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CSCI 150

Introduction to Digital and Computer System Design

Lecture 2: Combinational Logical Circuits IV



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2020 Winter Semester (S1)

Overview

- Focus: Boolean Algebra
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch2 2.4, 2.5; v5: Ch2 2.4, 2.5
- Core Ideas:
 1. Boolean Algebra III: K-Map

Boolean Algebra I&II

- AND, OR, NOT Operators and Gates
 - Simple digital circuit implementation
 - Algebraic manipulation using Binary Identities
- Standard Forms
 - Minterm & Maxterm
 - Sum of Products & Product of Sums

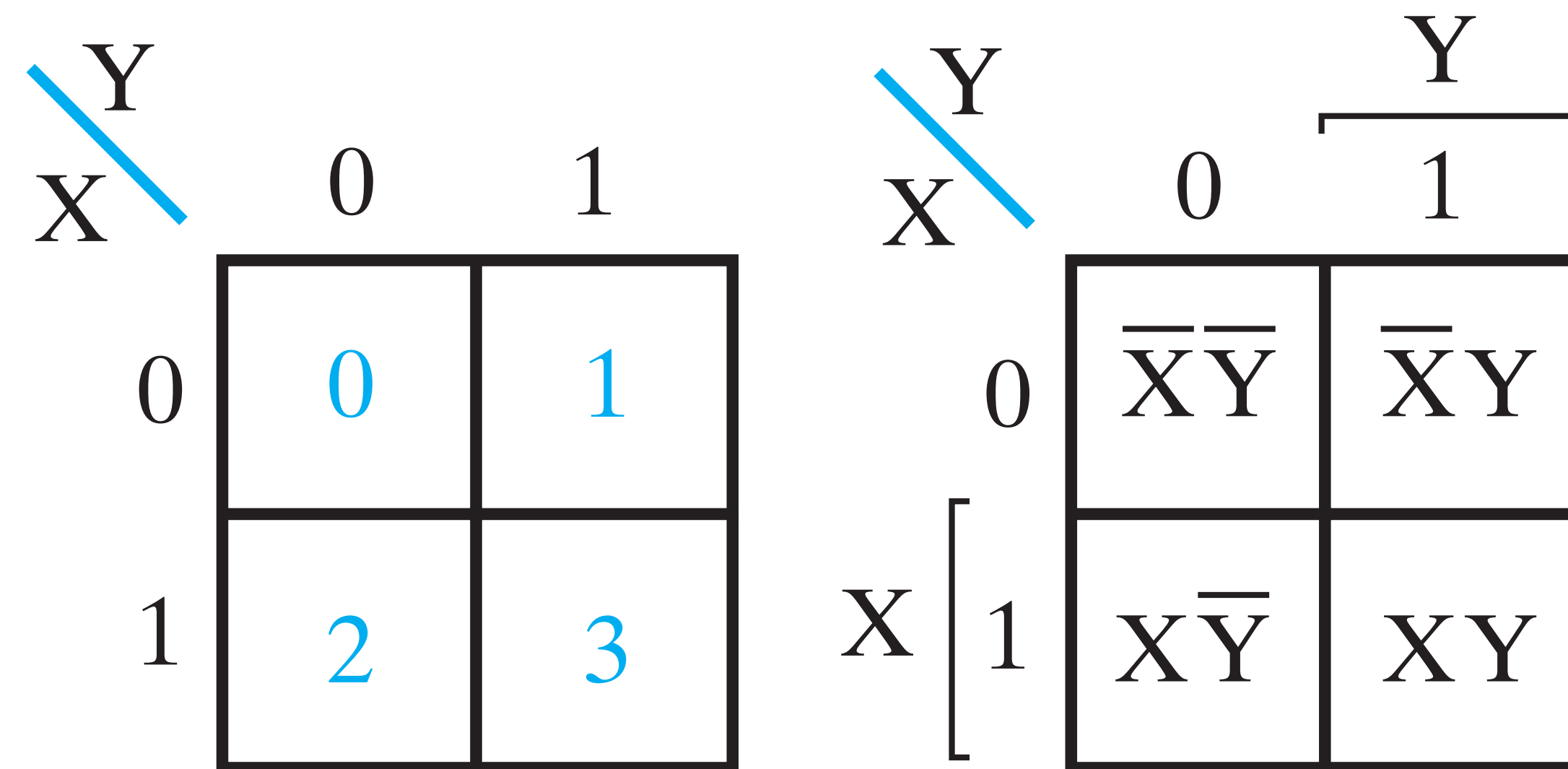
Boolean Algebra III: K-Map

Cost Criteria;
Map and Map Manipulation

K-Map

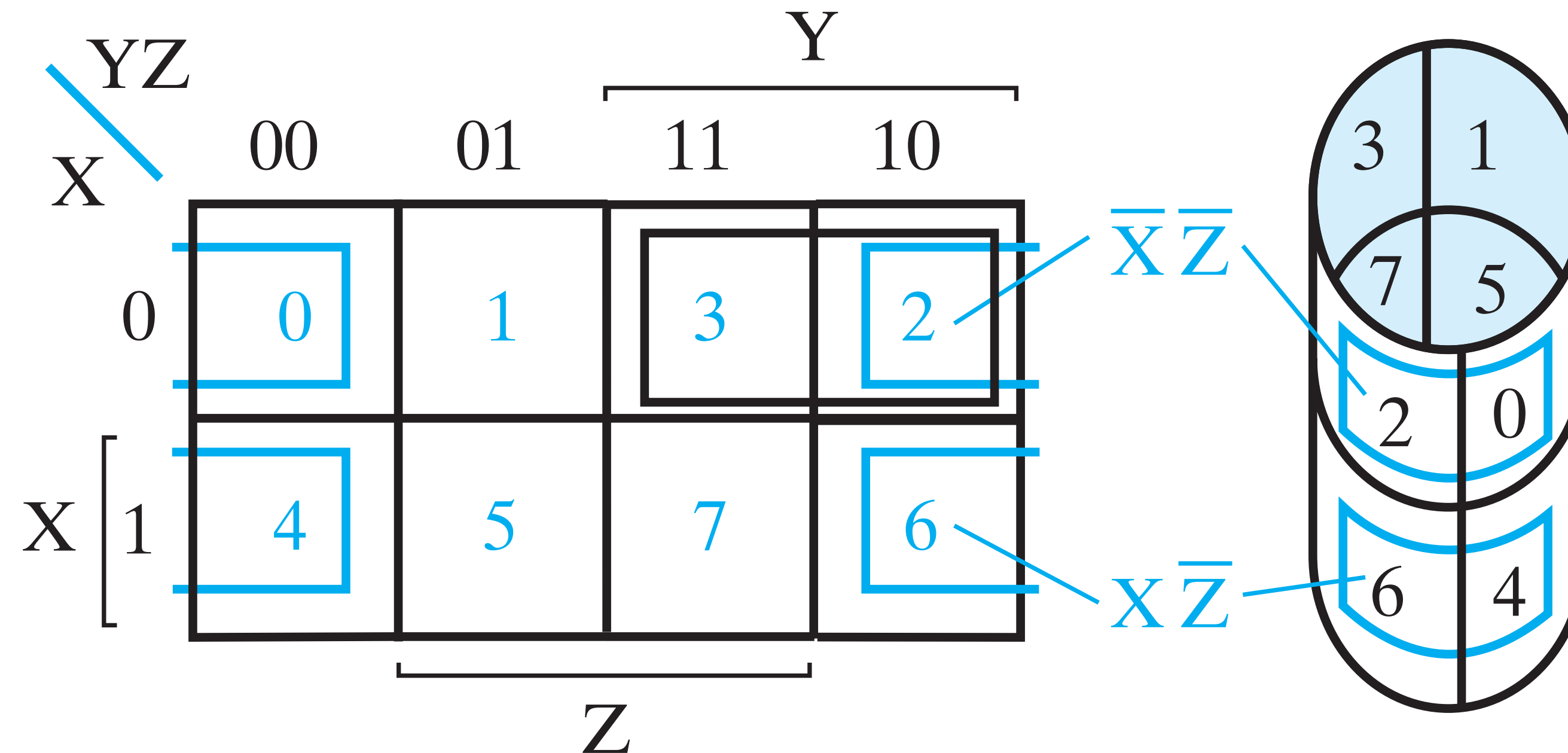
- Karnaugh Map, or just K-Map
- For optimising 2-4 variable boolean expressions
- Skip: 5,6 variable K-Maps can also be drawn but are not very intuitive to use

Two Variable Maps



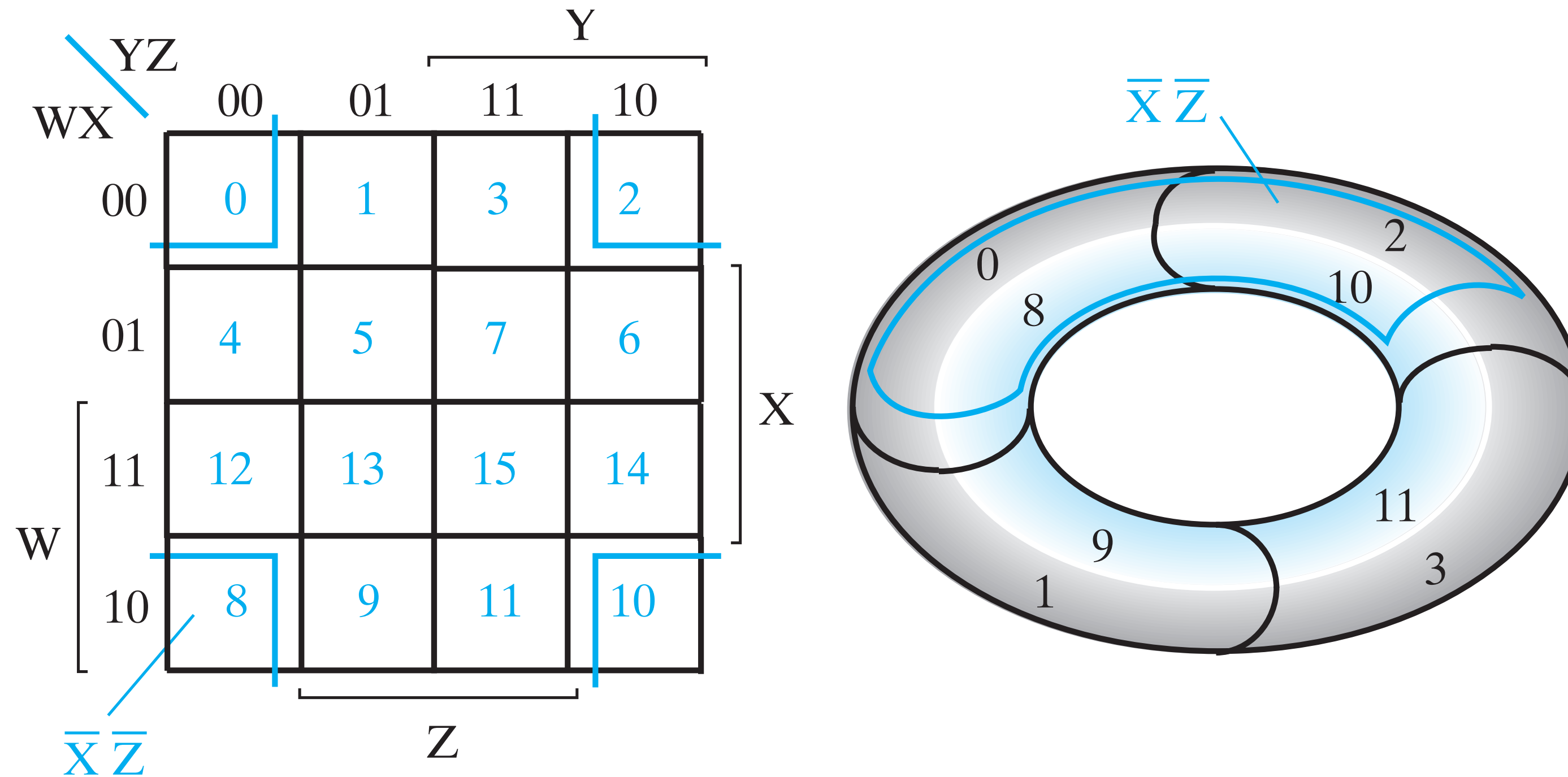
- Number of squares in each map is equal to the number of minterms for the same number of variables, light blue digit above is the index (of minterm)
- Two squares are adjacent if they only differ in one variable
- Binary value inside at each position indicates the truth table value for that term

Three Variable Maps



- Number of squares in each map is equal to the number of minterms for the same number of variables, light blue digit above is the index (of minterm)
- Two squares are adjacent if they only differ in one variable
- Binary value inside at each position indicates the truth table value for that term

Four Variable Maps



- Number of squares in each map is equal to the number of minterms for the same number of variables, light blue digit above is the index (of minterm)
- Two squares are adjacent if they only differ in one variable
- Binary value inside at each position indicates the truth table value for that term

Two Variable Maps Optimisation

		Y	
		0	1
X	0	0	1
	1	2	3

Truth Table

X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

Two Variable Maps Optimisation

		Y	
		0	1
X	0	0	1
	1	2	3

Truth Table

X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values

Two Variable Maps Optimisation

		Y	
		0	1
X	0	⁰ 0	¹ 1
	1	² 0	³ 1

Truth Table

X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values

Two Variable Maps Optimisation

		Y	
		0	1
X	0	0 0	1 1
	1	2 0	3 1

Truth Table

X	Y	F
0	0	0
0	1	1
1	0	0
1	1	1

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s

Two Variable Maps Optimisation

		Y	
		0	1
X	0	0 0	1 1
	1	2 0	3 1

Truth Table

X	Y	F
0	0	0
0	1	1
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- Step 1: Enter the values
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- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Two Variable Maps Optimisation

