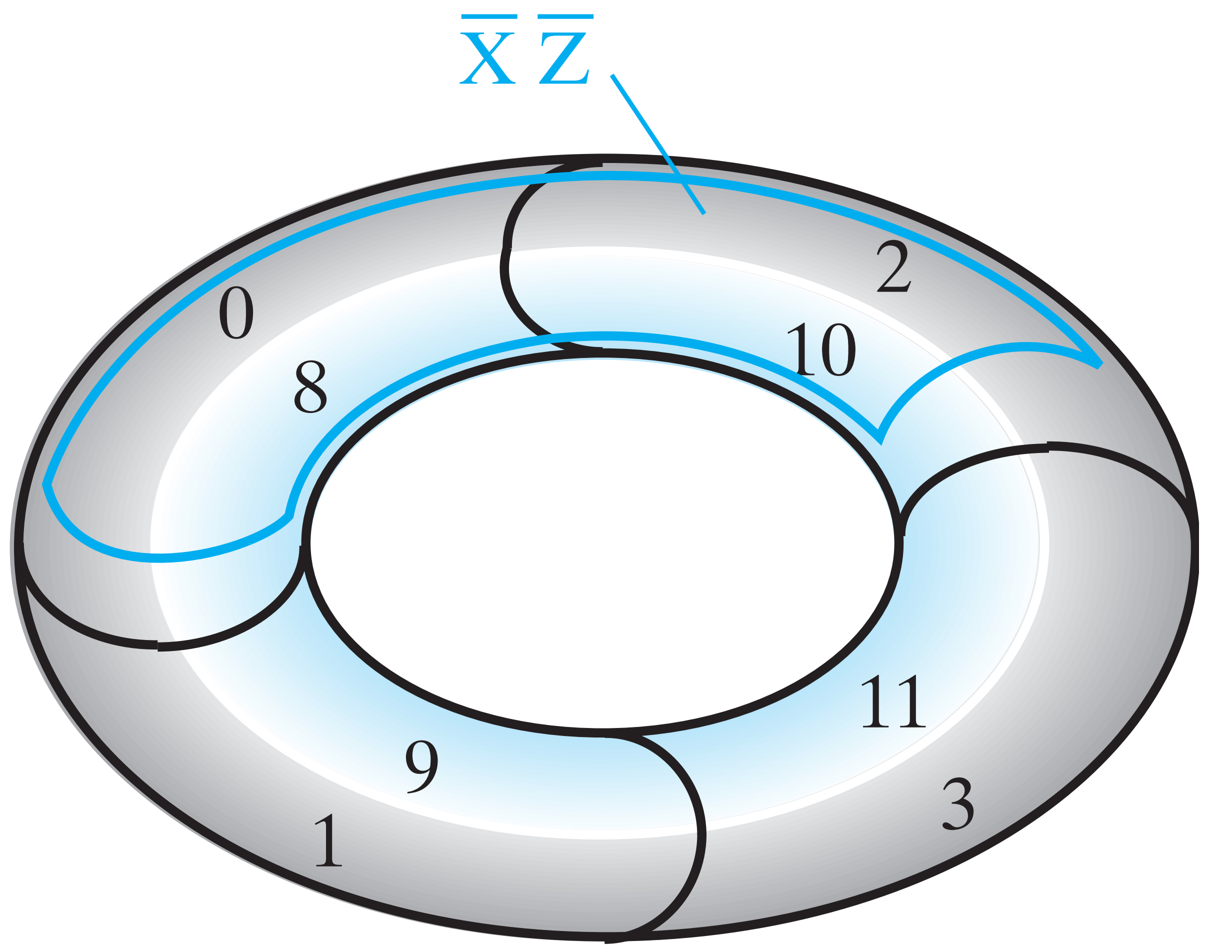
$$F(X, Y, Z) = \Sigma m(1,3,4,5,6)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Exercise

Four Variable Maps Optimisation





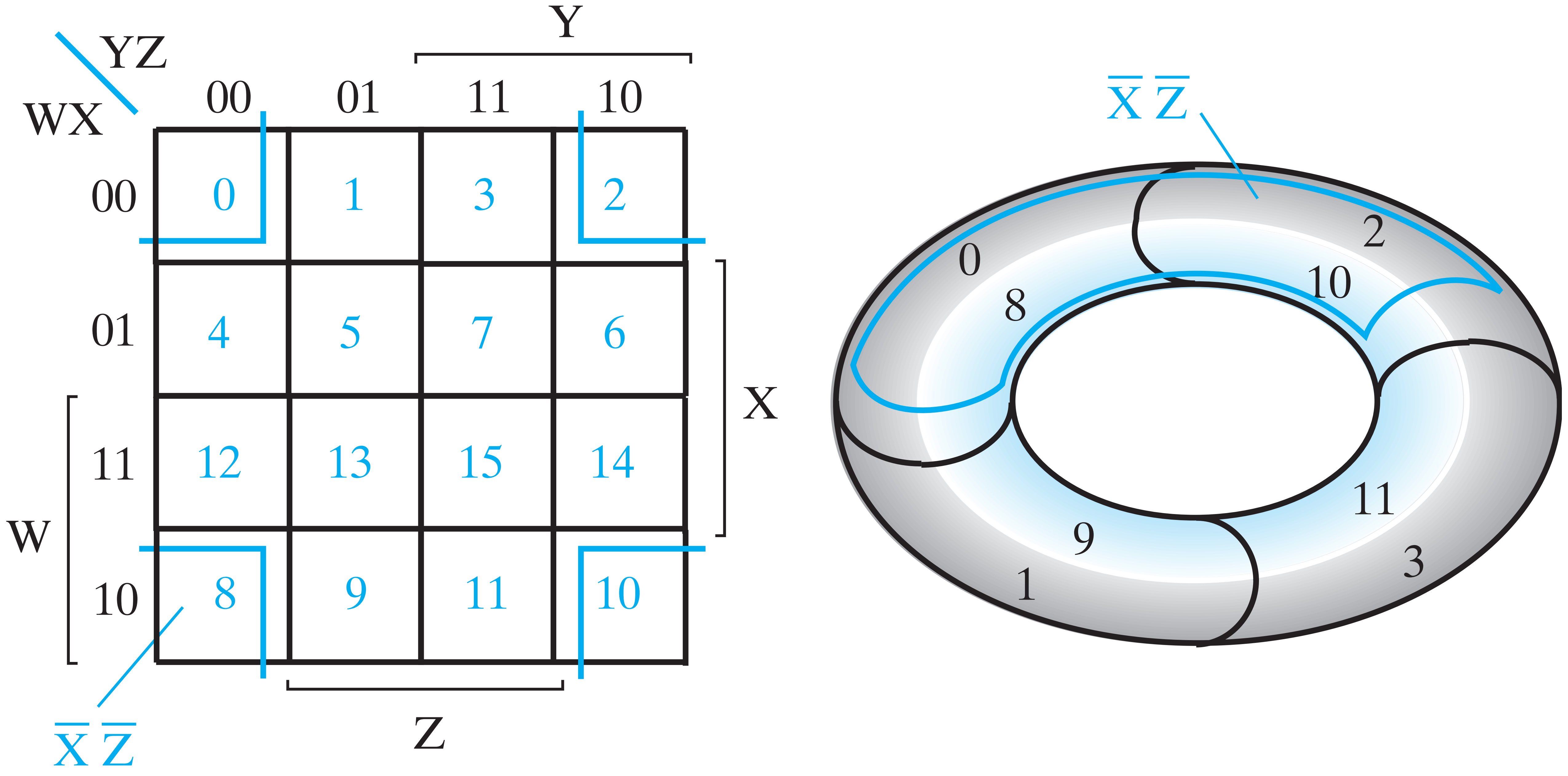
$$F(W, X, Y, Z) = \Sigma m(0,1,2,4,5,6,8,9,10,12,13)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Exercise

