CSCI 150
Introduction to Digital and Computer System Design
Lecture 2: Combinational Logical Circuits II

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Overview

- Focus: Boolean Algebra
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch2 2.2, 2.3; v5: Ch2 2.2, 2.3
- Core Ideas:
    1. Boolean Algebra I
    2. In-Class Exercises

Boolean Algebra

It’s just math

Intro. to Boolean Algebra

$$L(X_1, X_2, \ldots, X_n) = Y_1, Y_2, \ldots, Y_m$$

- **Boolean Expression**
  An algebraic expression formed by using binary variables, the constants 0 and 1, the logic operation symbols, and parentheses

- **Boolean Function**
  A Boolean equation consisting of a binary variable identifying the function, followed by an equals sign and a **Boolean Expression**

- **Single-Output / Multi-Output** Boolean Function
  Multiple Boolean function variables as input, **value 0/1 (single)** or **combinations of 0/1s (multi)** as output

Concept

Intro. to Boolean Algebra

$$L(X_1, X_2, \ldots, X_n) = Y_1, Y_2, \ldots, Y_m$$

- If a boolean function as n input variables and m output variables, how many rows will its Truth Table have?

Question

Basic Identities

1. $X + 0 =$
2. $X \cdot 1 =$
3. $X + 1 =$
4. $X \cdot 0 =$
5. $X + X =$
6. $X \cdot X =$
7. $X + \overline{X} =$
8. $X \cdot \overline{X} =$
9. $\overline{\overline{X}} =$

Concept

Basic Identities

- Communicative

10. $X + Y = Y + X$

11. $XY = YX$

- Associative

12. $X + (Y + Z) = (X + Y) + Z$

13. $X(YZ) = (XY)Z$

- Distributive

14. $X(Y + Z) = XY + XZ$

15. $X + (YZ) = (X + Y)(X + Z)$

- DeMorgan's

16. $\overline{X + Y} = \overline{X} \cdot \overline{Y}$

17. $\overline{X \cdot Y} = \overline{X} + \overline{Y}$

P1
Boolean Algebra

DeMorgan's

$$\overline{X+Y} = \overline{X} \cdot \overline{Y}$$

Truth Table

X	Y	$\overline{X+Y}$	$\overline{X} \cdot \overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

$$\overline{X \cdot Y} = \overline{X} + \overline{Y}$$

Truth Table-1

X	Y	$\overline{X \cdot Y}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Concept

Basic Identities

A. $X + XY = X$

B. $XY + X\overline{Y} = X$

C. $X + \overline{X}Y = X + Y$

D. $X(X + Y) = X$

E. $(X + Y)(X + \overline{Y}) = X$

F. $X(\overline{X} + Y) = XY$

Basic Identities

- Dual: change AND to OR; OR to AND; 0 to 1; 1 to 0
- Duality principle: a Boolean equation remains valid if we take the dual of the expressions on both sides of the equals sign.

1. $X + 0 = X$
2. $X \cdot 1 = X$
3. $X + 1 = 1$
4. $X \cdot 0 = 0$
5. $X + X = X$
6. $X \cdot X = X$

Concept

Basic Identities

- Prove
  - $\overline{X} \cdot \overline{Y} + \overline{X} \cdot Y + X \cdot Y = \overline{X} + Y$
  - $\overline{A} \cdot B + \overline{B} \cdot \overline{C} + A \cdot B + \overline{B} \cdot C = 1$
  - $Y + \overline{X} \cdot Z + X \cdot \overline{Y} = X + Y + Z$
  - $\overline{X} \cdot \overline{Y} + \overline{Y} \cdot Z + X \cdot Z = \overline{X} \cdot \overline{Y} + X \cdot Z$
    - $\overline{X}\overline{Y} + \overline{Y}Z + XZ + XY + Y\overline{Z} = \overline{X} \cdot \overline{Y} + X \cdot Z + Y \cdot \overline{Z}$