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Truth Table

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Two Variable Maps Optimisation

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Two Variable Maps Optimisation

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Two Variable Maps Optimisation

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X	0	0 1	1 1
	1	2 0	3 1

Truth Table

X	Y	F
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0	1	1
1	0	0
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- Step 1: Enter the values
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Two Variable Maps Optimisation

Truth Table

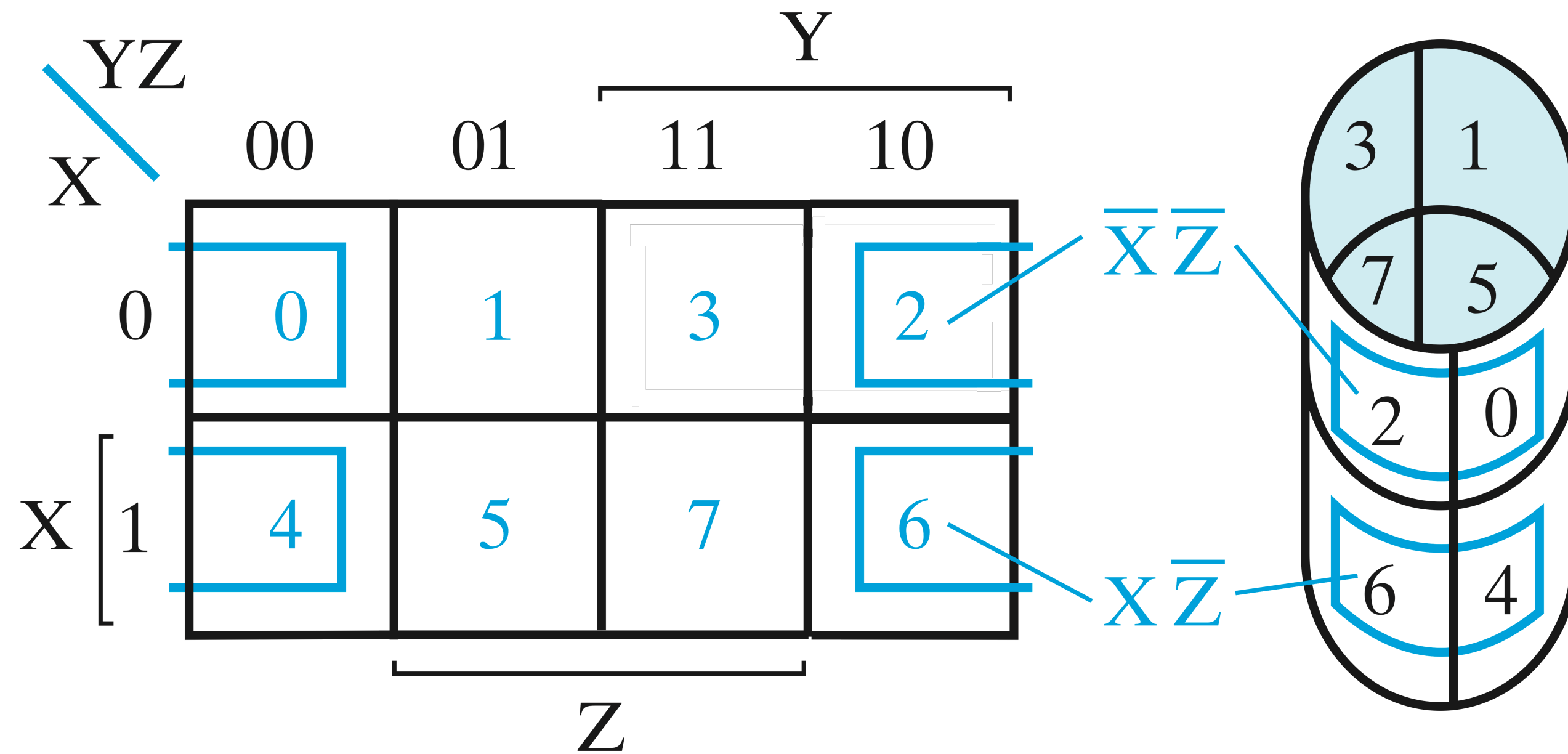
		Y	
		0	1
X	0	0 1	1 1
	1	2 0	3 1

$\bar{X} + Y$

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	1

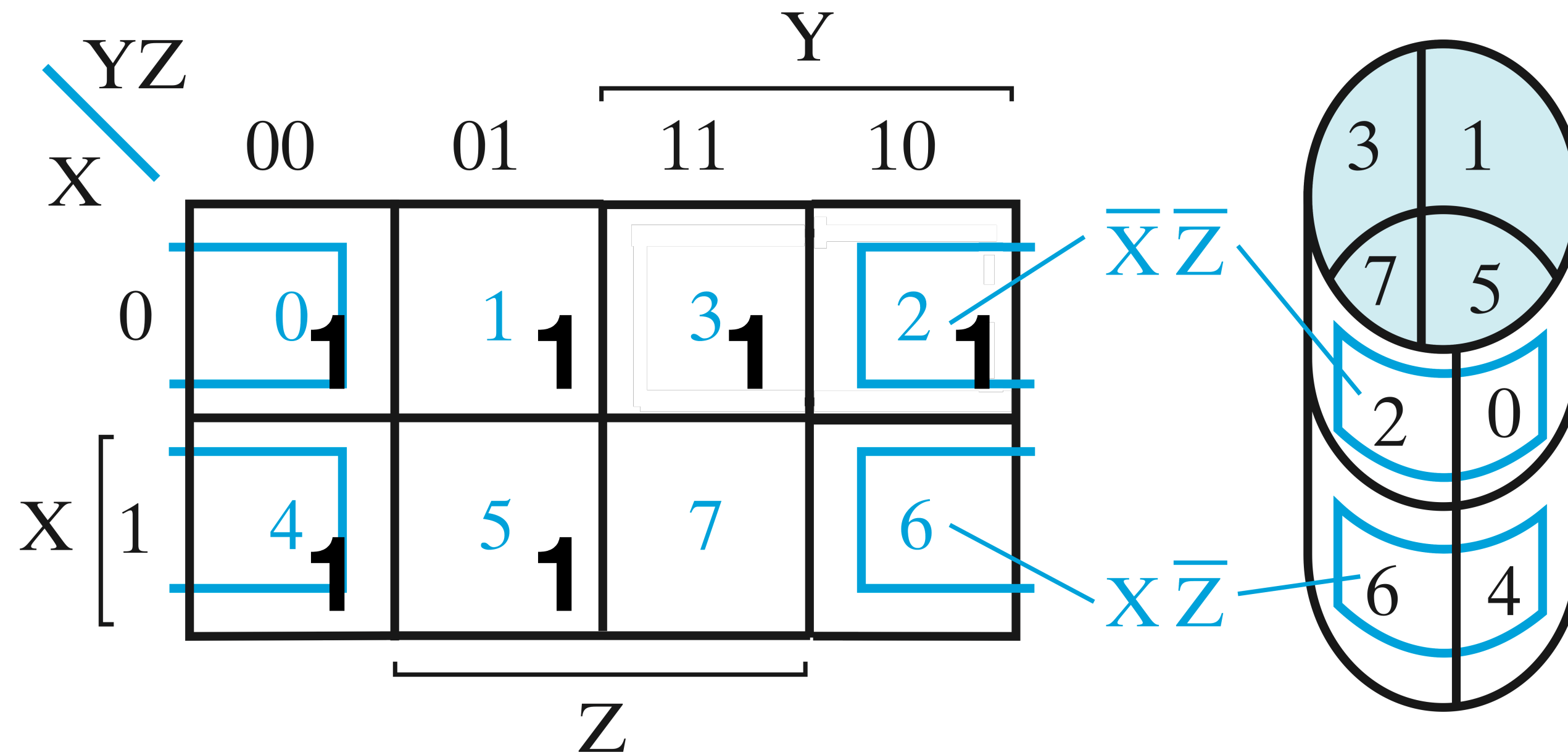
- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
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Three Variable Maps Optimisation



$$F(X, Y, Z) = \sum m(0, 1, 2, 3, 4, 5)$$

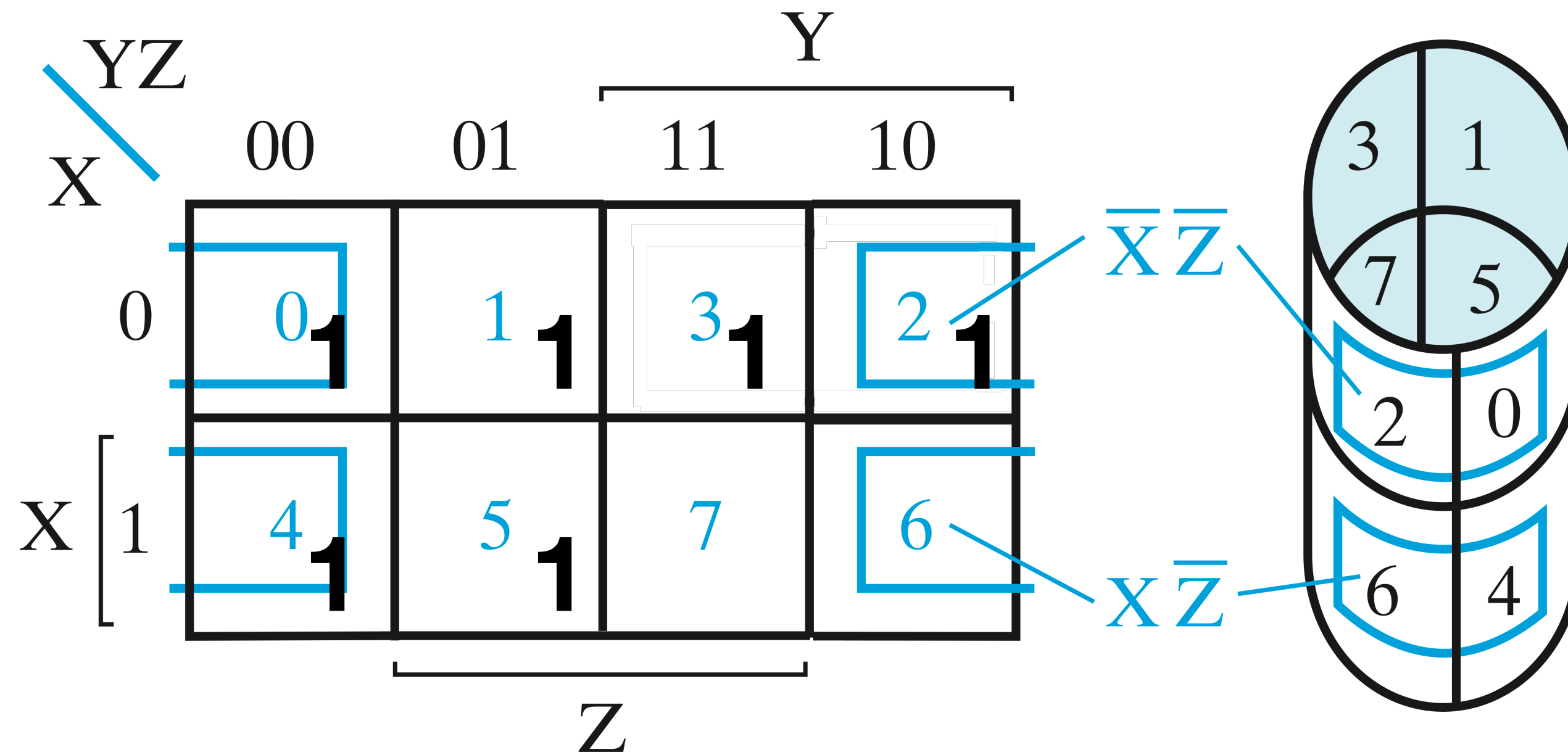
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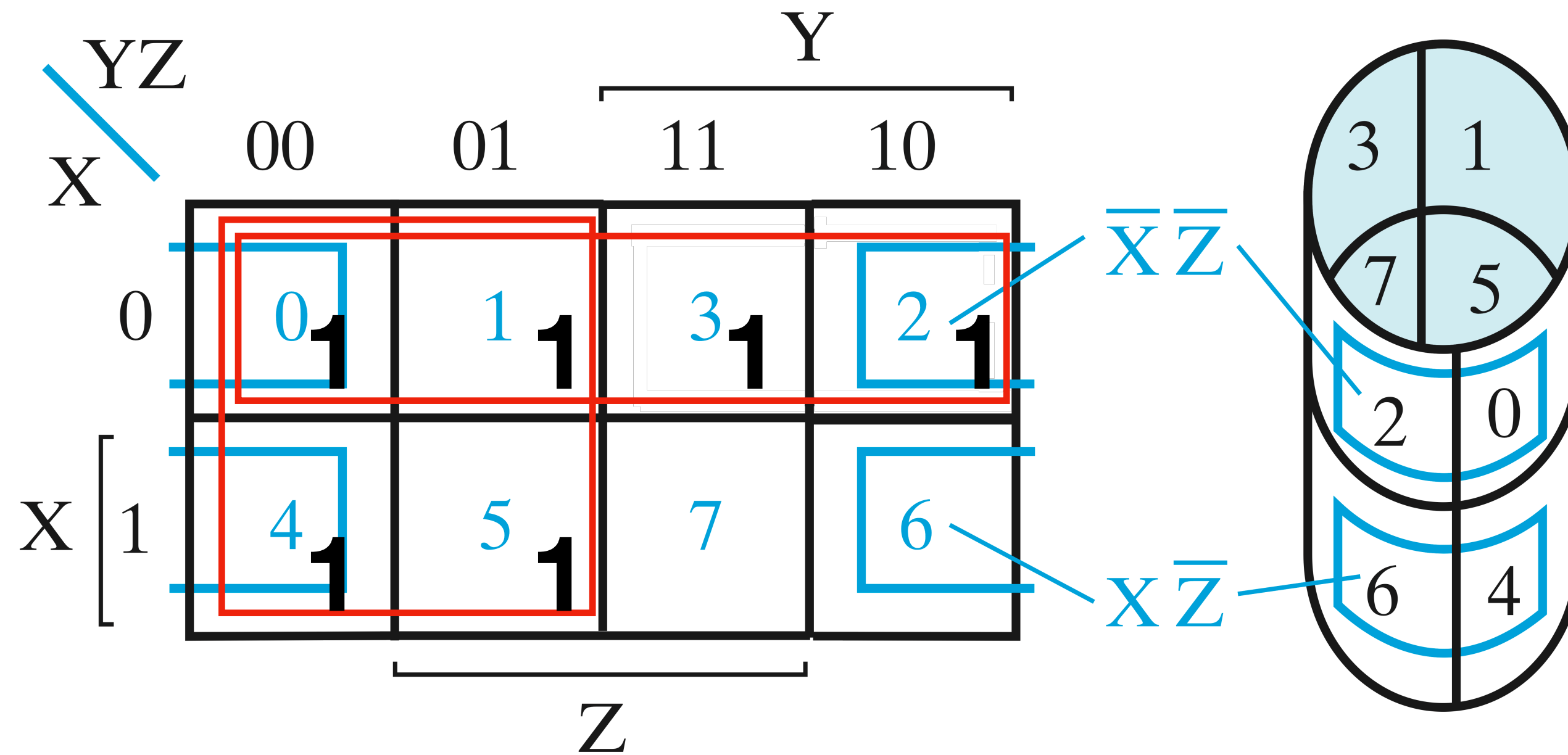
Three Variable Maps Optimisation