P1
Optimisation

Four Variable Maps Optimisation

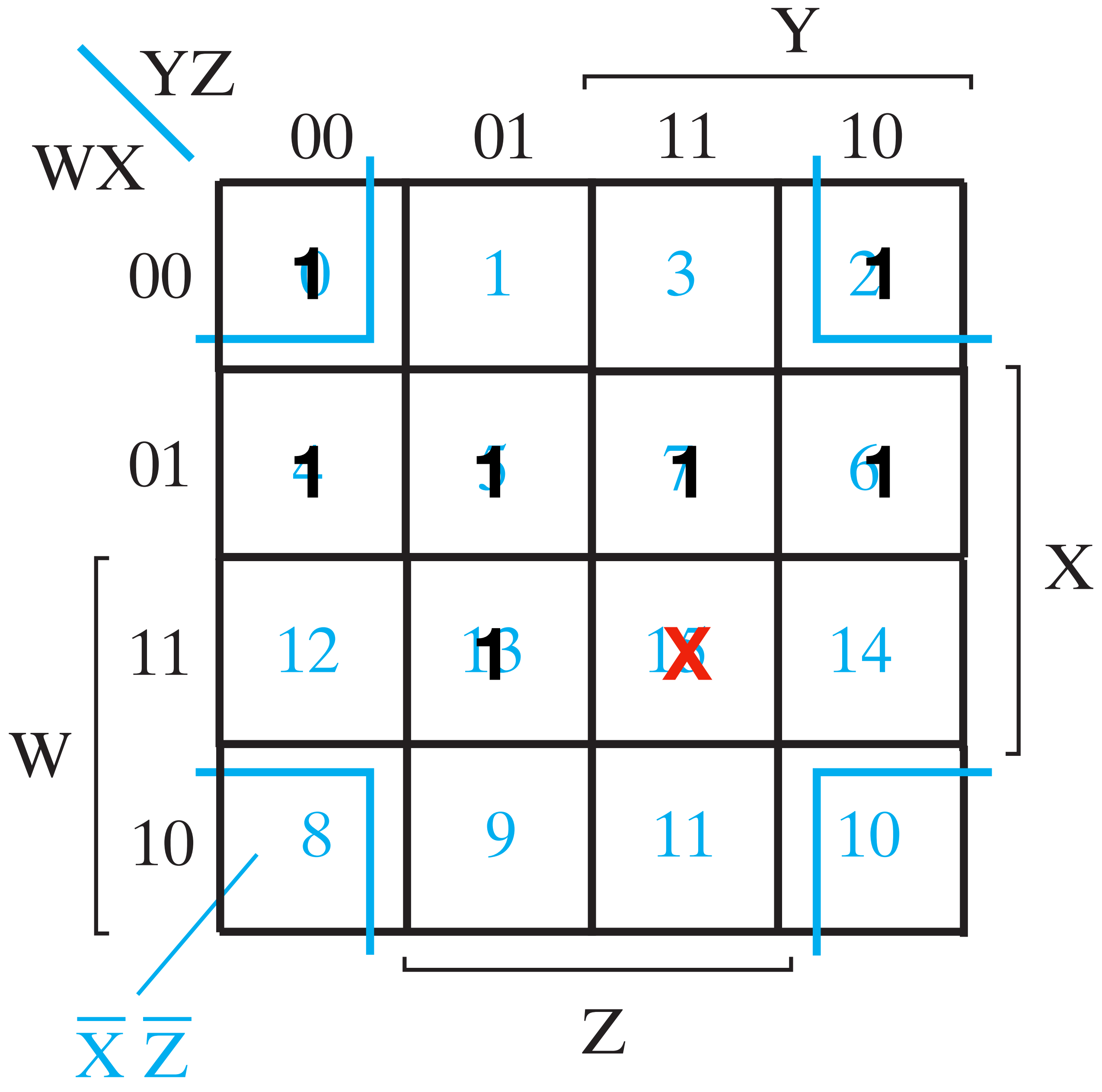


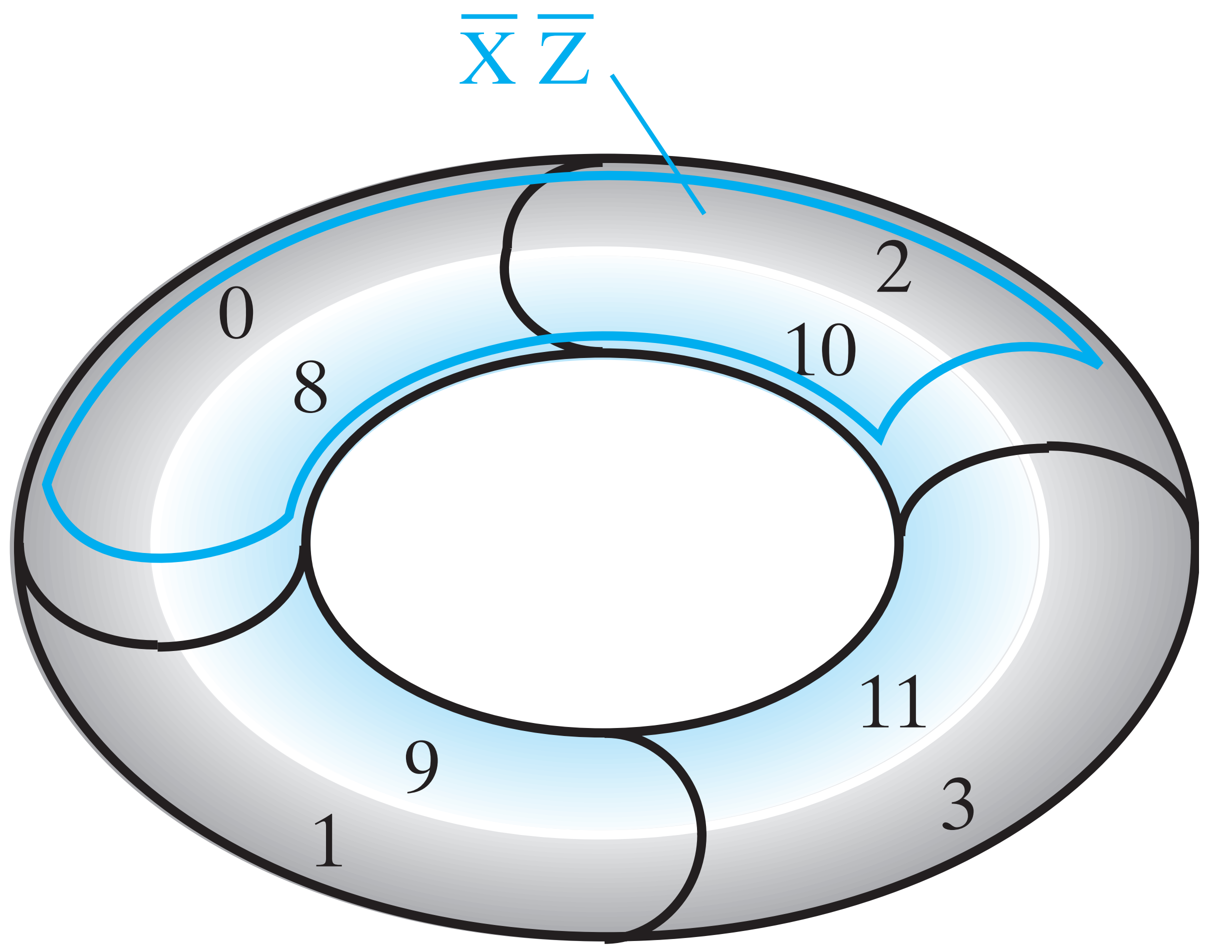
$$F(W,X,Y,Z) = \overline{W}\overline{Y}\overline{Z} + \overline{W}Z + \overline{X}Y + YZ + W\overline{X}\overline{Z}$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Exercise