Example

P1
Boolean Algebra

Basic Identities

- $\overline{X} \cdot \overline{Y} + \overline{Y} \cdot Z + X \cdot Z = \overline{X} \cdot \overline{Y} + X \cdot Z$
  - Since $\overline{Y} \cdot Z = X \cdot \overline{Y} \cdot Z + \overline{X} \cdot \overline{Y} \cdot Z$
  - $\overline{X} \cdot \overline{Y} + \overline{Y} \cdot Z + X \cdot Z = \overline{X} \cdot \overline{Y} + X \cdot Z + X \cdot \overline{Y} \cdot Z + \overline{X} \cdot \overline{Y} \cdot Z$ **Rule B**

$= \overline{X} \cdot \overline{Y} + \overline{X} \cdot \overline{Y} \cdot Z + X \cdot Z + X \cdot \overline{Y} \cdot Z$ **Rule A x 2**

$= \overline{X} \cdot \overline{Y} + X \cdot Z$

Example

Basic Identities

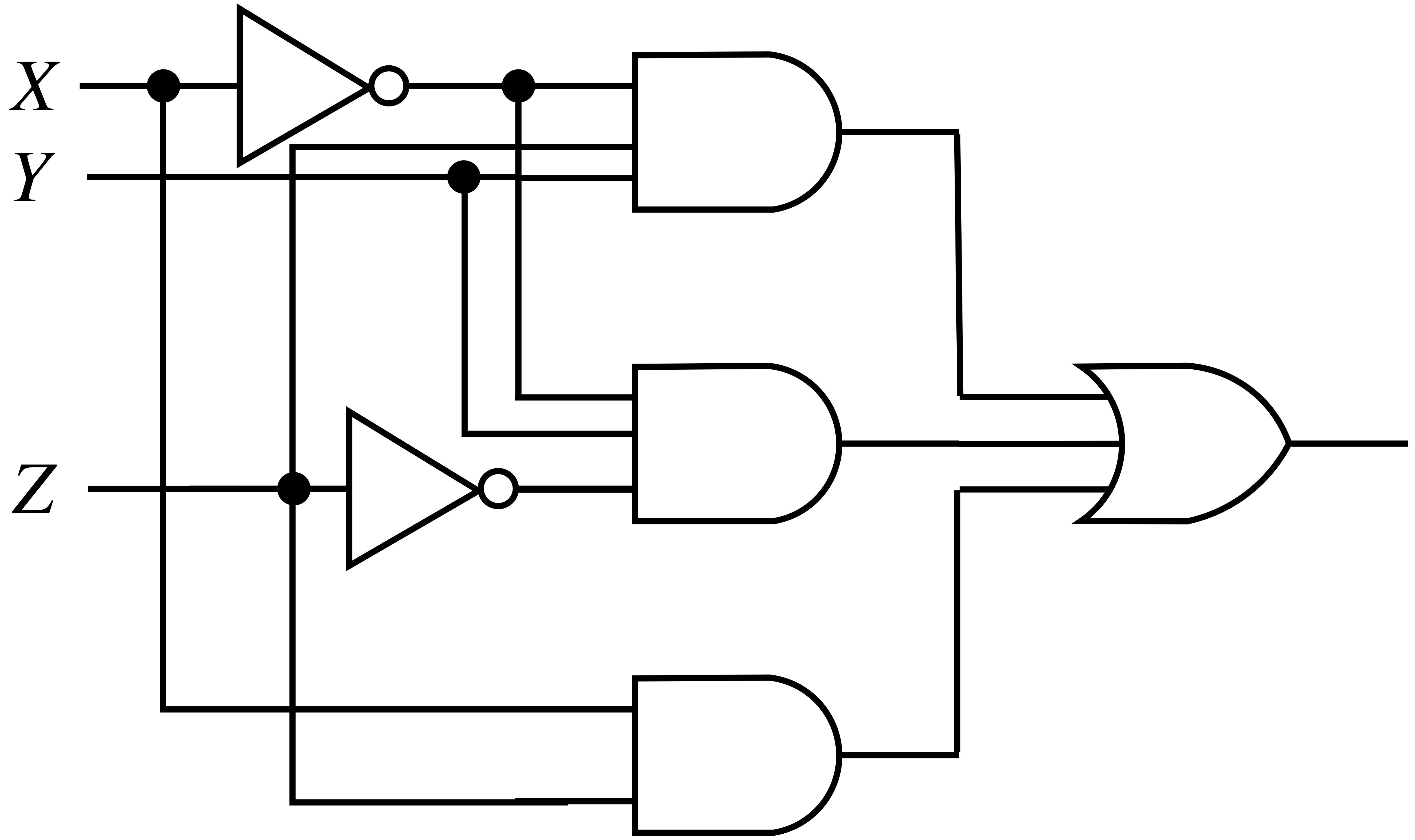
- Boolean Algebra solving
  - **Identify** rules **applicable** to the expression
  - **Apply** rules that can help you **simplify** the expression
    - **Simplification**: reducing the number of variables and operators in an expression without changing it's truth table values
    - **Atomic element:** an element that can't have the number of its variables and operators reduced any further