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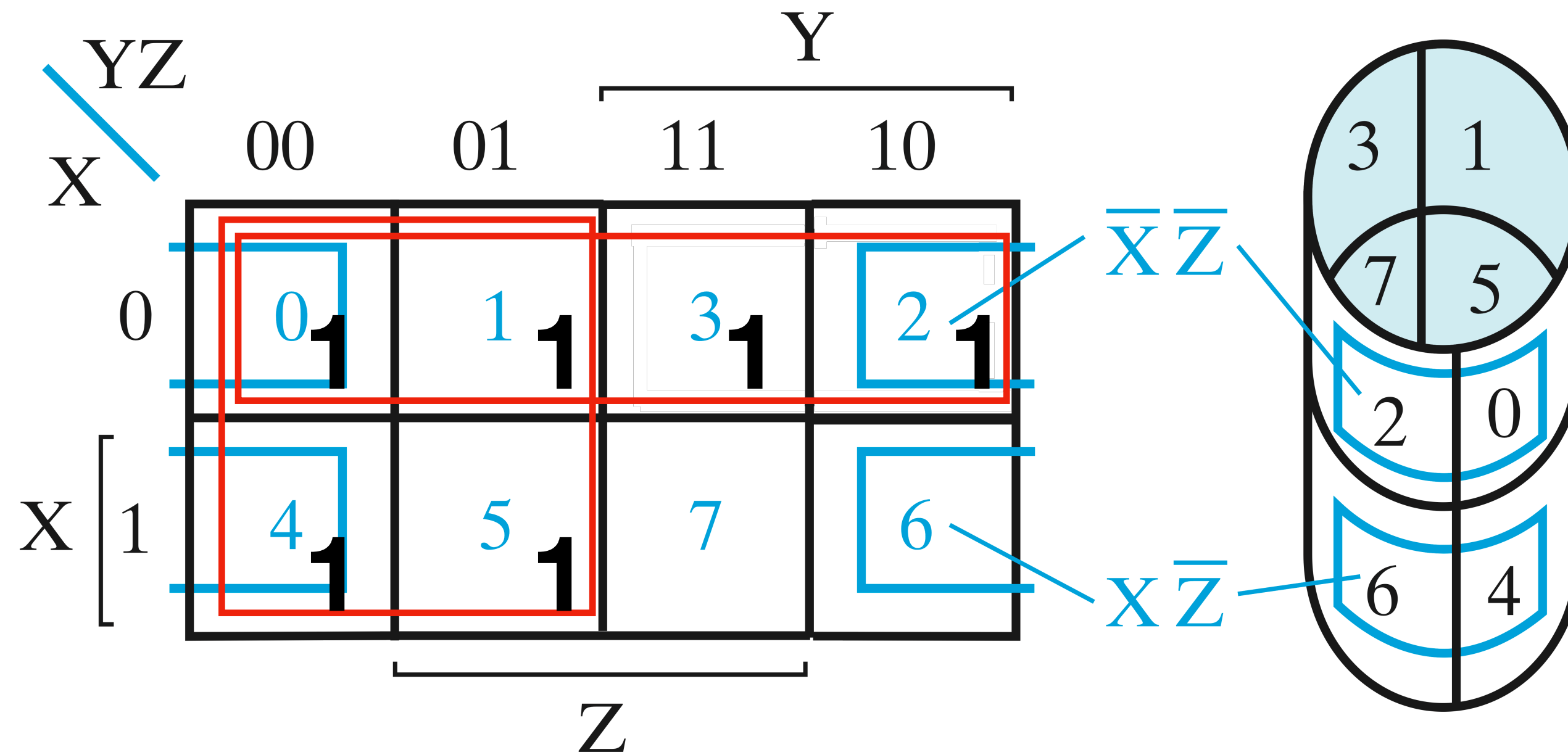
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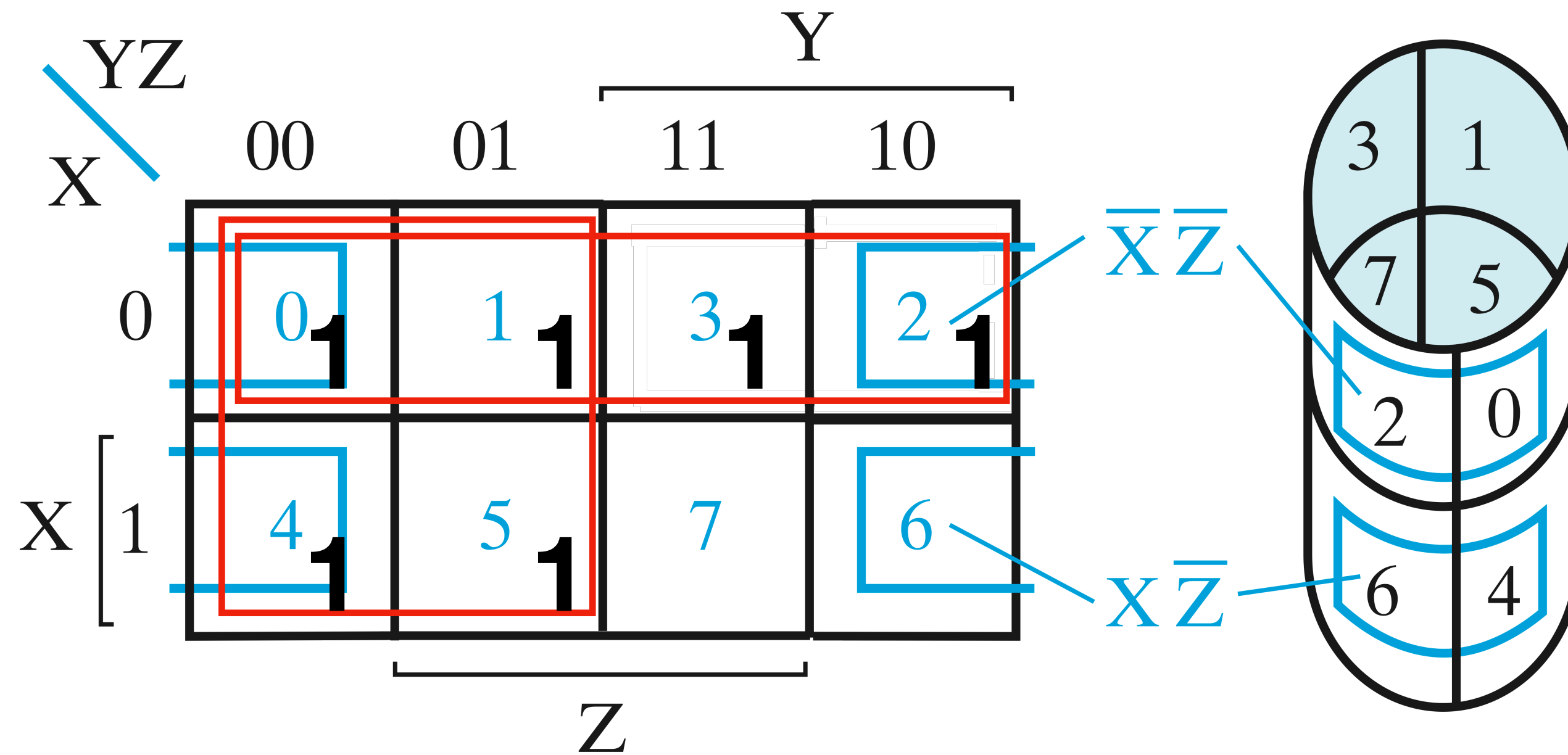
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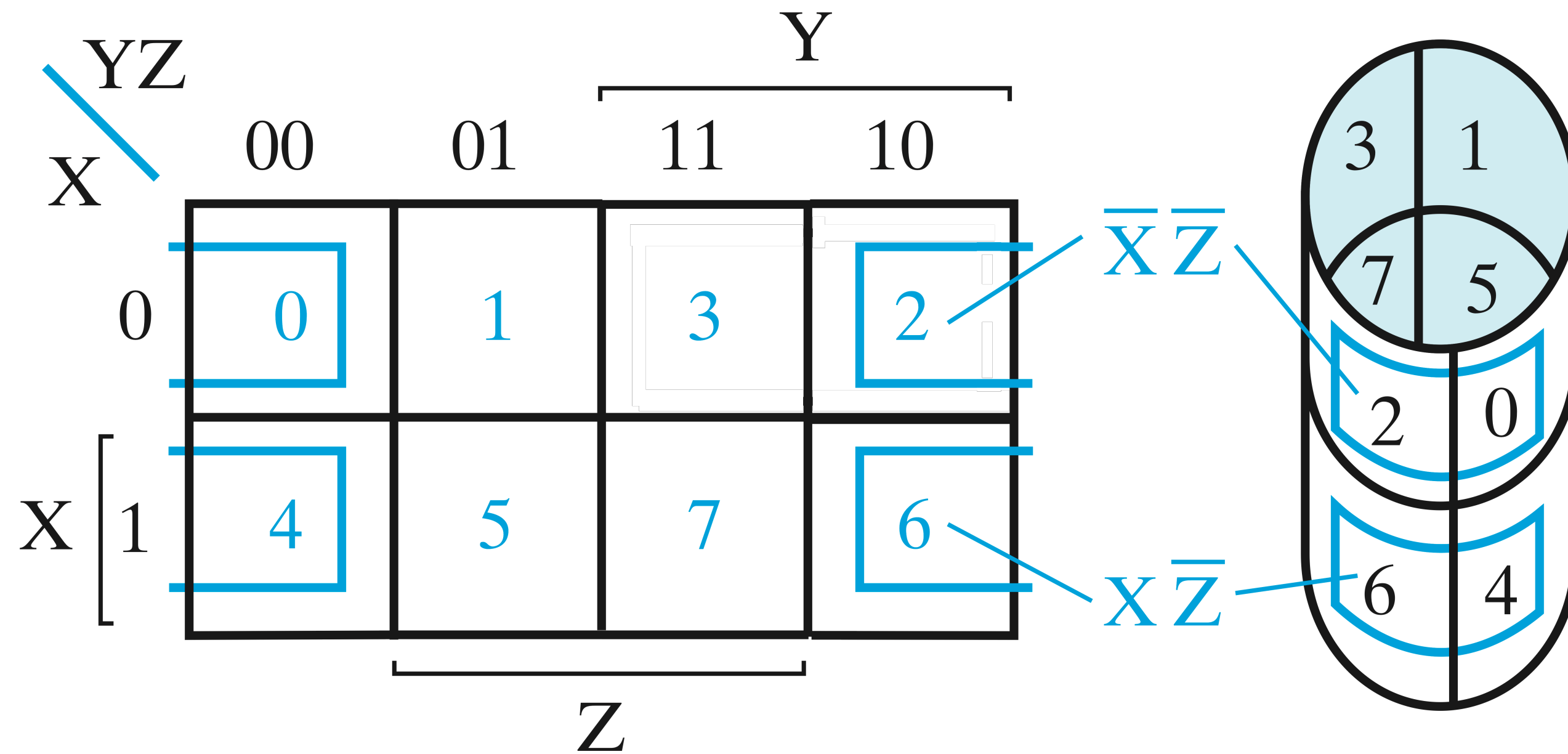
Three Variable Maps Optimisation