Don’t Care Condition



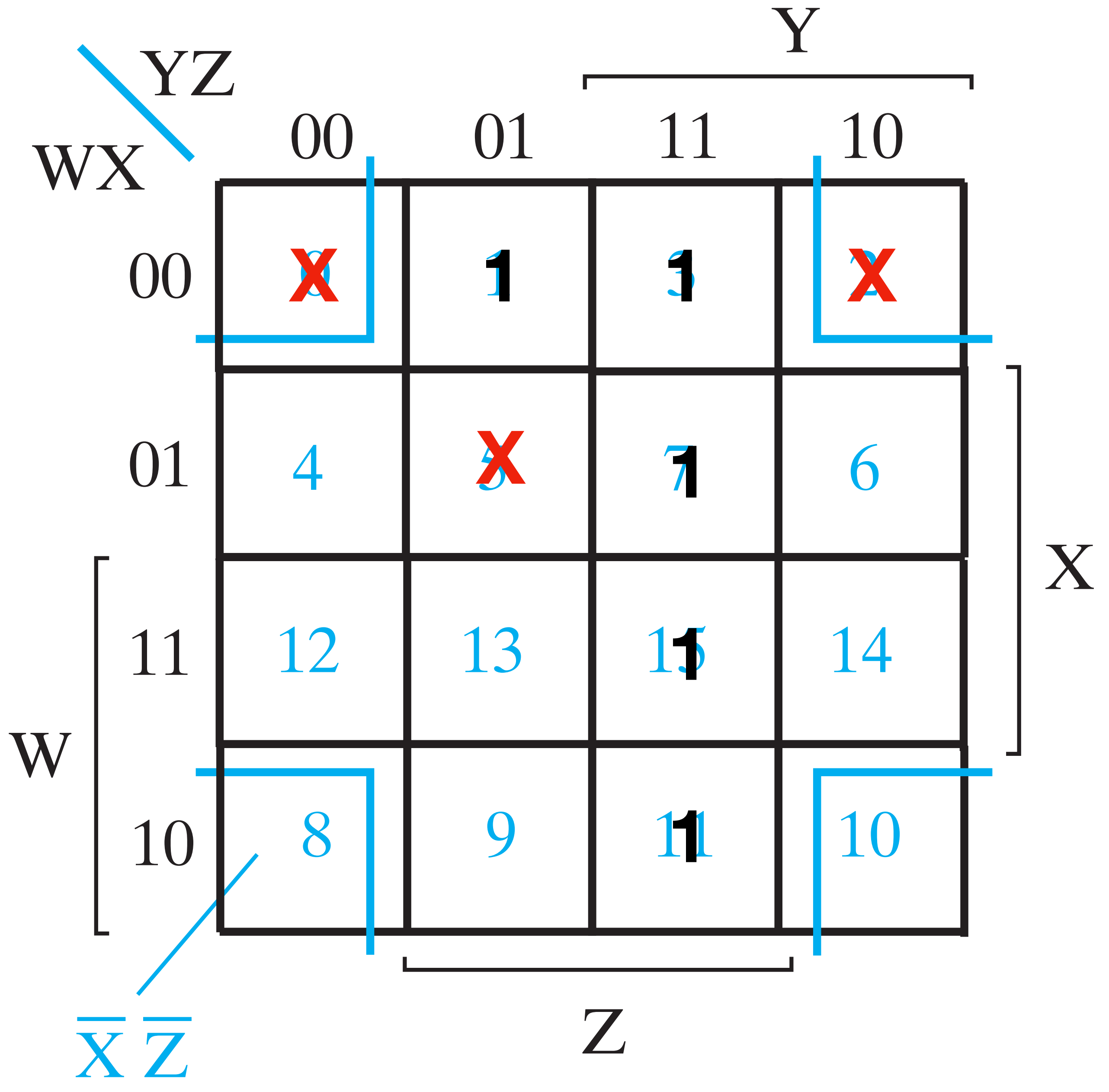


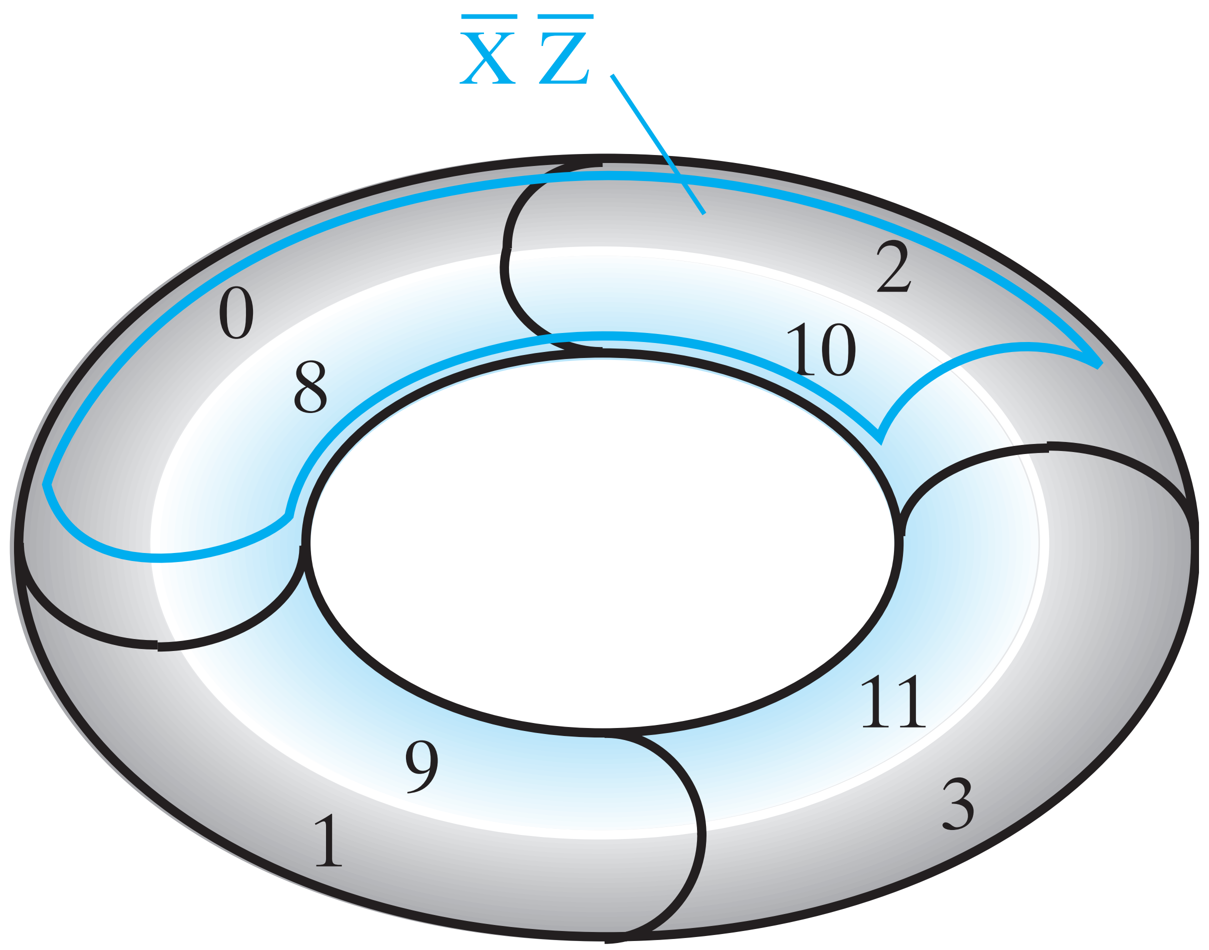
- Sometimes we don’t care what the output is when the inputs are in certain combinations

Concept

P1
Optimisation

Don’t Care Condition



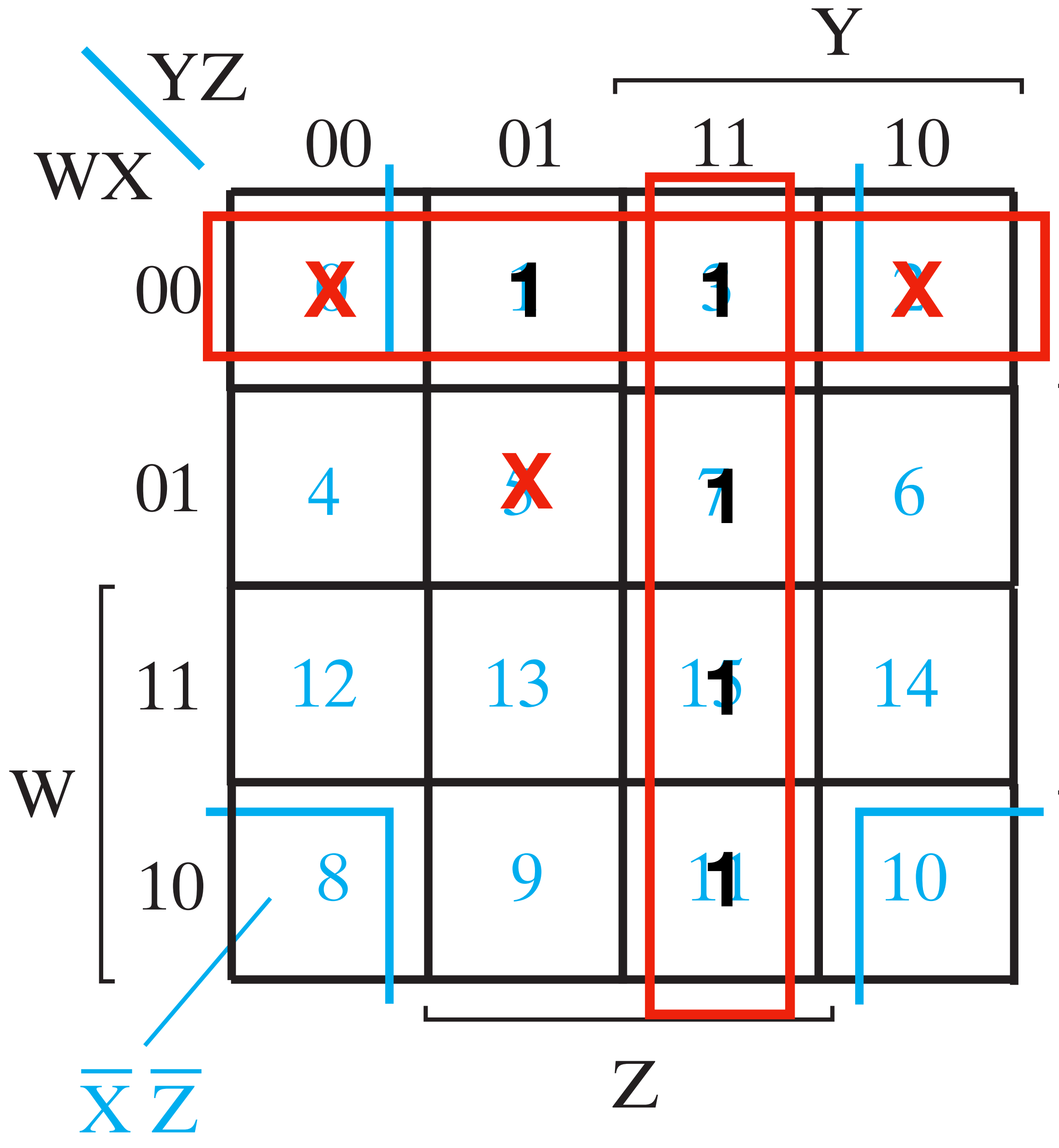


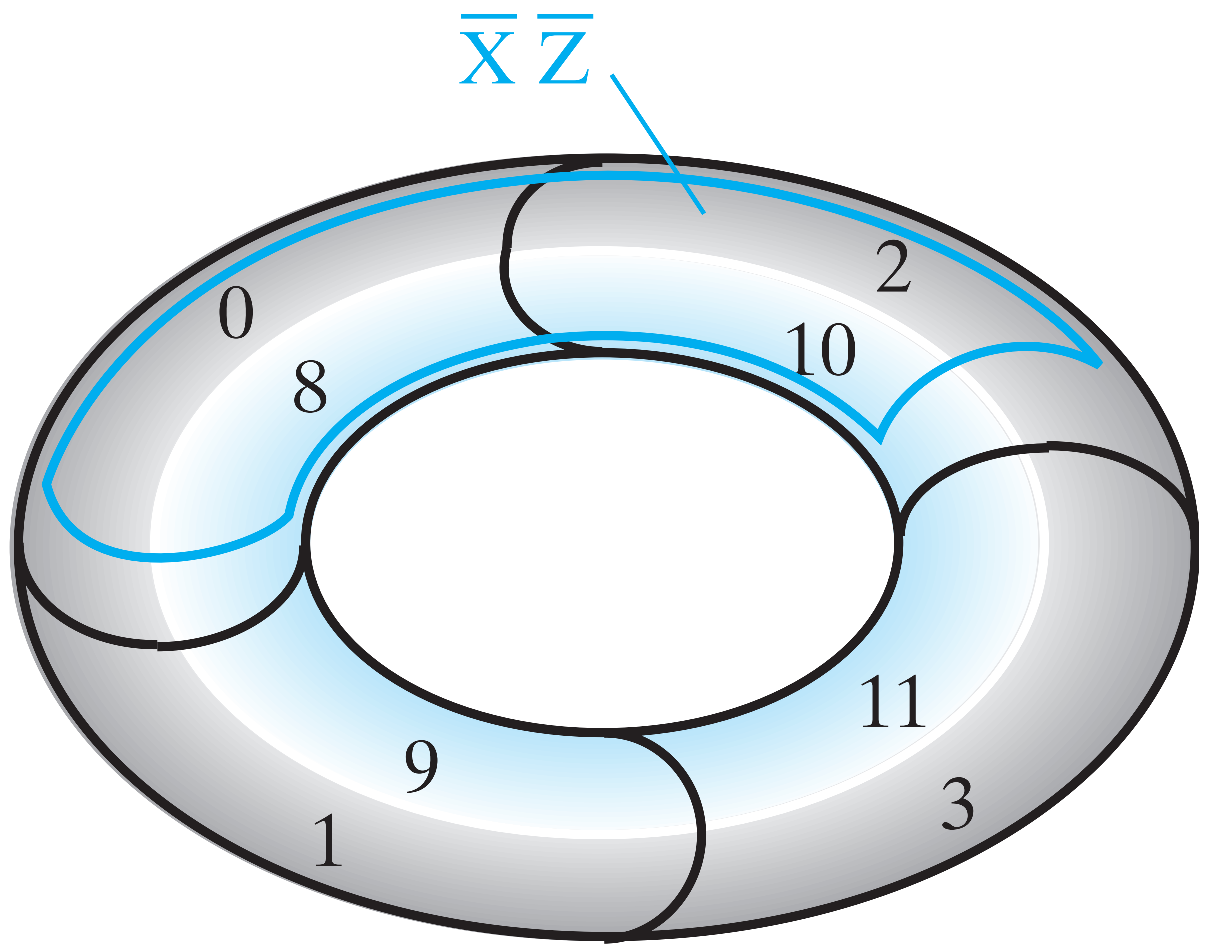
- Sometimes we don’t care what the output is when the inputs are in certain combinations