Concept

Algebraic Manipulation

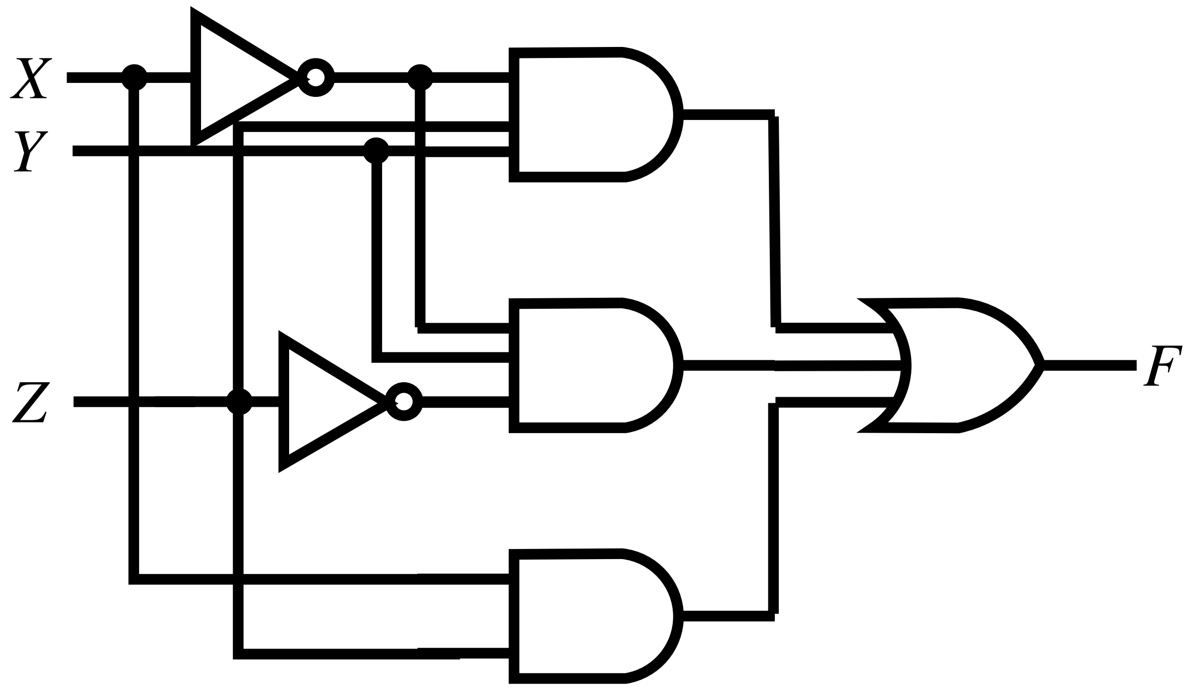
Truth Table

X	Y	Z	$F = \bar{X}YZ + \bar{X}Y\bar{Z} + XZ$
0	0	0	0
0	1	0	1
1	0	0	0
1	1	0	0
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1