$$F(X, Y, Z) = \sum m(0, 1, 2, 3, 4, 5) \\ = \bar{X} + \bar{Y}$$

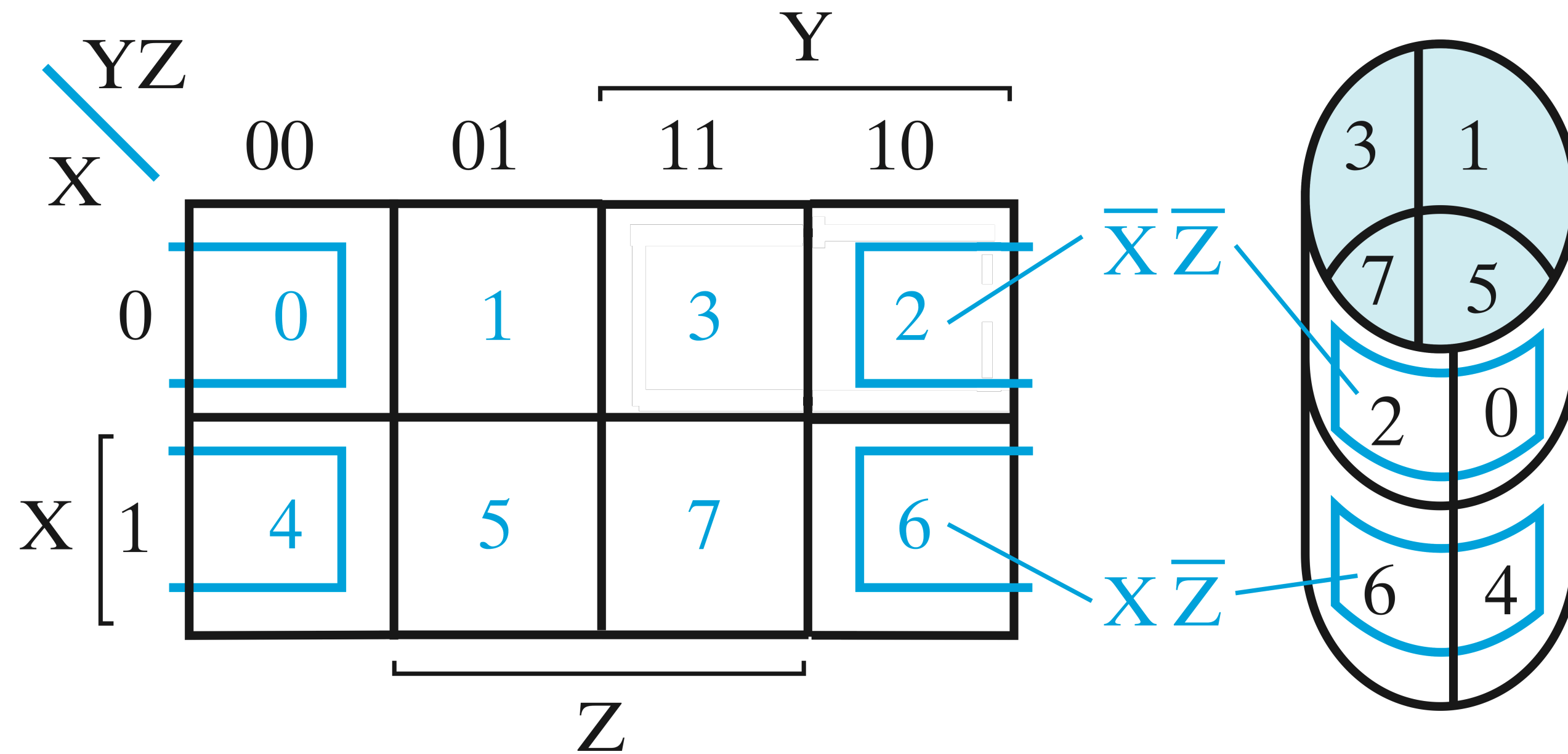
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Three Variable Maps Optimisation