Demo

P1
Optimisation

Don’t Care Condition



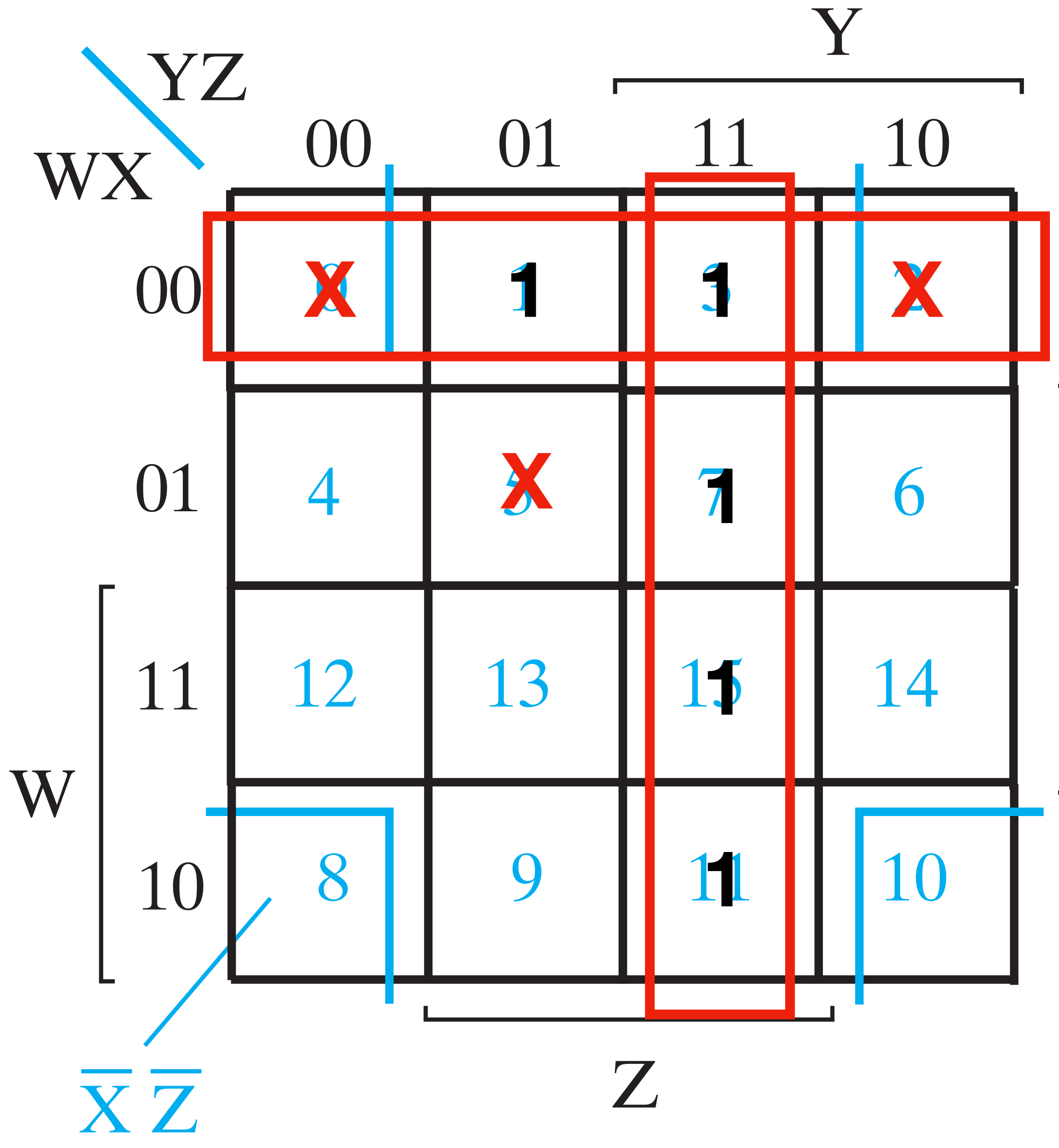


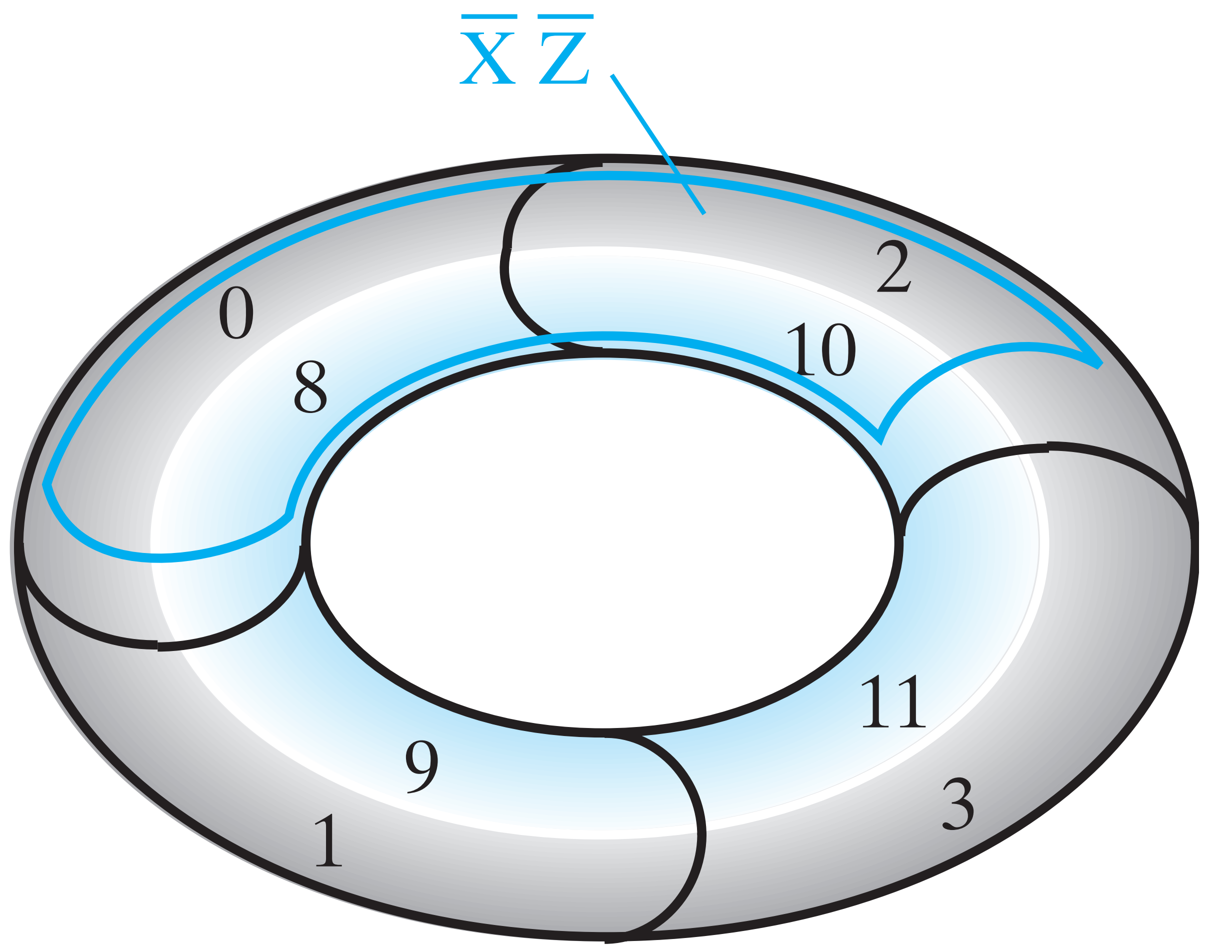
- Sometimes we don’t care what the output is when the inputs are in certain combinations