Example

Algebraic Manipulation



Truth Table

X	Y	Z	$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$
0	0	0	0
0	1	0	1
1	0	0	0
1	1	0	0
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

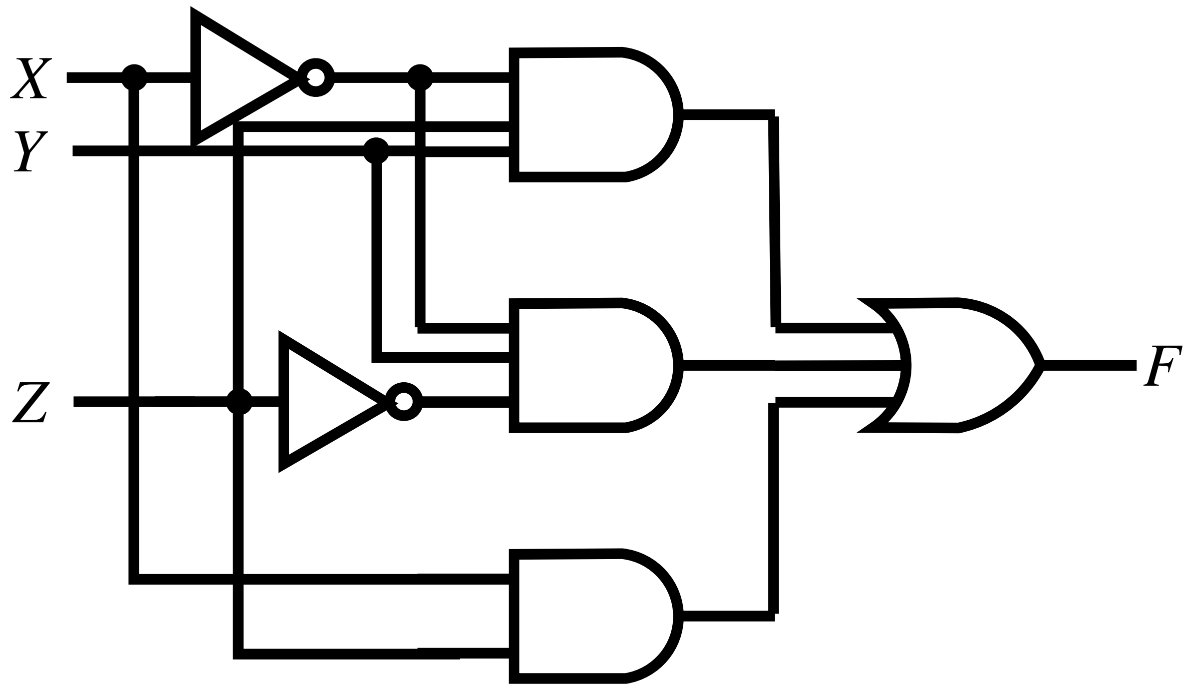
$$= \overline{X}Y(Z + \overline{Z}) + XZ$$ **Rule 14**