$$F(X, Y, Z) = \Sigma m(0, 2, 4, 5, 6)$$

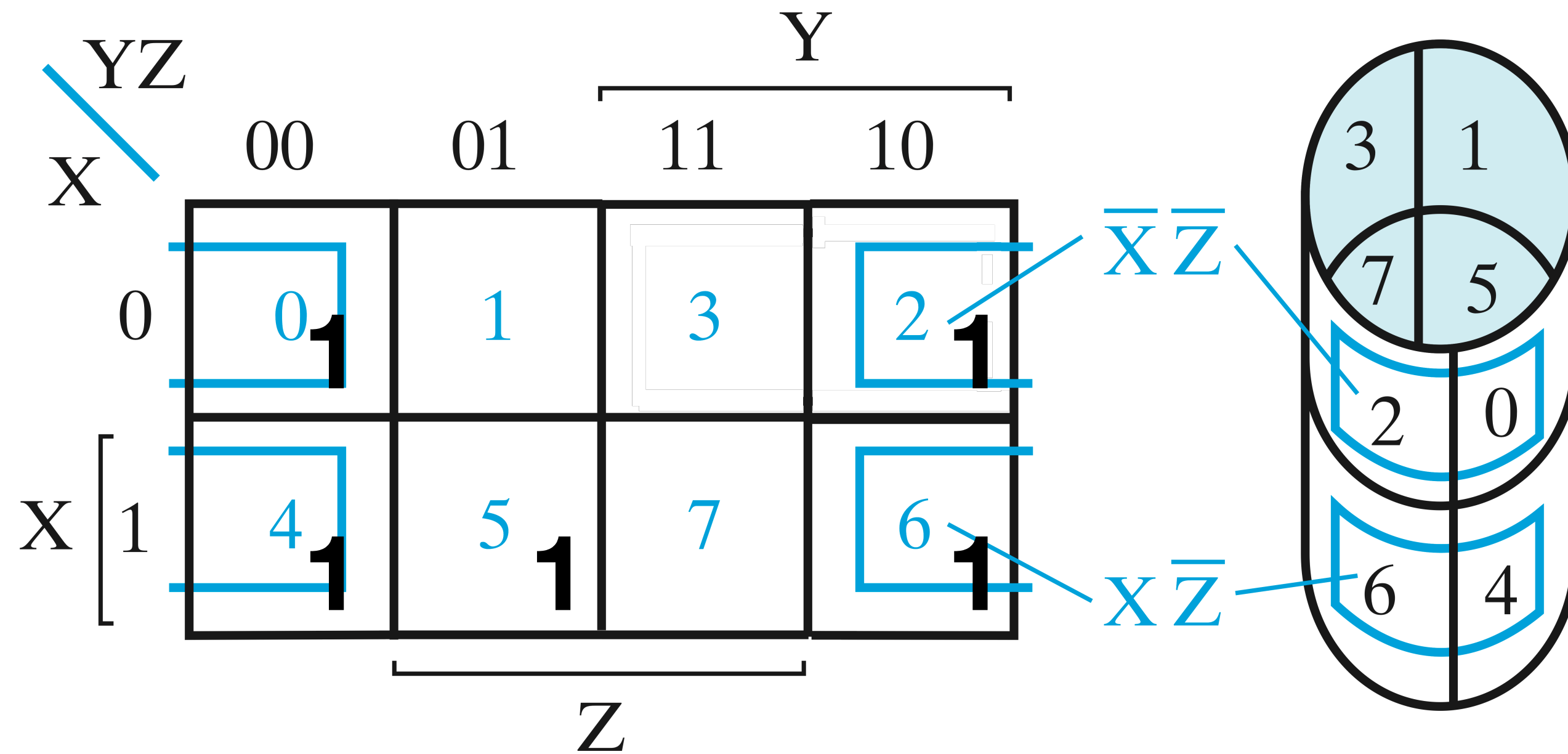
Three Variable Maps Optimisation



- Step 1: Enter the values

$$F(X, Y, Z) = \Sigma m(0, 2, 4, 5, 6)$$

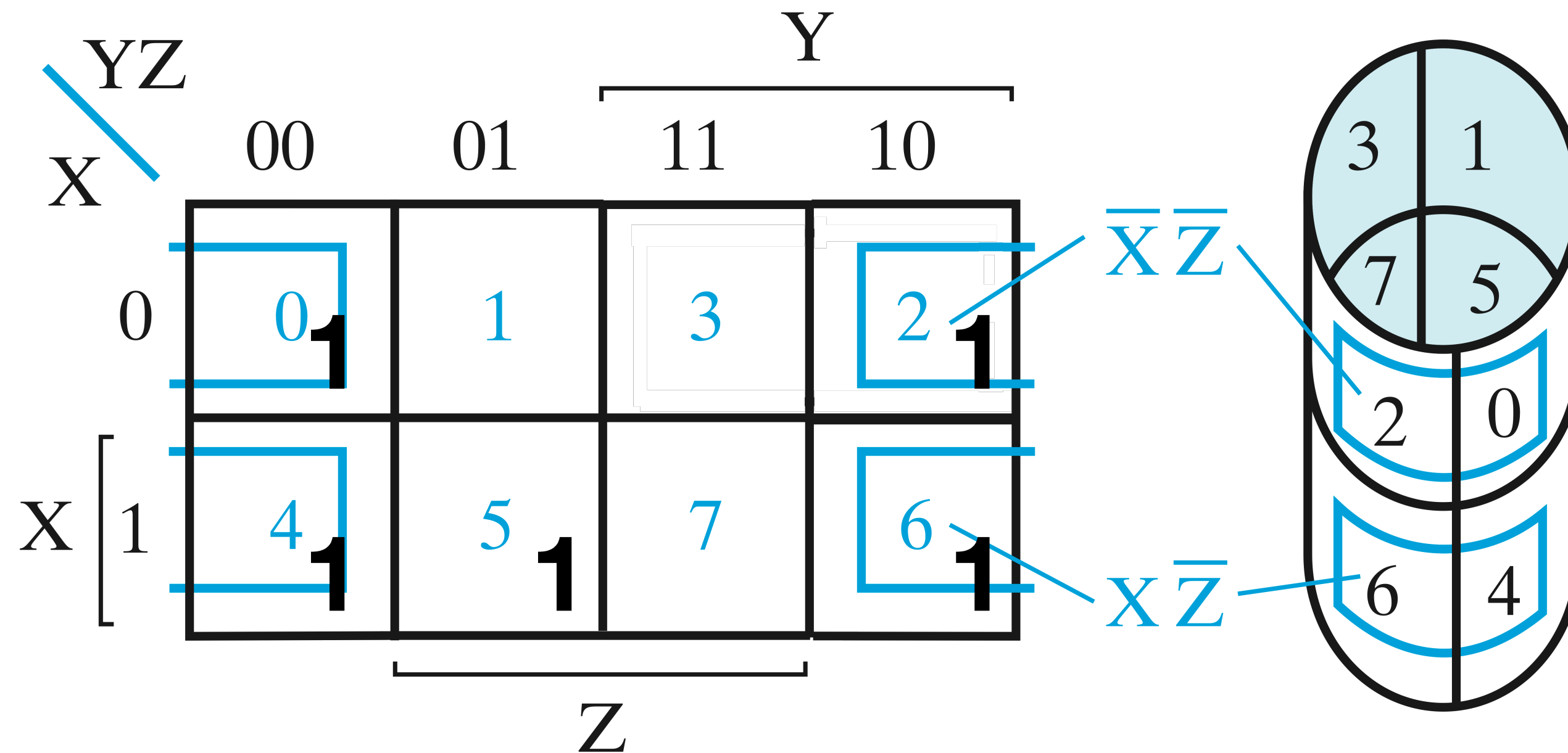
Three Variable Maps Optimisation



- Step 1: Enter the values

$$F(X, Y, Z) = \Sigma m(0, 2, 4, 5, 6)$$

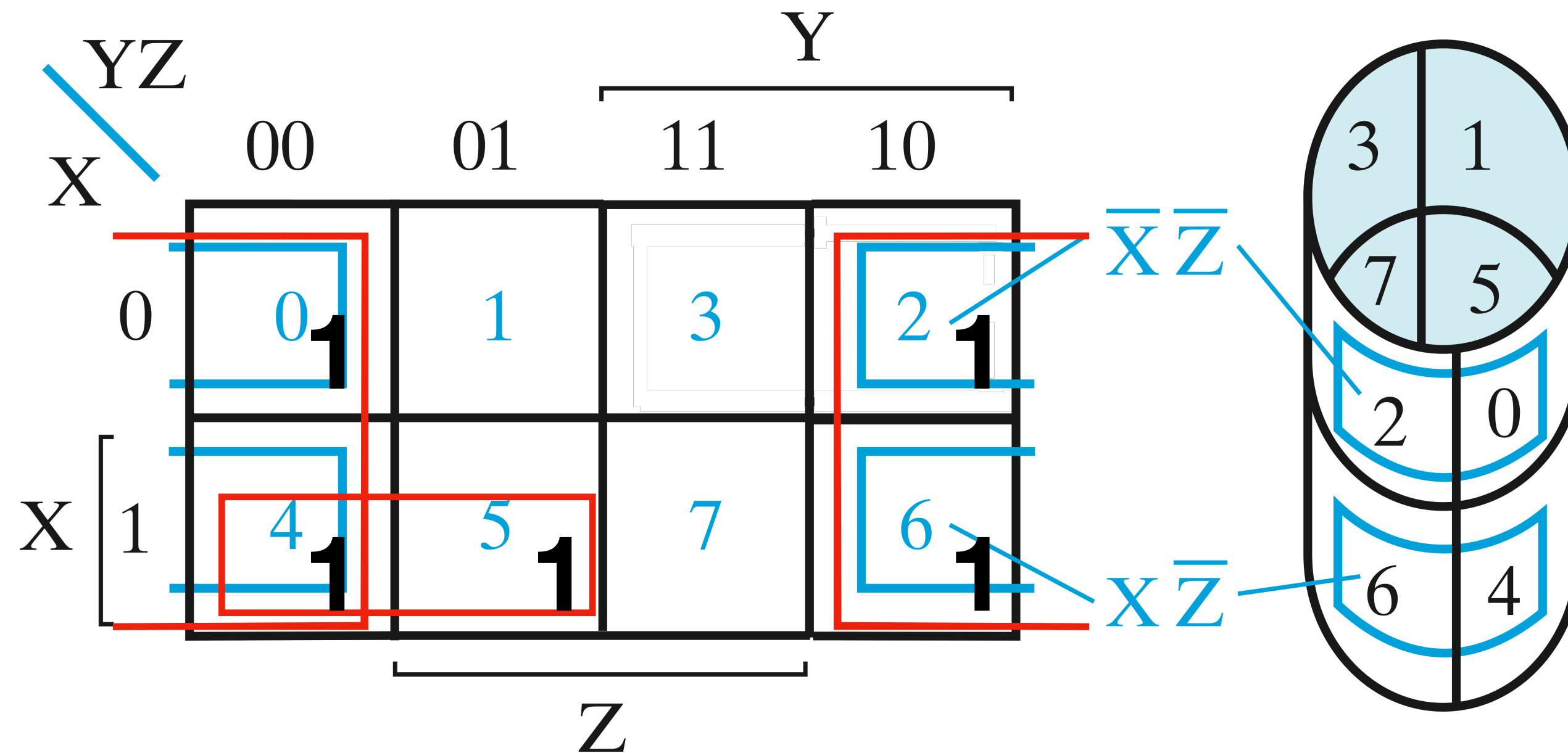
Three Variable Maps Optimisation



$$F(X, Y, Z) = \sum m(0, 2, 4, 5, 6)$$

- Step 1: Enter the values
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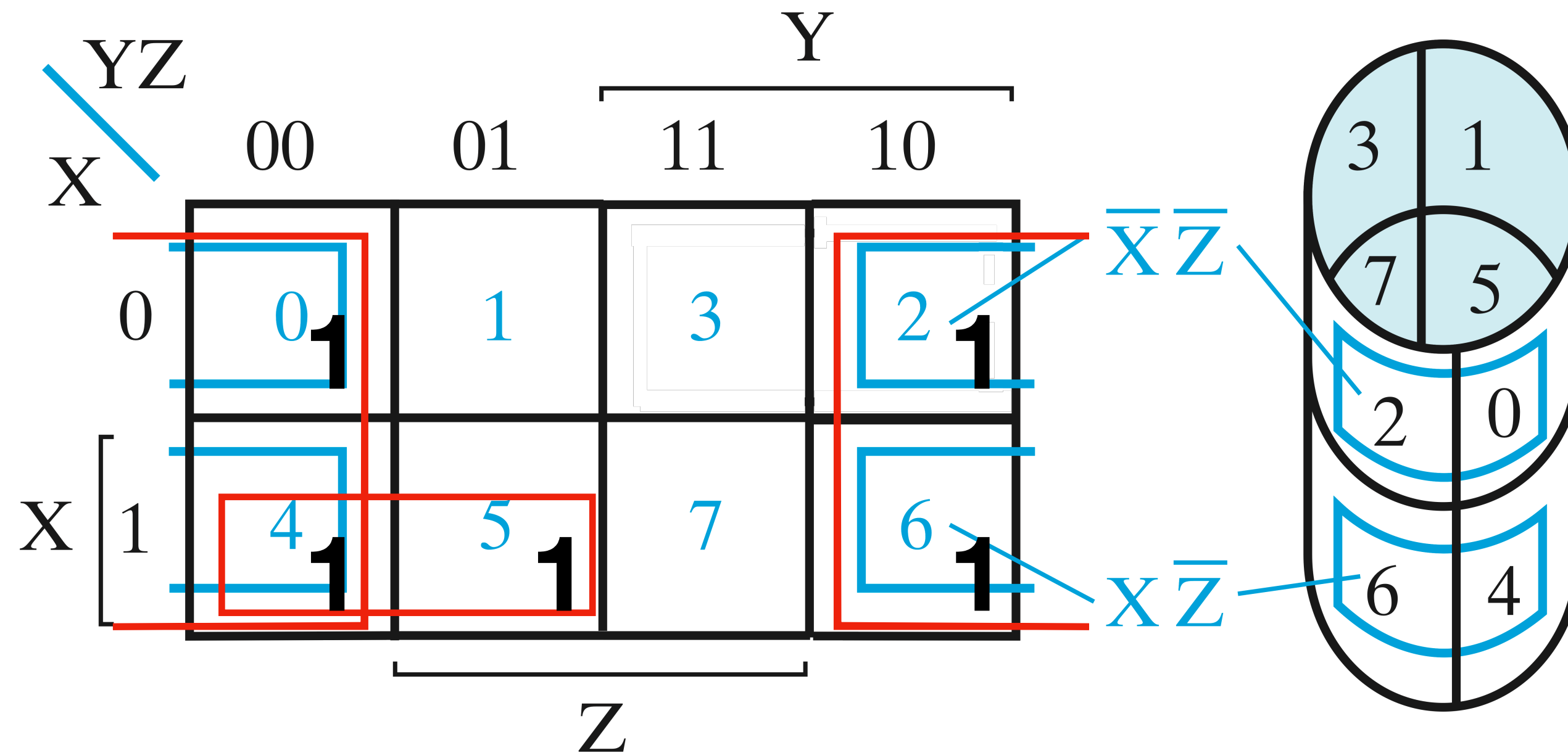
Three Variable Maps Optimisation



- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s

$$F(X, Y, Z) = \Sigma m(0, 2, 4, 5, 6)$$

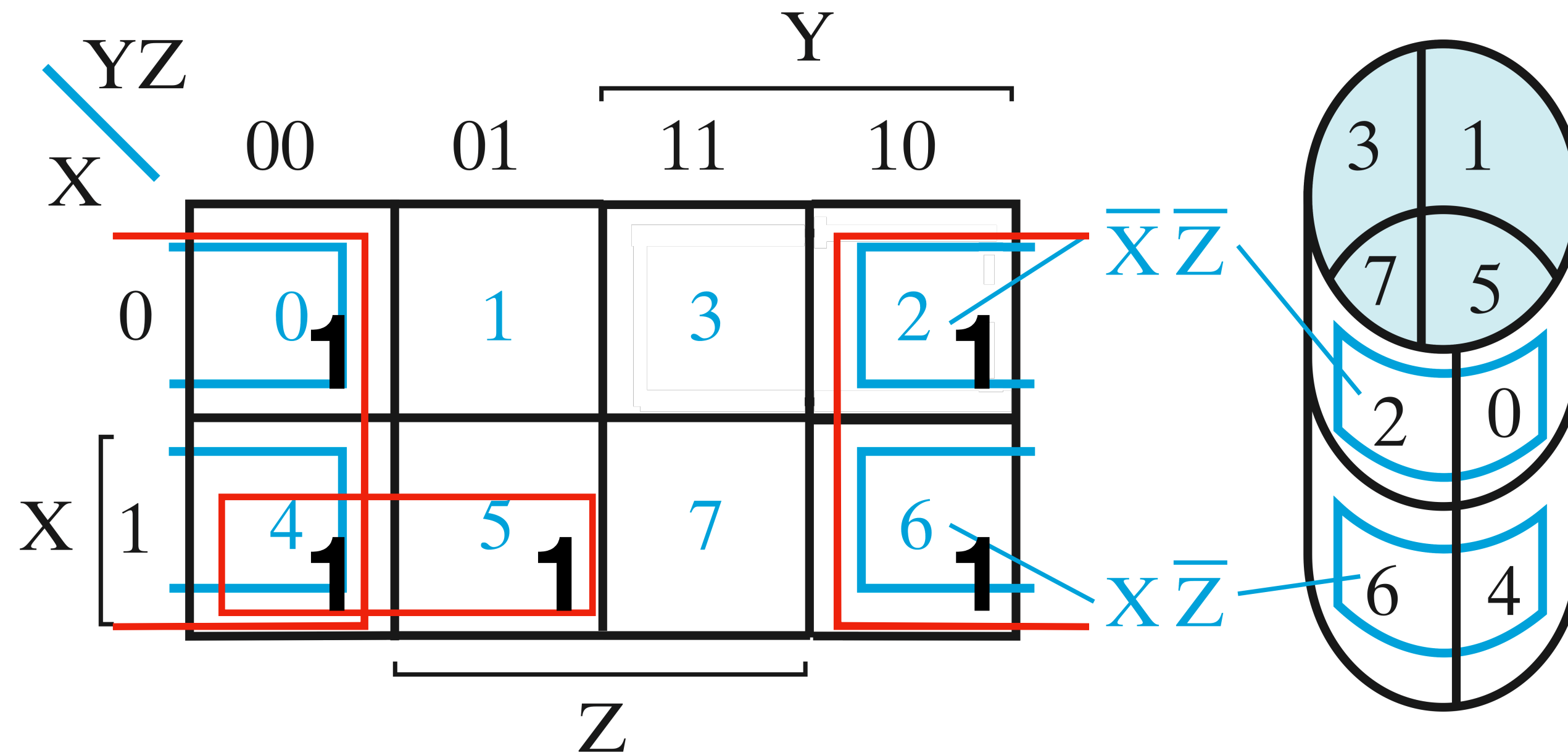
Three Variable Maps Optimisation



$$F(X, Y, Z) = \sum m(0, 2, 4, 5, 6)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
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Three Variable Maps Optimisation