Demo

P1 Optimisation

Don’t Care Condition



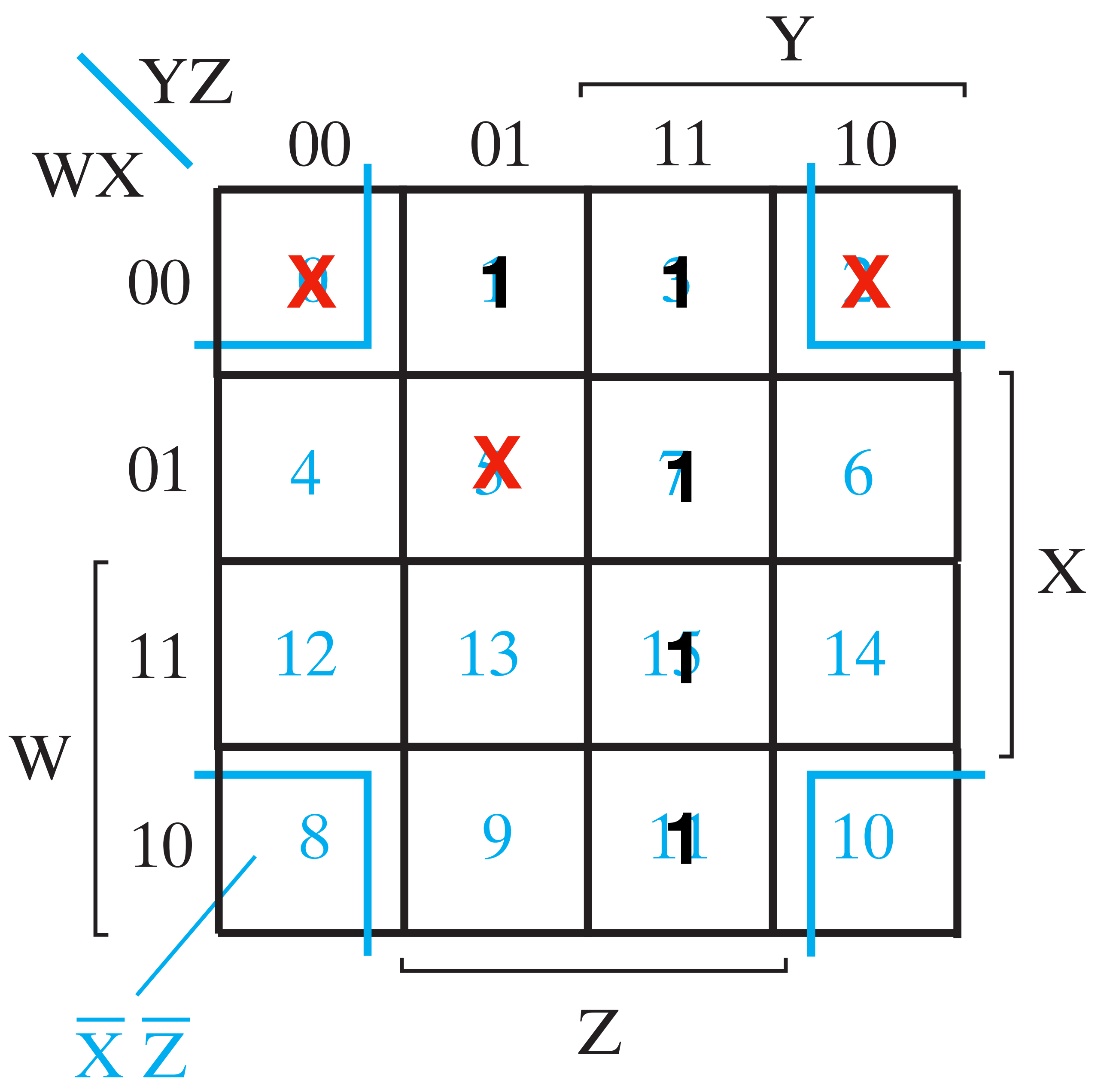


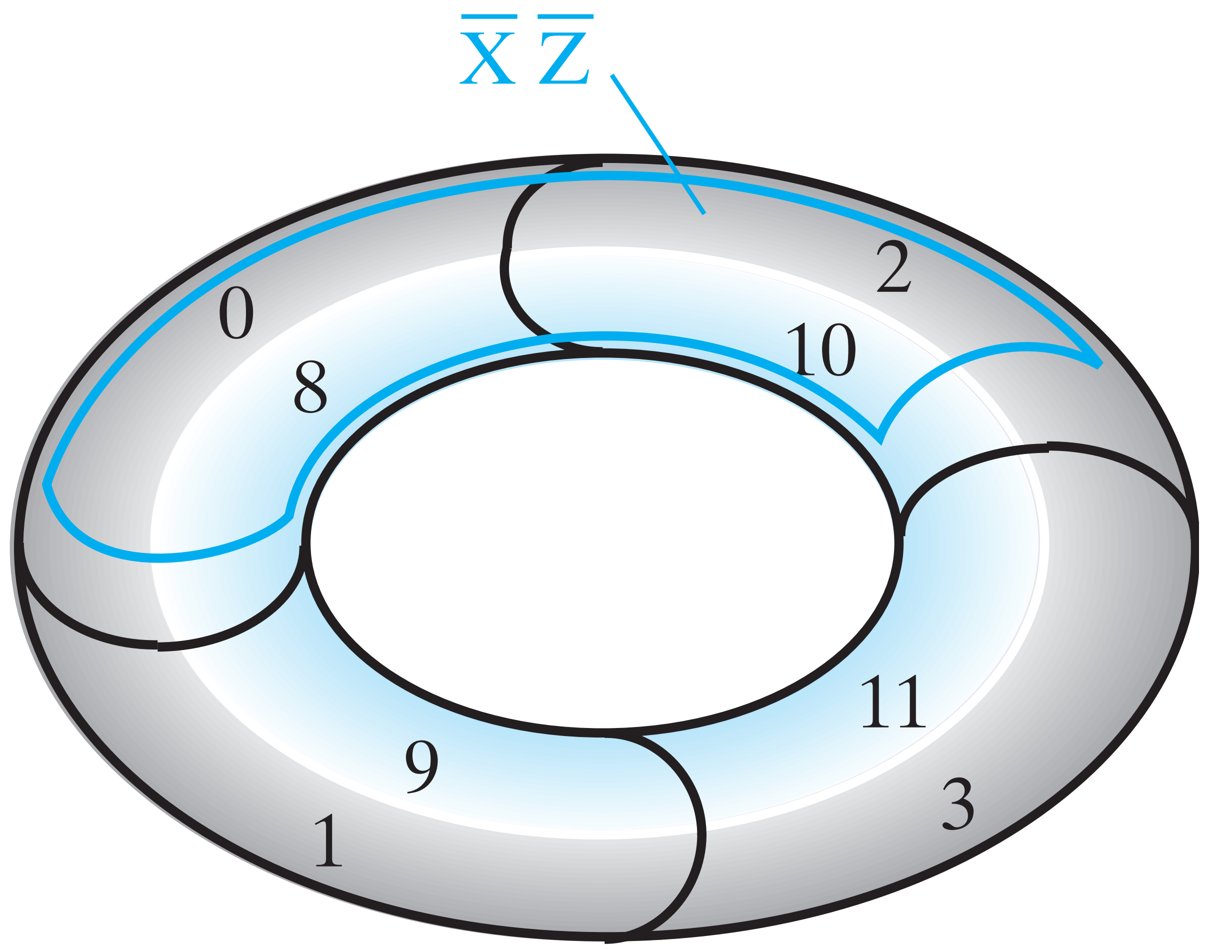
- Sometimes we don’t care what the output is when the inputs are in certain combinations