$$= \overline{X}Y \cdot 1 + XZ$$ **Rule 7**

$$= \overline{X}Y + XZ$$ **Rule 2**

Example

Algebraic Manipulation



Truth Table

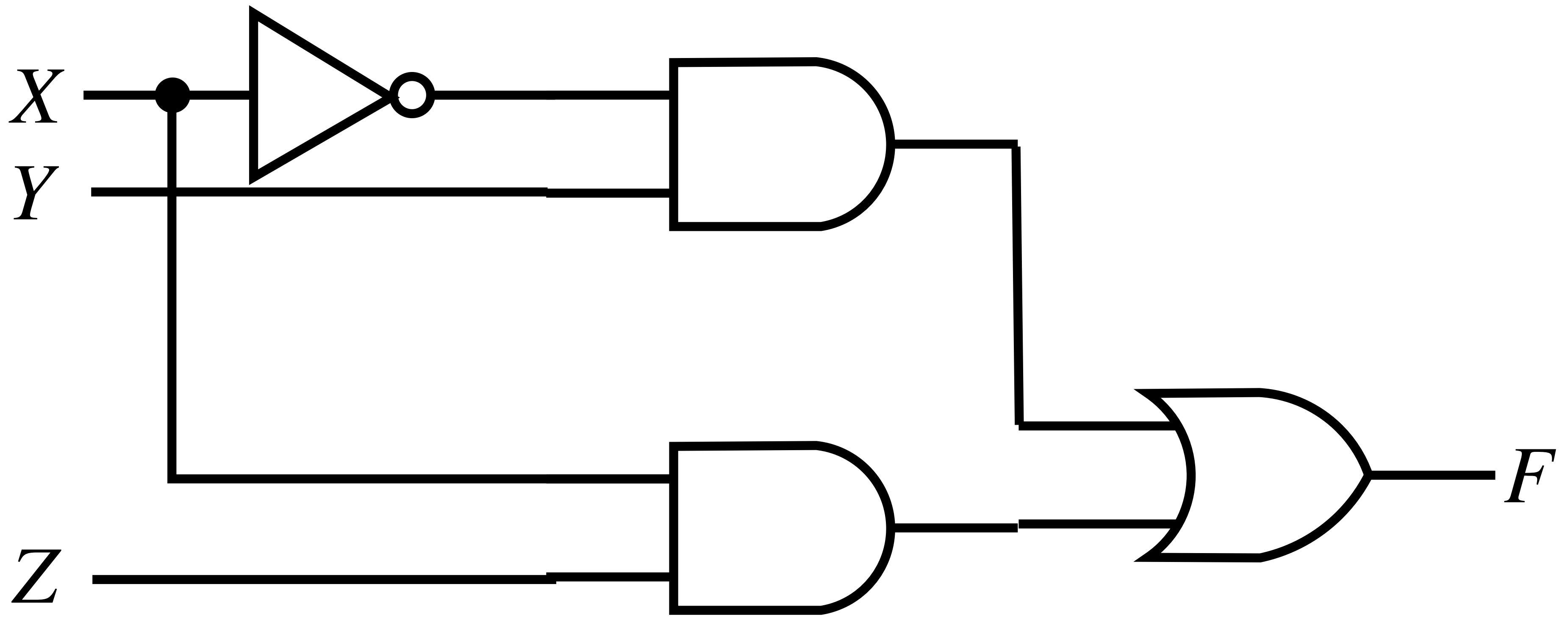
X	Y	Z	$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$
0	0	0	0
0	1	0	1
1	0	0	0
1	1	0	0
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

$$= \overline{X}Y(Z + \overline{Z}) + XZ \quad \textbf{Rule 14}$$

$$= \overline{X}Y \cdot 1 + XZ \quad \textbf{Rule 7}$$

$$= \overline{X}Y + XZ \quad \textbf{Rule 2}$$



Example

Algebraic Manipulation

- Algebraic Manipulation can help reduce the number of gates in a circuit
  - easier to implement and debug
  - more efficient

Concept

Complementation

- $\overline{F}$: complement (invert) representation for a function F, obtained from an interchange of 1s to 0s and 0s to 1s for the values of F in the truth table

- Apply DeMorgan's Rule

16. $\overline{X_1 + X_2 + \ldots + X_n} = \overline{X_1} \cdot \overline{X_2} \cdot \ldots \cdot \overline{X_n}$