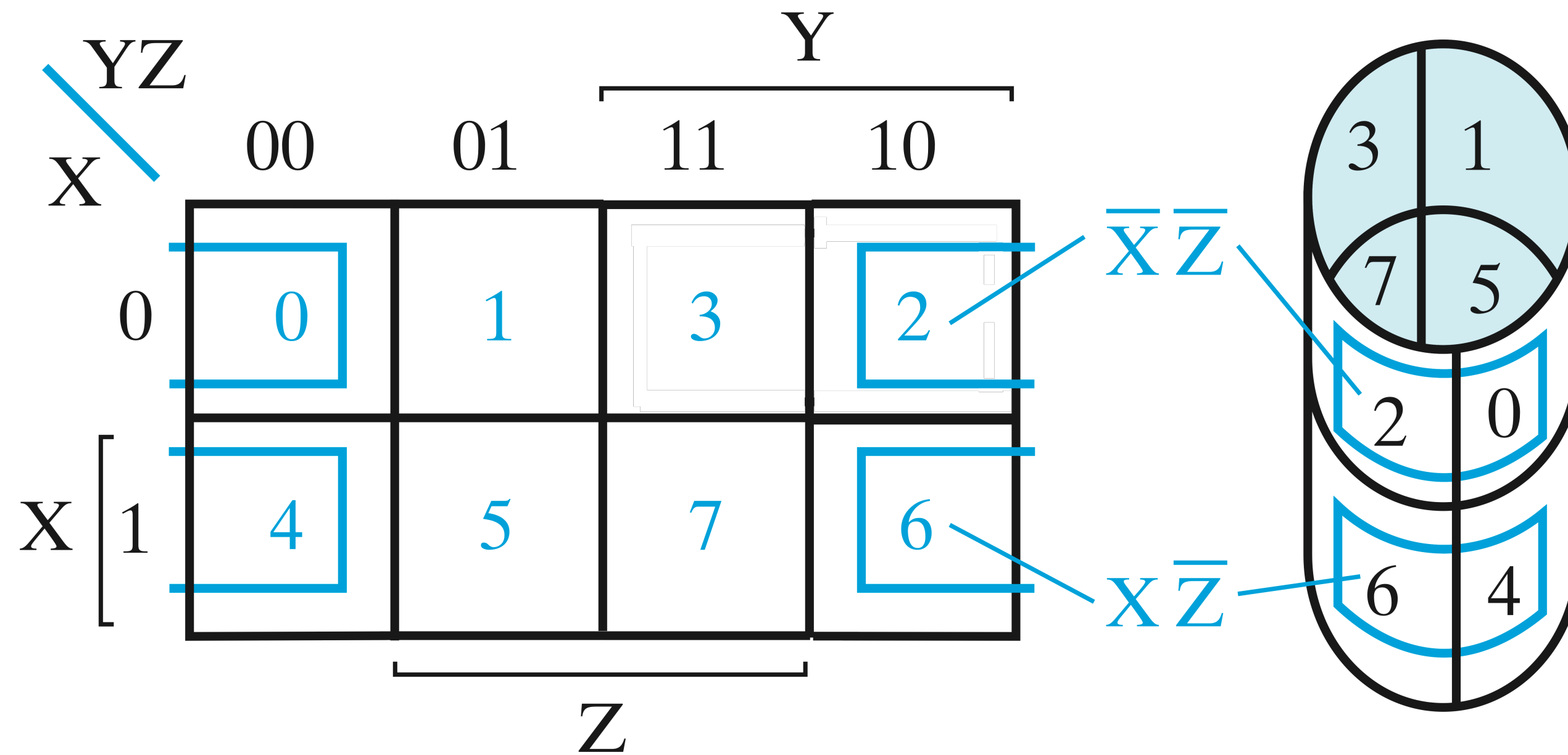


$$F(X, Y, Z) = \sum m(0, 2, 4, 5, 6)$$

$$= X\bar{Y} + \bar{Z}$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

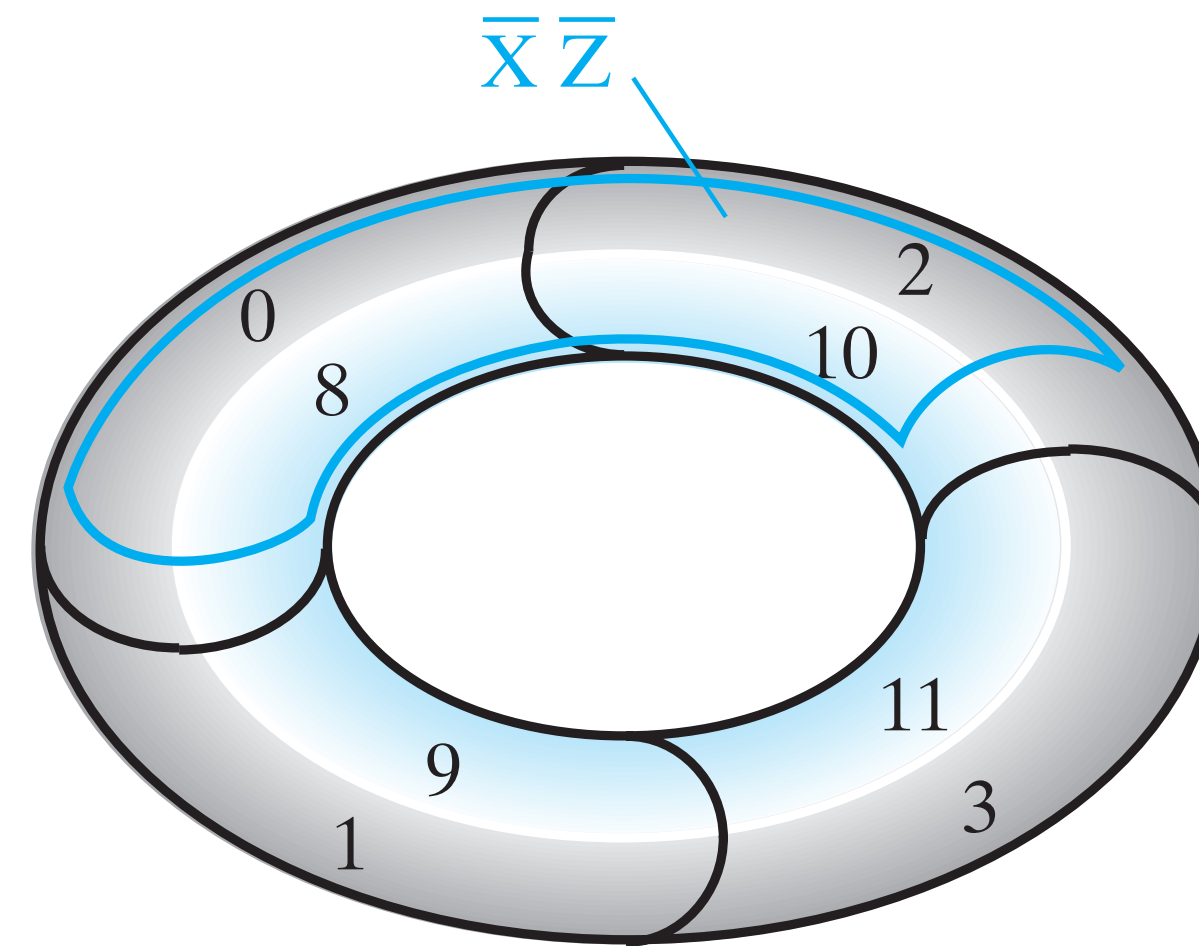
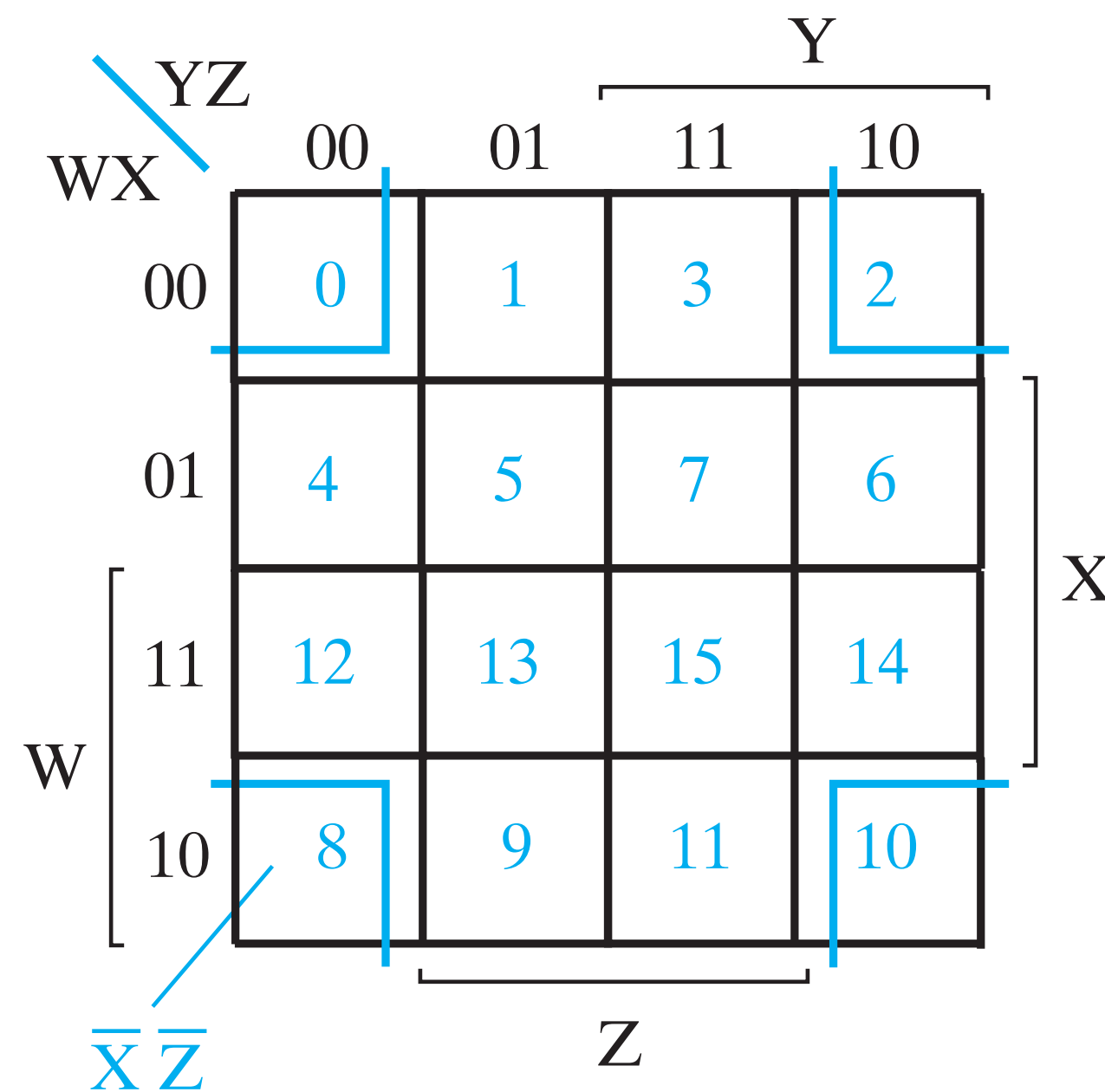
Three Variable Maps Optimisation



$$F(X, Y, Z) = \sum m(1, 3, 4, 5, 6)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Four Variable Maps Optimisation



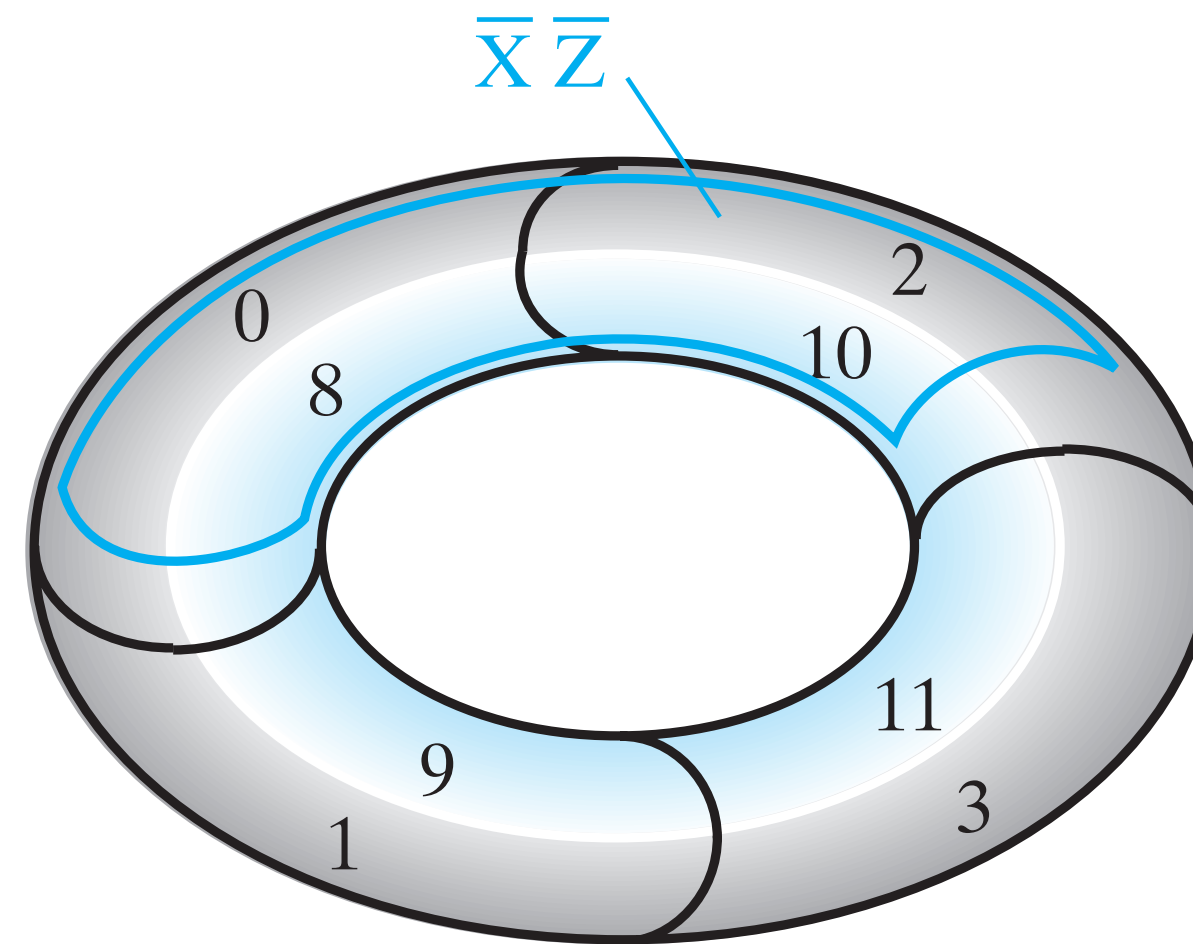
- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

$$F(W, X, Y, Z) = \overline{W}\overline{Y}\overline{Z} + \overline{W}Z + \overline{X}Y + YZ + W\overline{X}\overline{Z}$$

Don't Care Condition

		Y			
		00	01	11	10
WX	00	1	1	3	2
	01	4	5	7	6
W	11	12	13	15	14
	10	8	9	11	10

YZ
WX
X
Z
 $\bar{X}\bar{Z}$

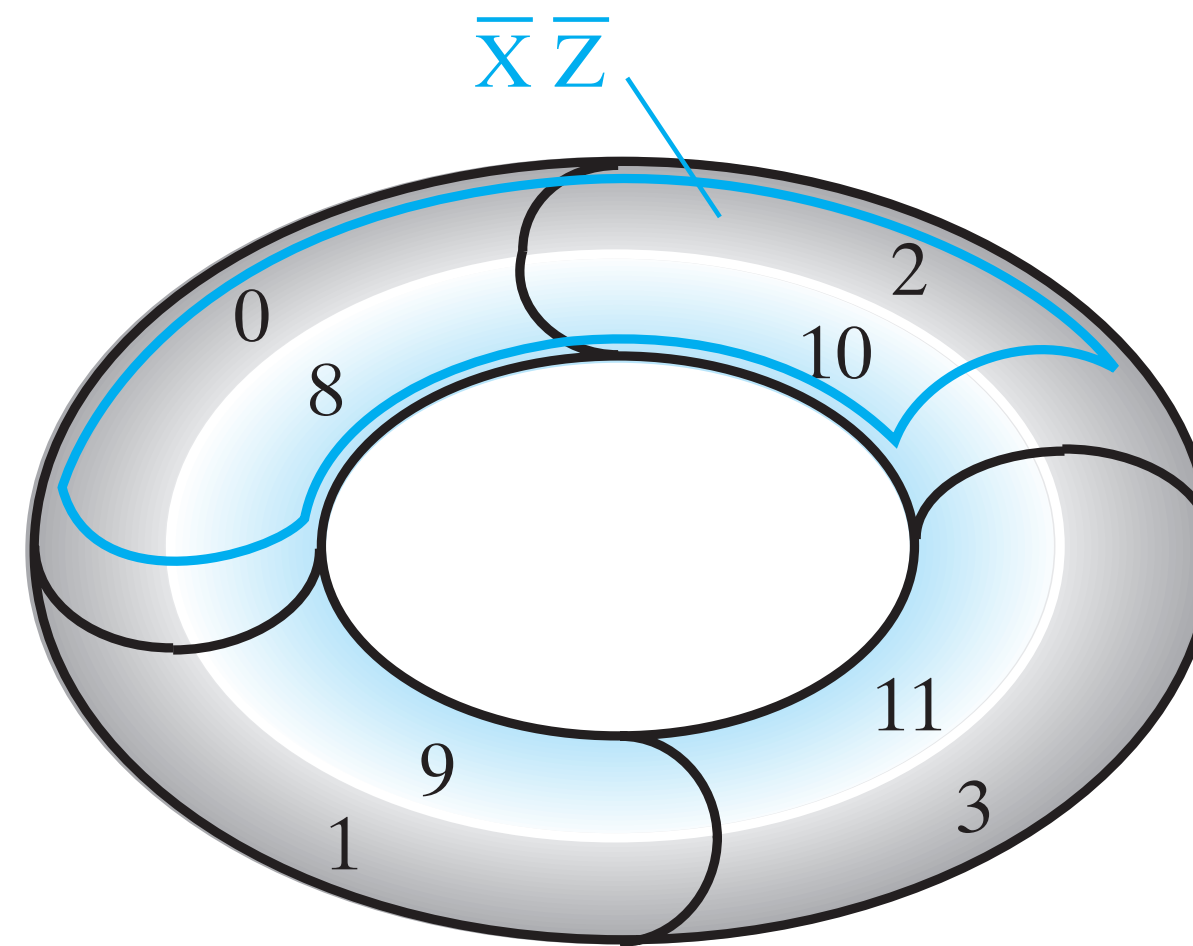


- Sometimes we don't care what the output is when the inputs are in certain combinations