$$F = YZ + \overline{W}\,\overline{X}$$

Demo

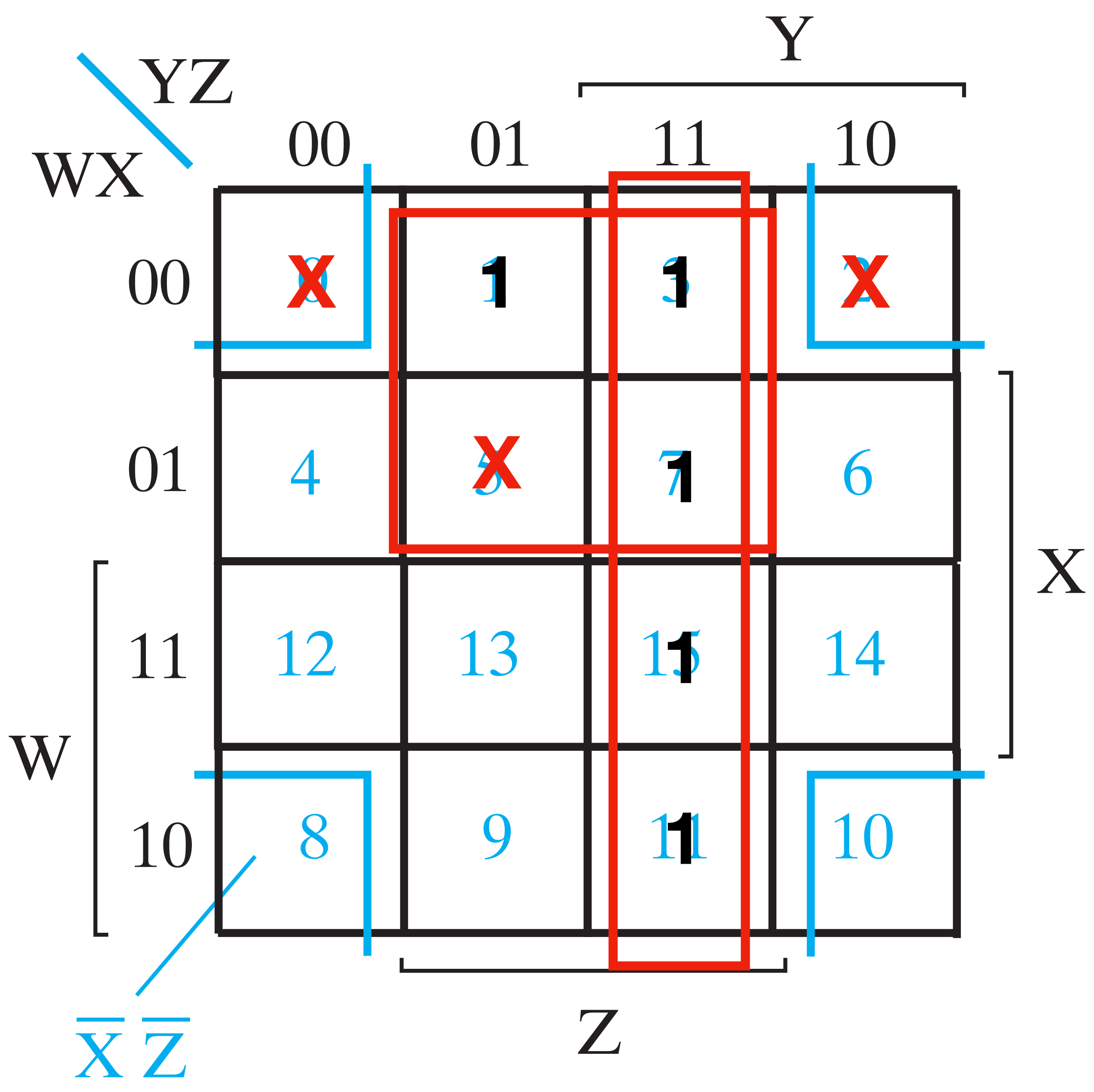
Don’t Care Condition

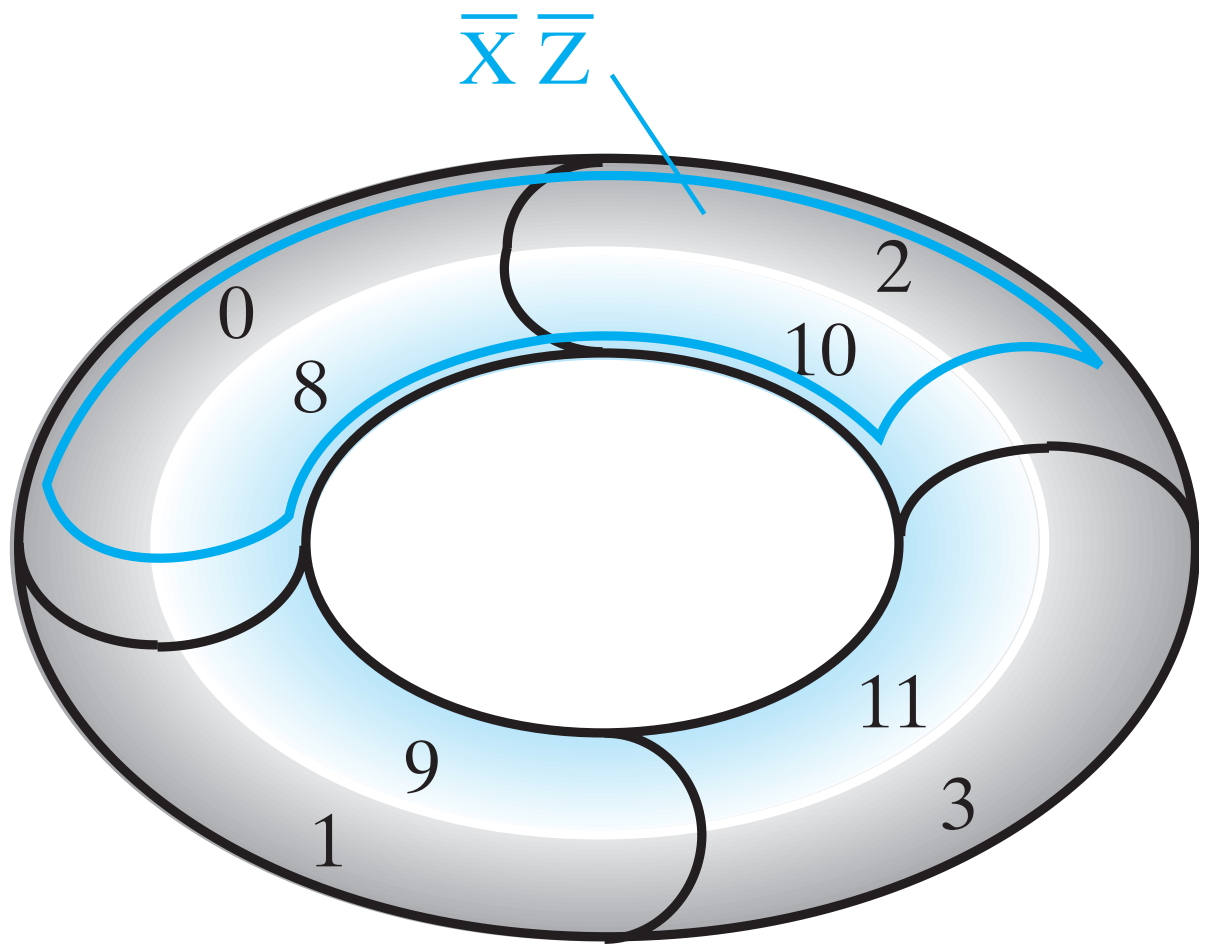




- Sometimes we don’t care what the output is when the inputs are in certain combinations