17. $\overline{X_1 \cdot X_2 \cdot \ldots \cdot X_n} = \overline{X_1} + \overline{X_2} + \ldots + \overline{X_n}$

Concept

Complementation

- $F_1 = \overline{X}Y\overline{Z} + \overline{X}\overline{Y}Z$
- $F_2 = X(\overline{Y}\overline{Z} + YZ)$

Example

Algebra Solving 1

- Known $BC + D = 1; \overline{C} = 1$
  - Calculate: $\overline{D}(\overline{B} + \overline{C})$
  - Calculate: $(D + B)(D + C)$

P1
Boolean Algebra

Algebra Solving 2

- Known $AB + C = 1; A + \overline{D} = 0$
  - Calculate: $C + AB$
  - Calculate: $\overline{C}\overline{A} + \overline{C}\overline{B}$

Algebra Solving 3

- Known $A + B + \overline{C} = 1; \overline{A}\,\overline{C} + \overline{A}D = 1$
  - Calculate: $(\overline{C} + B)(\overline{C} + D)$
  - Calculate: $\overline{C} + BD$

P2
Exercise

Boolean Algebra

Exercises! Use #04-2020-1000-401 to help you!

Summary

P2
Exercise

Boolean Algebra

Difficulty: Simple

Prove by truth table that

- $\overline{X}Y + \overline{Y}Z + X\overline{Z} = X\overline{Y} + Y\overline{Z} + \overline{X}Z$

Exercise

P2
Exercise

Algebraic Manipulation

Difficulty: Simple

Use DeMorgen's Rules to transform the following expression to one WITHOUT AND operator

- $\overline{ABC} + CD$
- $A\overline{B}C + \overline{A} \cdot \overline{C} + AB$

Exercise

P2
Exercise

Logic Diagram

Difficulty: Simple

Draw the logic diagram for the following expression (you don't have to perform any manipulation/transformation)

- $XYZ + \overline{X}\overline{Y} + \overline{X}\overline{Z}$
- $B(\overline{A} \cdot \overline{C} + AC) + \overline{B}(A + \overline{B}C)$

Exercise

P2
Exercise

Algebraic Manipulation

Difficulty: Simple

Simplify the following expressions

- $\overline{X} \cdot \overline{Y} + XYZ + \overline{X}Y$
- $X + Y(Z + \overline{X + Z})$

Exercise

P2
Exercise

Algebraic Manipulation

Difficulty: Mid

Simplify the following expressions

- $\overline{W}X(\overline{Z} + \overline{Y}Z) + X(W + \overline{W}YZ)$
- $(AB + \overline{A}\overline{B})(\overline{C}\overline{D} + CD) + AC$

Exercise

P2
Exercise

Algebraic Manipulation

Difficulty: Mid

Simplify the following expressions

- $\overline{A} \cdot \overline{C} + \overline{A}BC + \overline{B}C$
- $\overline{A + B + C} \cdot \overline{ABC}$

Exercise

P2
Exercise

Algebraic Manipulation

Difficulty: Mid

Simplify the following expressions

- $AB\overline{C} + AC$
- $\overline{A} \cdot \overline{B}D + \overline{A} \cdot \overline{C}D + BD$

Exercise

P2
Exercise

Algebraic Manipulation

Difficulty: HARDCORE

Given that $AB = 0$ and $A + B = 1$, prove that

- $(A + C)(\overline{A} + B)(B + C) = BC$

Exercise

P2 Exercise

Algebraic Manipulation

Difficulty: HARDCORE

Prove the identity of each of the following Boolean equations

- $AB\overline{C} + B\overline{C} \cdot \overline{D} + BC + \overline{C}D = B + \overline{C}D$
- $WY + \overline{W}Y\overline{Z} + WXZ + \overline{W}X\overline{Y} = WY + \overline{W}X\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}Z$
- $A\overline{D} + \overline{A}B + \overline{C}D + \overline{B}C = (\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + C + D)$

Exercise