Don't Care Condition

		Y			
		00	01	11	10
W	00	X	1	1	X
	01	4	X	1	6
	11	12	13	1	14
	10	8	9	1	10

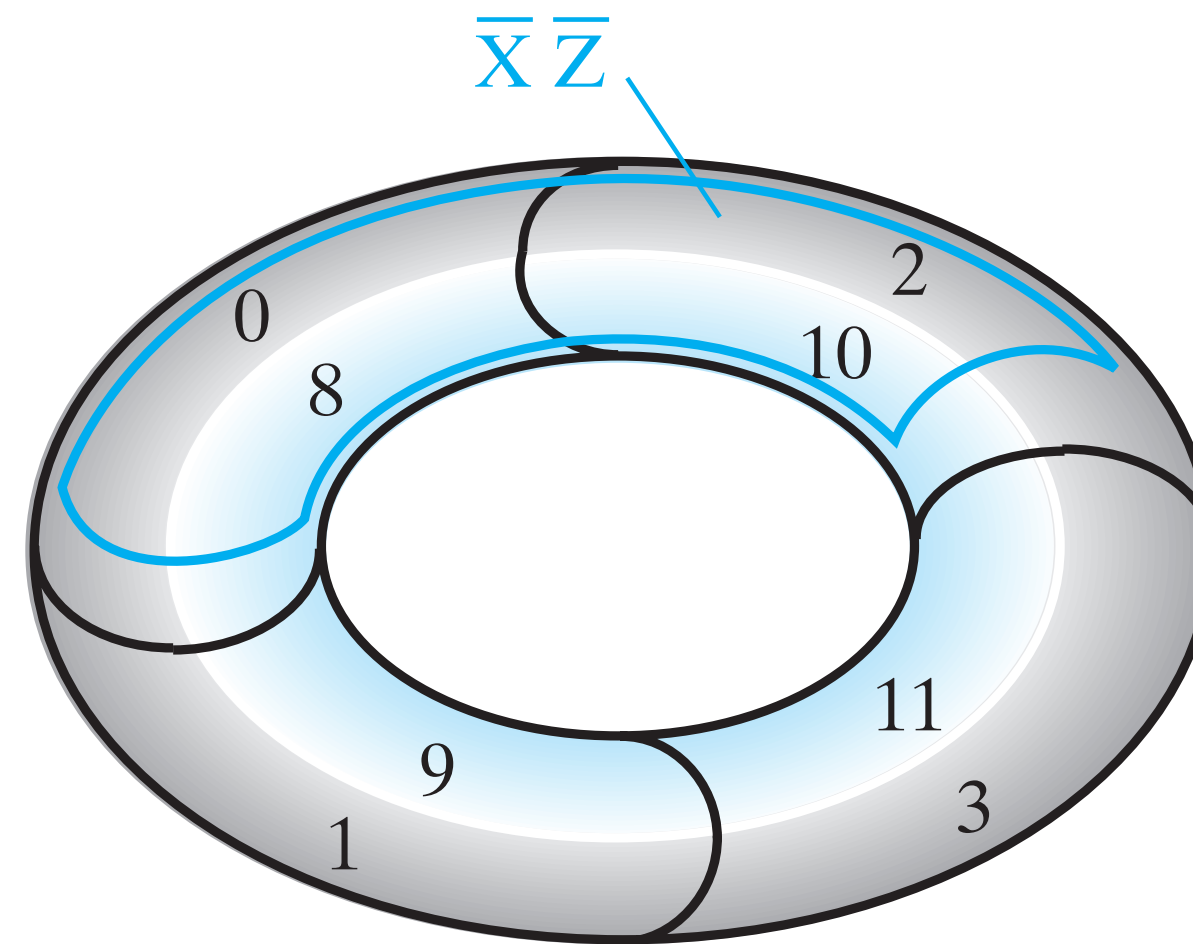
YZ
WX
X
Z
 $\bar{X}\bar{Z}$



- Sometimes we don't care what the output is when the inputs are in certain combinations

Don't Care Condition

		Y			
		00	01	11	10
W	YZ				
	WX				
	00	X	1	1	X
	01	4	X	7	6
	11	12	13	15	14
	10	8	9	11	10
		Z			
		X			

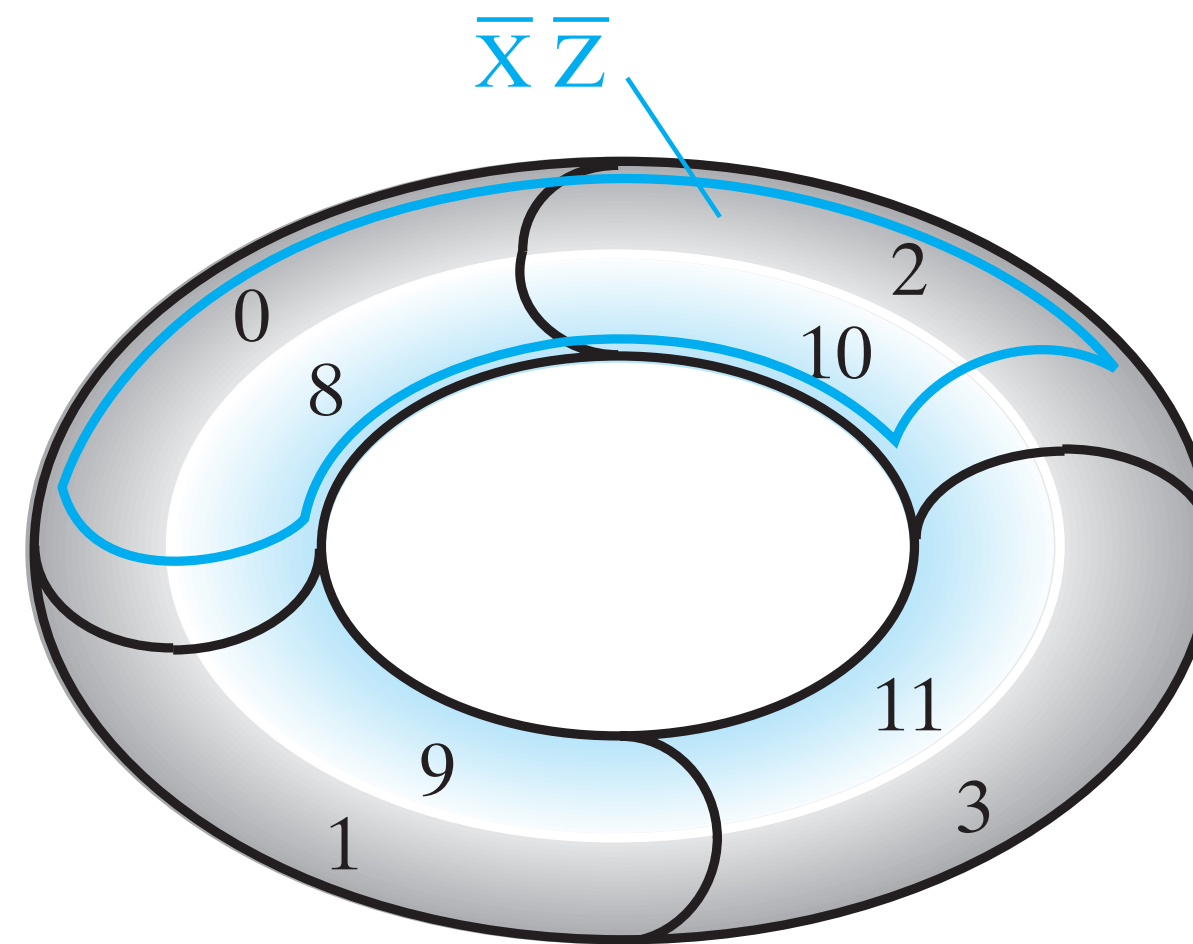


- Sometimes we don't care what the output is when the inputs are in certain combinations

Don't Care Condition

		Y			
		00	01	11	10
W	00	X	1	1	X
	01	4	X	7	6
	11	12	13	15	14
	10	8	9	11	10
		Z			

Diagram illustrating a 4x4 Karnaugh map for a function F. The map is labeled with variables W, X, Y, and Z. The top row (W=00) contains values X, 1, 1, X. The bottom row (W=10) contains values 8, 9, 11, 10. The left column (X=00) contains values X, 4, 12, 8. The right column (X=10) contains values X, 6, 14, 10. The map is divided into four quadrants by W and X. The top-left quadrant (W=00, X=00) contains values X, 4, 12, 8. The top-right quadrant (W=00, X=10) contains values X, 6, 14, 10. The bottom-left quadrant (W=10, X=00) contains values 8, 9, 11, 10. The bottom-right quadrant (W=10, X=10) contains values 11, 10, 14, 10. The map is labeled with YZ (00, 01, 11, 10) and WX (00, 01, 11, 10). The map is also labeled with X and Z.



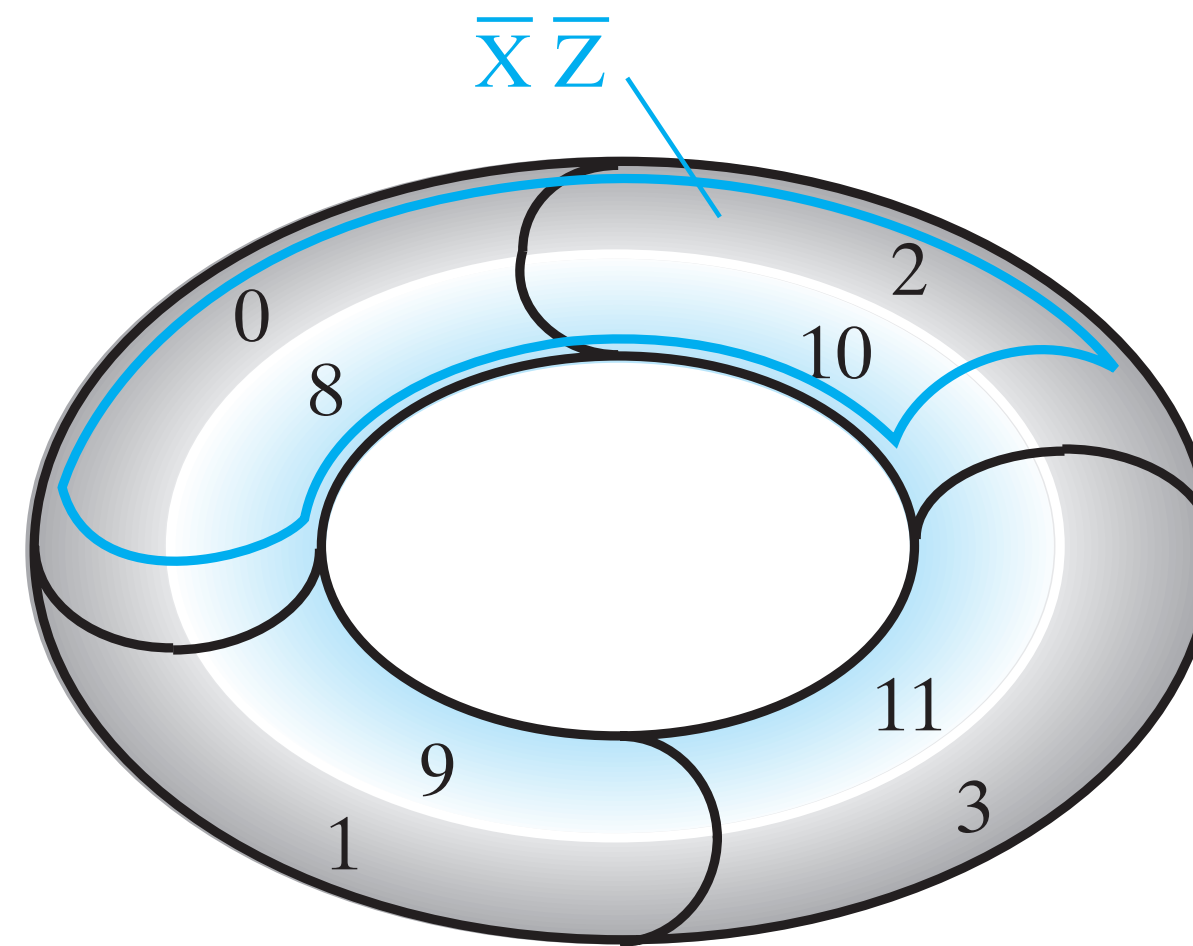
- Sometimes we don't care what the output is when the inputs are in certain combinations