Don’t Care Condition



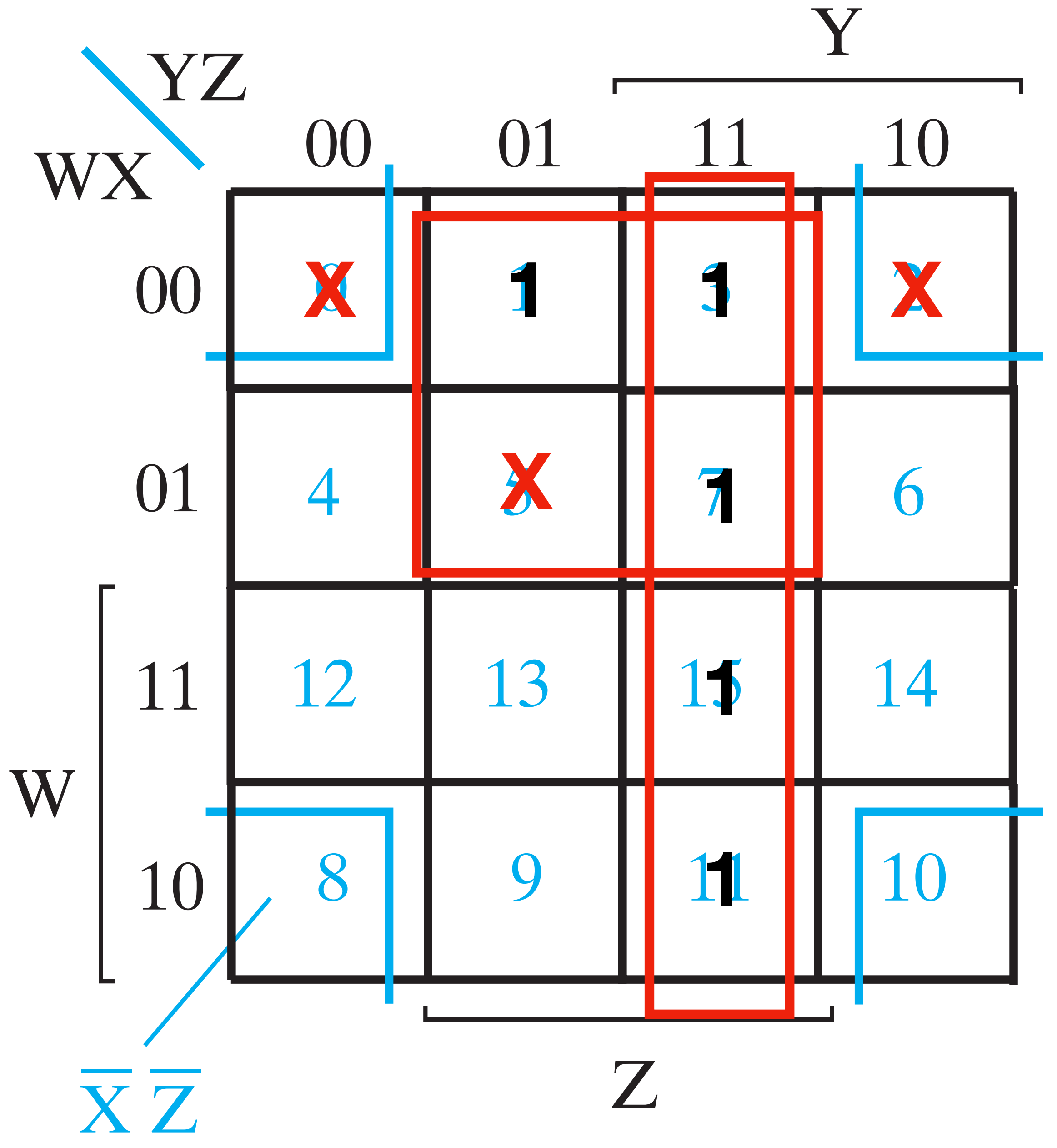


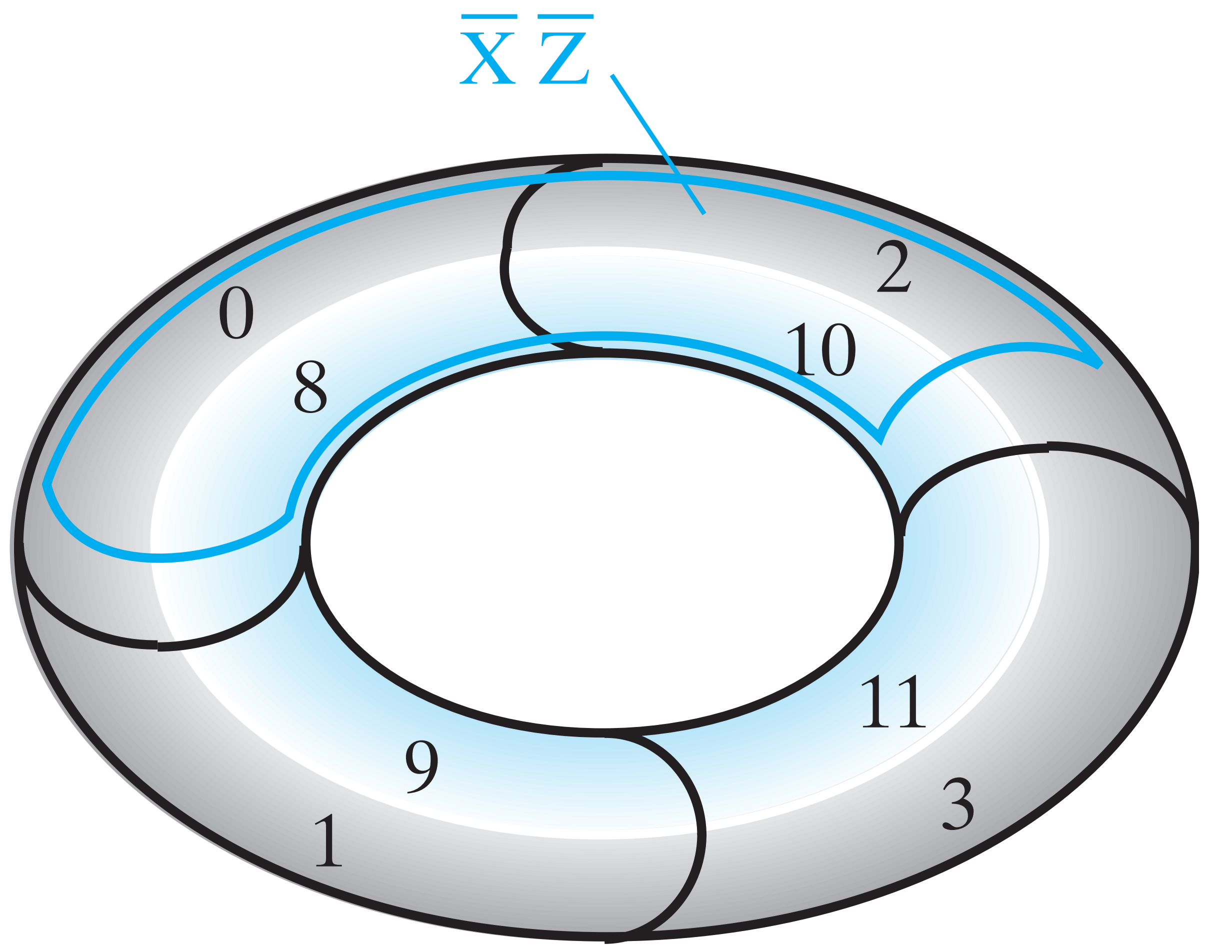
- Sometimes we don’t care what the output is when the inputs are in certain combinations

Demo

P1
Optimisation

Don’t Care Condition





- Sometimes we don’t care what the output is when the inputs are in certain combinations

$$F = YZ + \overline{W}Z$$

Demo

Summary

Summary

- Boolean Algebra III: K-Map

Summary

- Boolean Algebra III: K-Map
  - Two Variable K-Map

Review

Summary

- Boolean Algebra III: K-Map
  - Two Variable K-Map
  - Three Variable K-Map

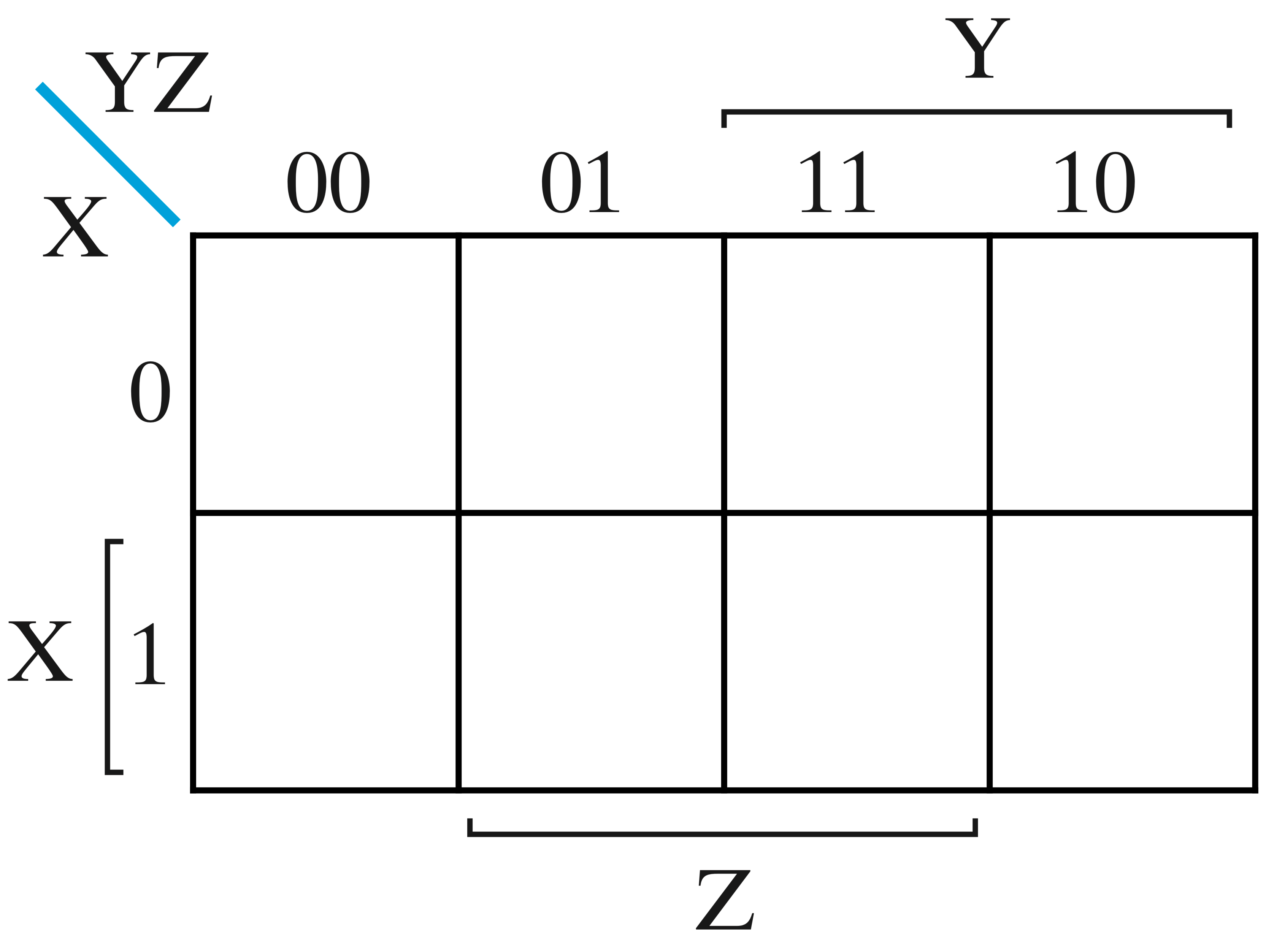
Summary

- Boolean Algebra III: K-Map
  - Two Variable K-Map
  - Three Variable K-Map
  - Four Variable K-Map

Summary

- Boolean Algebra III: K-Map
  - Two Variable K-Map
  - Three Variable K-Map
  - Four Variable K-Map
  - Don’t care optimisation

Exercises

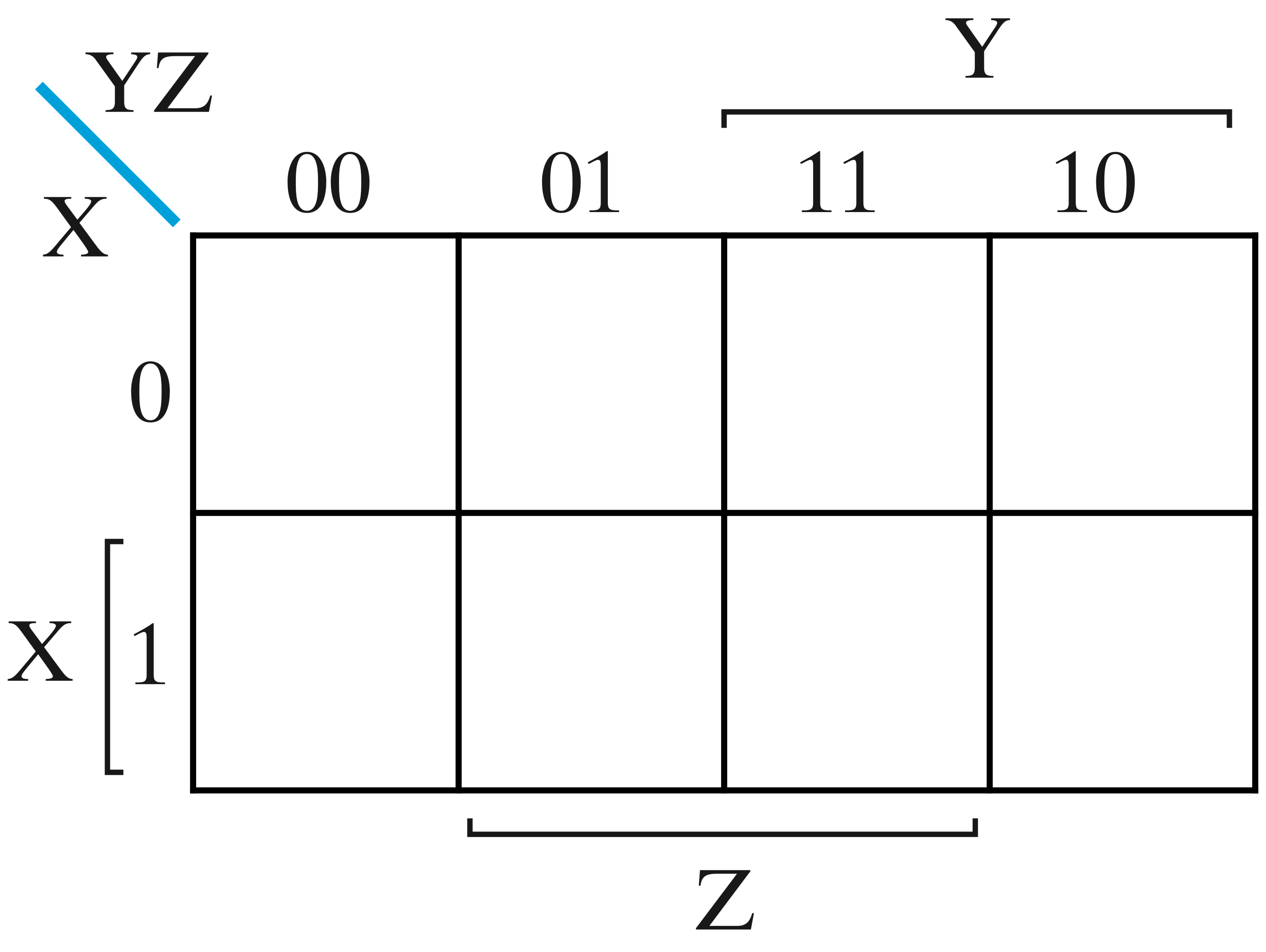


$$F(X, Y, Z) = \Sigma m(0,2,6,7)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Exercise

Exercises

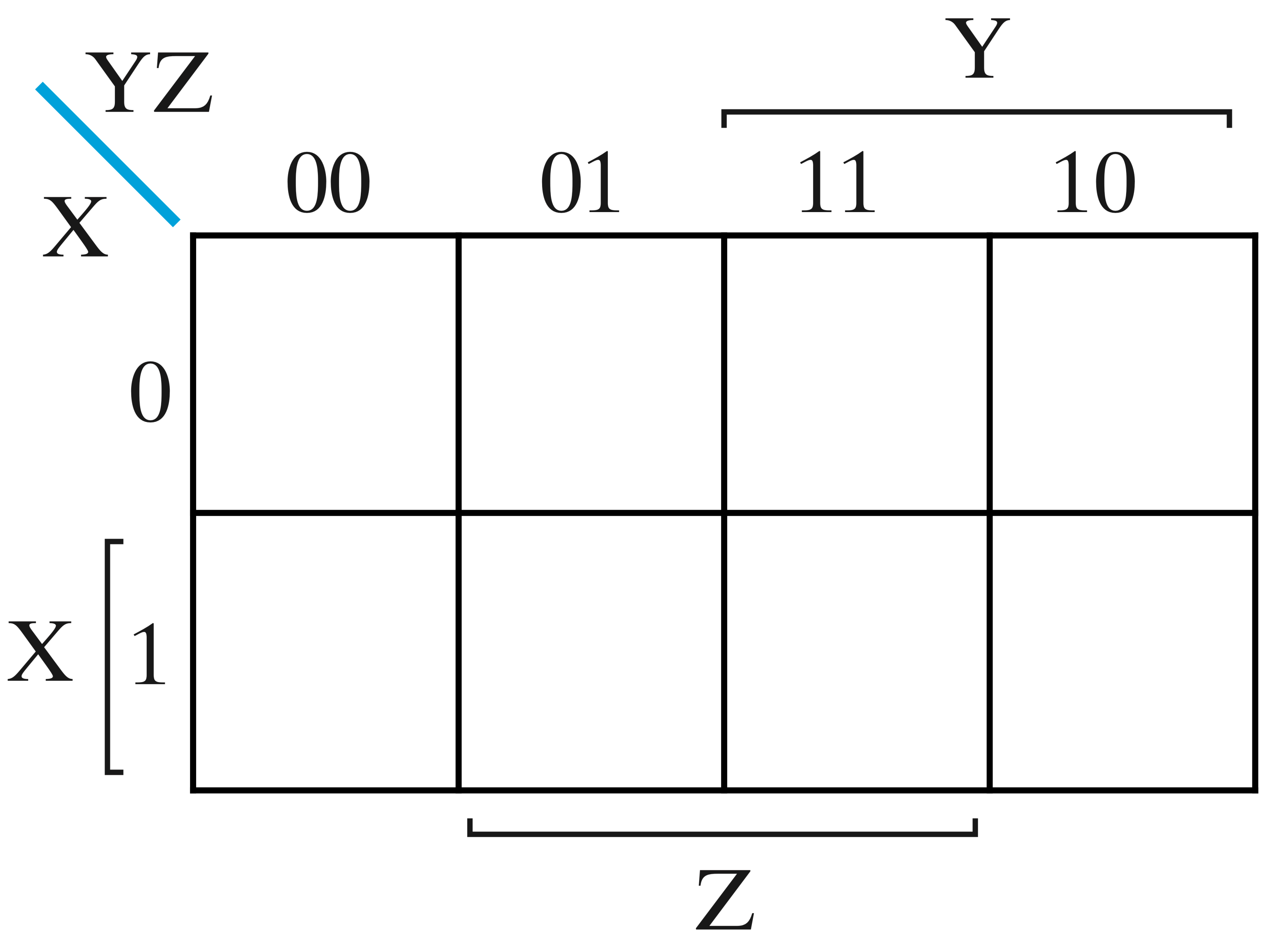


$$F(X, Y, Z) = \Sigma m(0,1,2,4)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Exercise

Exercises

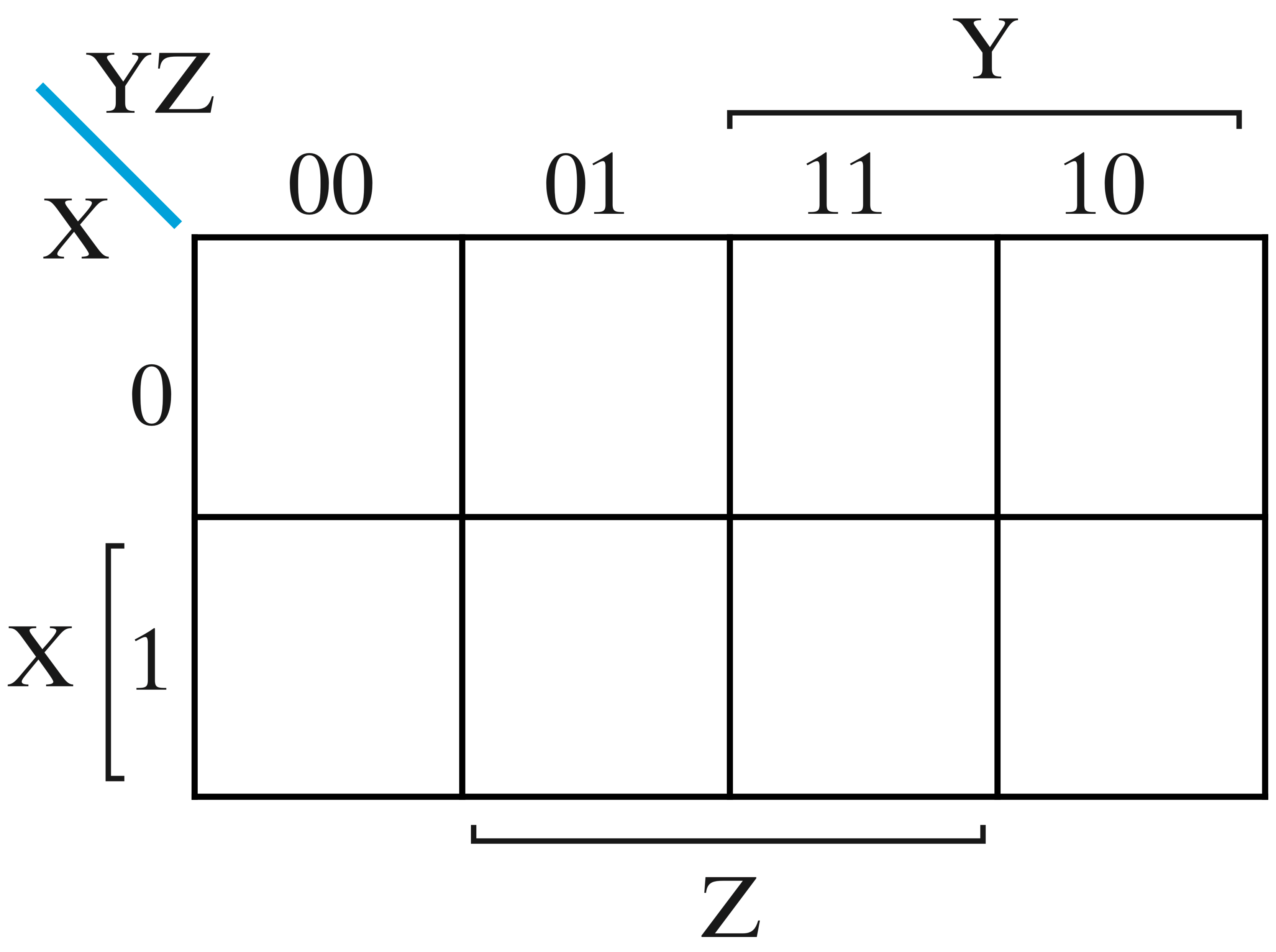


$$F(X, Y, Z) = \Sigma m(0,2,3,4,6)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Exercise

Exercises



$$F(X, Y, Z) = \Sigma m(0,2,3,4,5,7)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Exercise