$$F = YZ + \overline{W}\overline{X}$$

Don't Care Condition

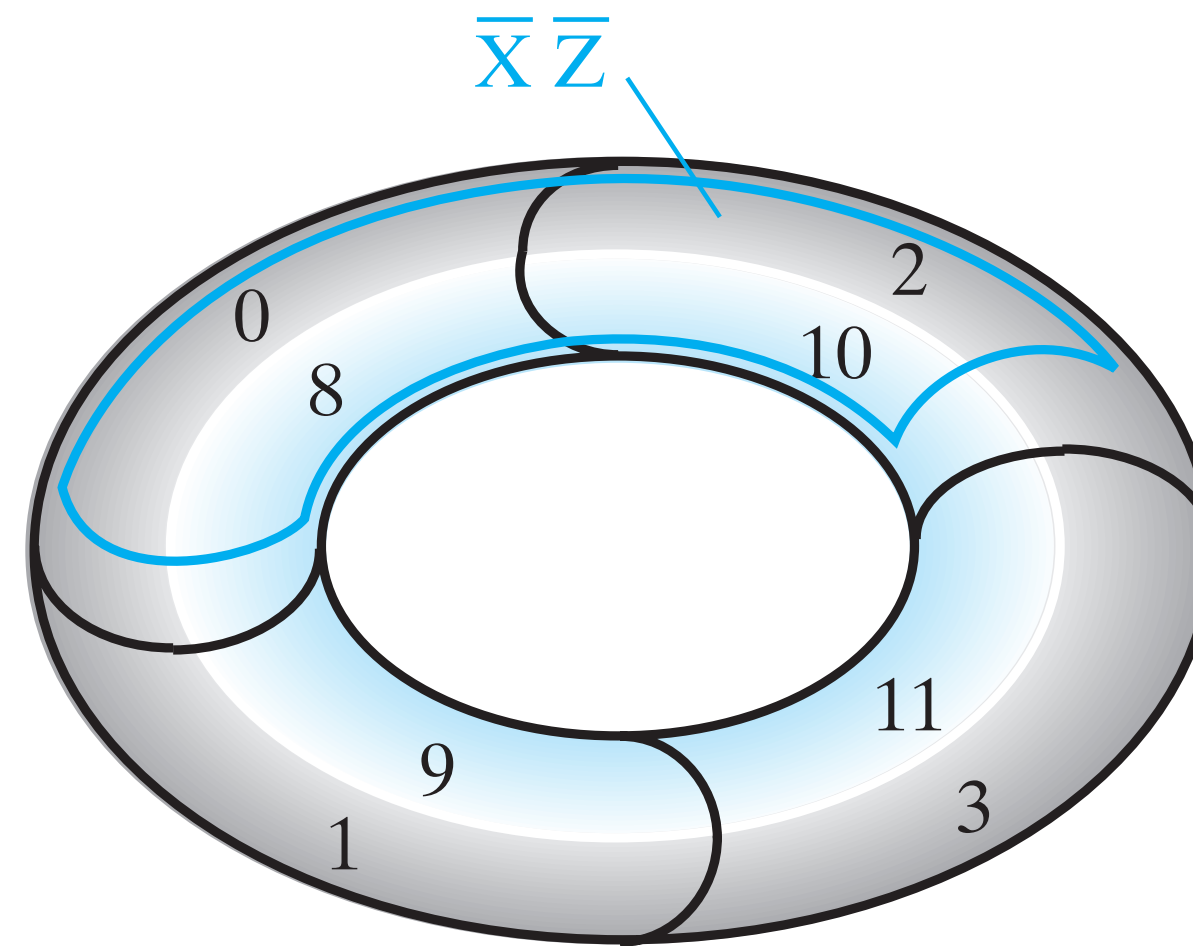
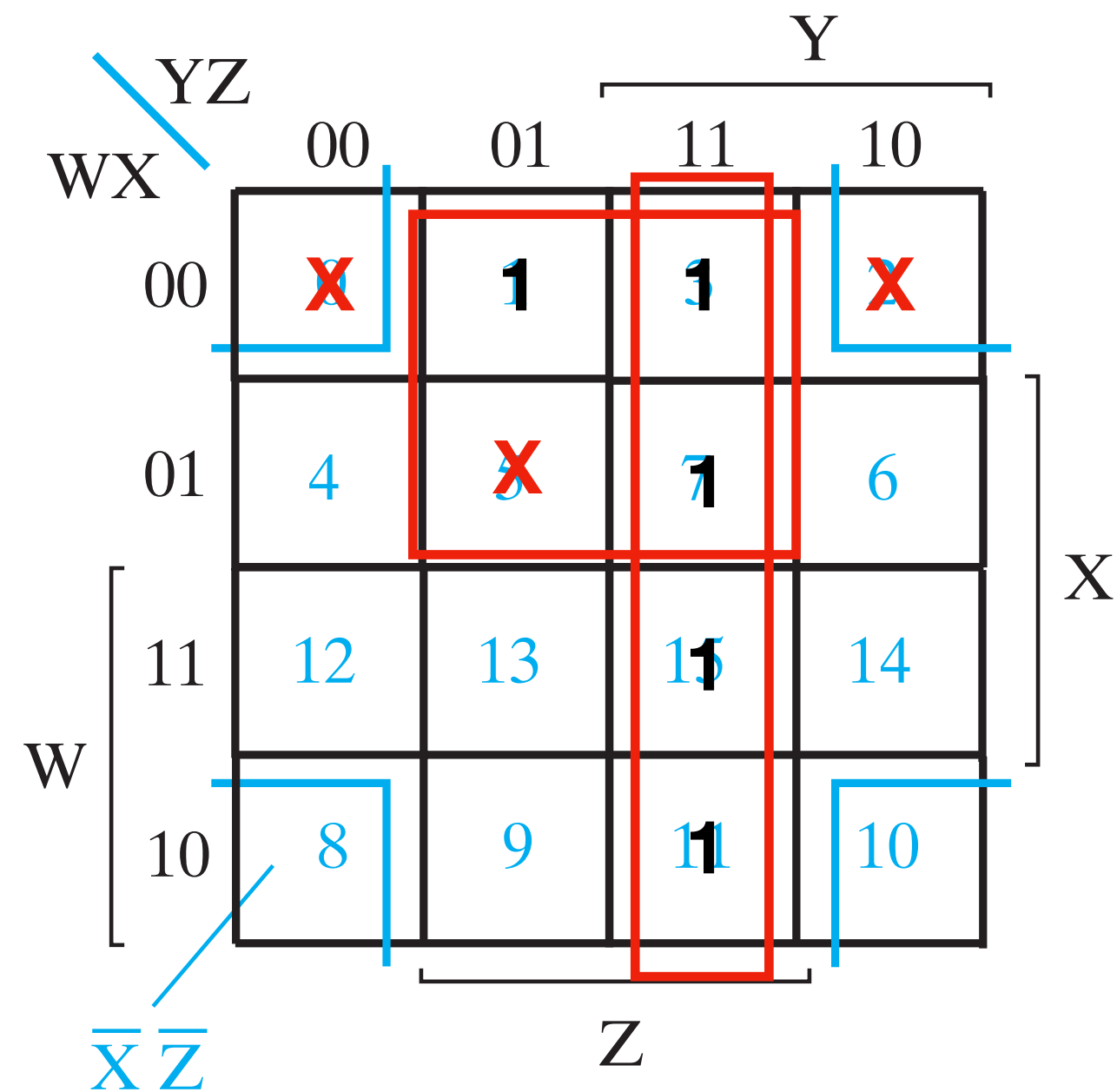
		Y			
		00	01	11	10
W	00	X	1	1	X
	01	4	X	1	6
	11	12	13	1	14
	10	8	9	1	10

YZ
WX
X
Z
 $\bar{X}\bar{Z}$



- Sometimes we don't care what the output is when the inputs are in certain combinations

Don't Care Condition

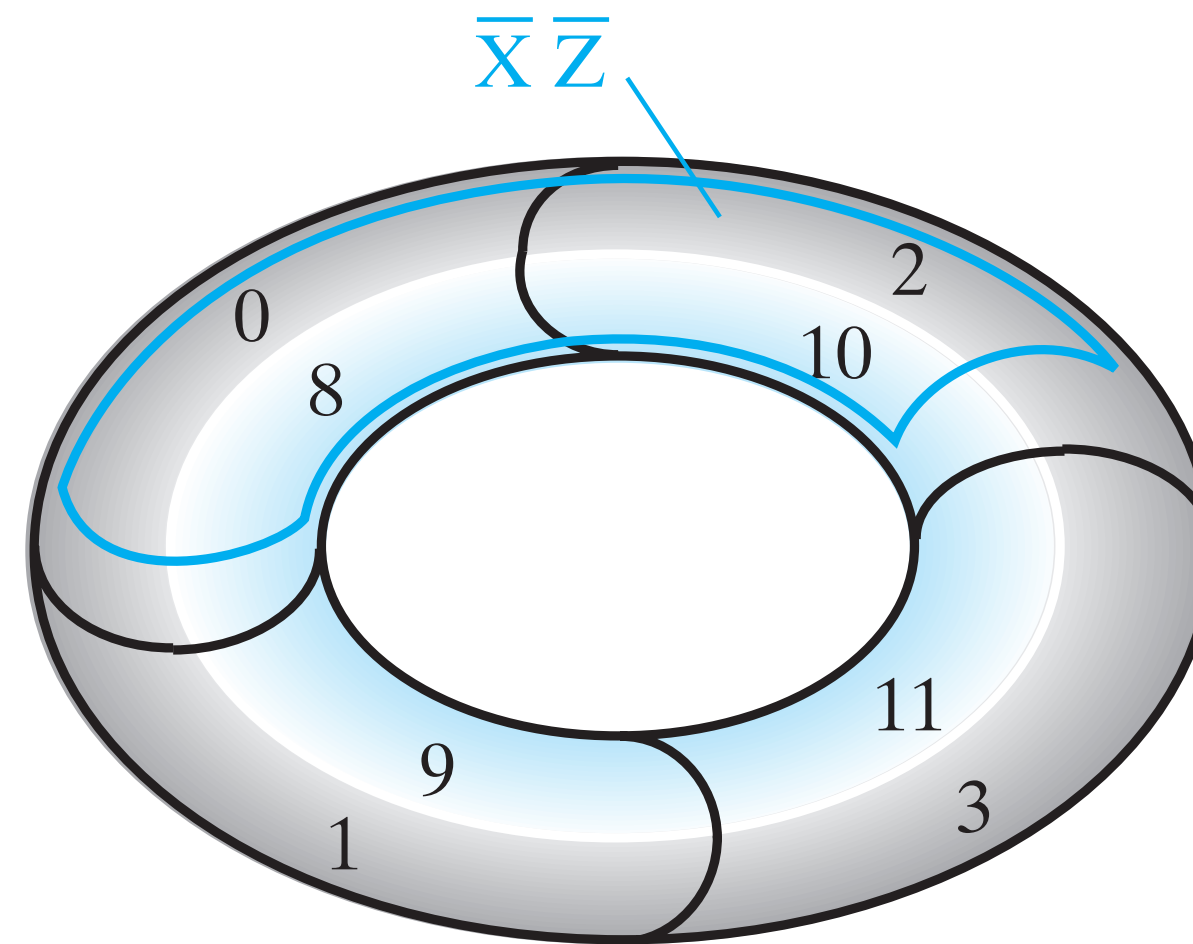


- Sometimes we don't care what the output is when the inputs are in certain combinations

Don't Care Condition

		Y			
		00	01	11	10
W	00	X	1	1	X
	01	4	X	7	6
	11	12	13	15	14
	10	8	9	11	10
		Z			

Diagram illustrating a 4x4 Karnaugh map for a function F. The map is labeled with variables W, X, Y, and Z. The top row is labeled YZ (00, 01, 11, 10) and the left column is labeled WX (00, 01, 11, 10). The map shows cells with values 1, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15. Cells with 'X' (Don't Care) are at (W=00, YZ=00), (W=00, YZ=10), (W=01, YZ=01), and (W=10, YZ=10). A red box highlights the cells (W=00, YZ=01), (W=00, YZ=11), (W=01, YZ=01), and (W=01, YZ=11). A blue box highlights the cells (W=00, YZ=00), (W=00, YZ=10), (W=10, YZ=00), and (W=10, YZ=10). The expression $\bar{X}\bar{Z}$ is shown with a blue line connecting the blue box to the text.



- Sometimes we don't care what the output is when the inputs are in certain combinations

$$F = YZ + \bar{W}Z$$

Summary

Summary

- Boolean Algebra III: K-Map

Summary

- Boolean Algebra III: K-Map
 - Two Variable K-Map

Summary

- Boolean Algebra III: K-Map
 - Two Variable K-Map
 - Three Variable K-Map

Summary

- Boolean Algebra III: K-Map
 - Two Variable K-Map
 - Three Variable K-Map
 - Four Variable K-Map

Summary

- Boolean Algebra III: K-Map
 - Two Variable K-Map
 - Three Variable K-Map
 - Four Variable K-Map
- Don't care optimisation

Exercises

		Y			
		YZ		11	10
X	0	00	01	11	10
	1	00	01	11	10
		Z			

$$F(X, Y, Z) = \Sigma m(0, 2, 6, 7)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Exercises

	YZ	00	01	11	10
X	0				
X	1				
		Z		Y	

$$F(X, Y, Z) = \Sigma m(0, 1, 2, 4)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
- Step 3: Read off the selected rectangles. If rectangle has odd length edges (excluding 1), split

Exercises

		Y			
		YZ		11	10
X	0	00	01	11	10
	1				
		Z			

$$F(X, Y, Z) = \Sigma m(0, 2, 3, 4, 6)$$

- Step 1: Enter the values
- Step 2: Identify the set of largest rectangles in which all values are 1, covering all 1s
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Exercises

		Y			
		YZ			
		00	01	11	10
X	0				
	1				
		Z			

$$F(X, Y, Z) = \Sigma m(0, 2, 3, 4, 5, 7)$$

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