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CSCI 150
Introduction to Digital and Computer System Design
Lecture 1: Digital Information Representations II

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2020 Winter Semester (S1)

Overview

- Focus: Number Systems
- Architecture: Digital Circuits
- Textbook v4: Ch1 1.3, 1.4; v5: Ch1 1.4, 1.5
- Core Ideas:
    1. Arithmetic Operations, Ranges
    2. Digital & Analog Conversion

P0
Review

Digital and Analog Circuits

- In Digital Circuits, information are represented by discrete values, usually 0/1s defined by HIGH/LOW voltages ranges
- In Analog Circuits, information representation is continuous
- Von Neumann computers: all-purpose design
  - CPU (control unit and functional/datapath unit), IO, memory
- Embedded systems: specific functions only, compact and robust

Review

P0
Review

Number Systems

- Base N number systems
- Binary, Octal, Hexadecimal systems
- Basic conversions
- Bit, Byte, Representation ranges

Review

Arithmetic Operations

Additions, Subtractions, Conversions

Summary

Arithmetics

- The same as decimal (mostly)
-

$$\begin{array}{r} 0010 \\ +0011 \\ \hline 0101 \end{array} \qquad \begin{array}{r} 0101 \\ -0011 \\ \hline 0010 \end{array}$$

Example (binary)

Concept

Arithmetics

Binary additions and subtractions

Carries	11010
Augend	01101
Addend	+00101
Sum	10010

Borrows	00110
Minuend	10110
Subtrahend	−10011
Difference	00011

Demo

Arithmetics

OCTAL Multiplication

$$
\begin{array}{r}
762 \\
\times \quad 54 \\
\hline
4672 \\
3710 \\
\hline
43772
\end{array}
$$

$$5 \times 2 = 12$$

$$5 \times 6 + 1 = 37$$

$$5 \times 7 + 3 = 46$$

$$\cdots$$

Demo

Arithmetics

OCTAL Multiplication

Octal

$$
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762 \\
\times \quad 54 \\
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Octal

$$5 \times 2 = 12$$

$$5 \times 6 + 1 = 37$$

$$5 \times 7 + 3 = 46$$

$$\ldots$$

Decimal

$$10 = (12)_8$$

$$31 = (37)_8$$

$$38 = (46)_8$$

$$\ldots$$

Demo

Arithmetics

Convert 41 to binary

$$41 \div 2 = 20 \mod 1$$
$$20 \div 2 = 10 \mod 0$$
$$10 \div 2 = 5 \mod 0$$
$$5 \div 2 = 2 \mod 1$$
$$2 \div 2 = 1 \mod 0$$
$$1 \div 2 = 0 \mod 1$$

Least significant digit

Most significant digit

$41 = (1\,0\,1\,0\,0\,1)$

Demo

Arithmetics

Convert 0.6875 to binary

$$0.6875 \times 2 = 1 + 0.3750$$

$$0.3750 \times 2 = 0 + 0.7500$$

$$0.7500 \times 2 = 1 + 0.5000$$

$$0.5000 \times 2 = 1 + 0.0000$$

Most significant digit

↓

Least significant digit

0.6875= (1 0 1 1)

Demo

Signed Integers

- Integers in digital circuits have limited bits
  - e.g. 8bit for every number
- How do we represent negative values in digital circuits?

$$\begin{array}{r} 00000101 \\ -\,00001011 \\ \hline 0000????\end{array}$$

(binary, 8bit, unsigned)

Concept

P1
Arithmetics

Signed & Unsigned Integers

- Unsigned 8bit:
  - $(11111111)_2 = 255$
- Signed 8bit (only in digital circuits):
  - 127 -> '01111111'
  - -127 -> '11111111'

First digit:
- **0 for positive**
- **1 for negative**

1**0**0001111

(binary, 8bit, signed)

Concept

P1
Arithmetics

Signed & Unsigned Integers

- Unsigned 8bit integer: 0 - 255
  - Signed 8bit integer: -128 - 127
- Unsigned 32bit integer: 0 - 4,294,967,295
  - Signed 32bit integer: -2,147,483,648 - 2,147,483,647
- Unless otherwise specified, treat as unsigned

Concept

Summary

- Additions, Subtractions, Multiplications in Binary, Octal, Hexadecimal systems

Review

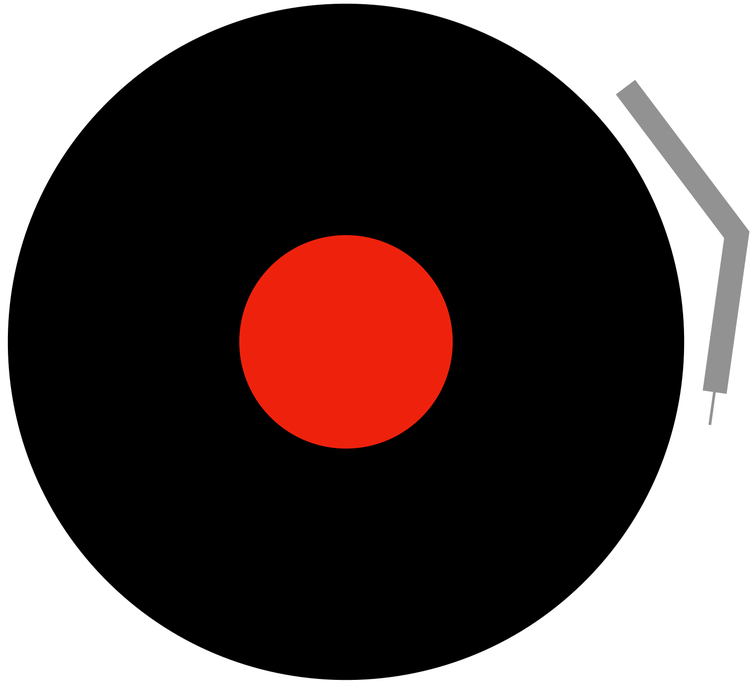
Digital & Analog Conversions

Demonstration

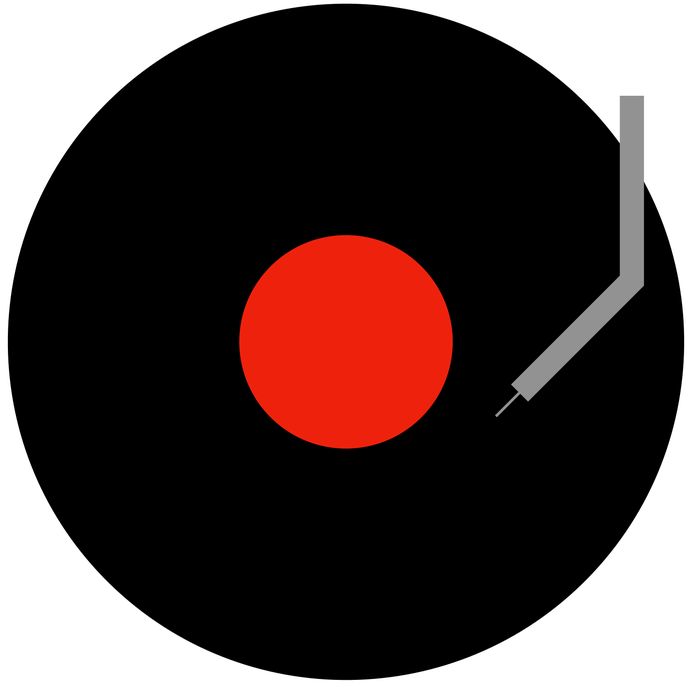
WARNING: inbound signed Binary/Hexadecimal operations

Summary

Case Study: Audio Signal Representation

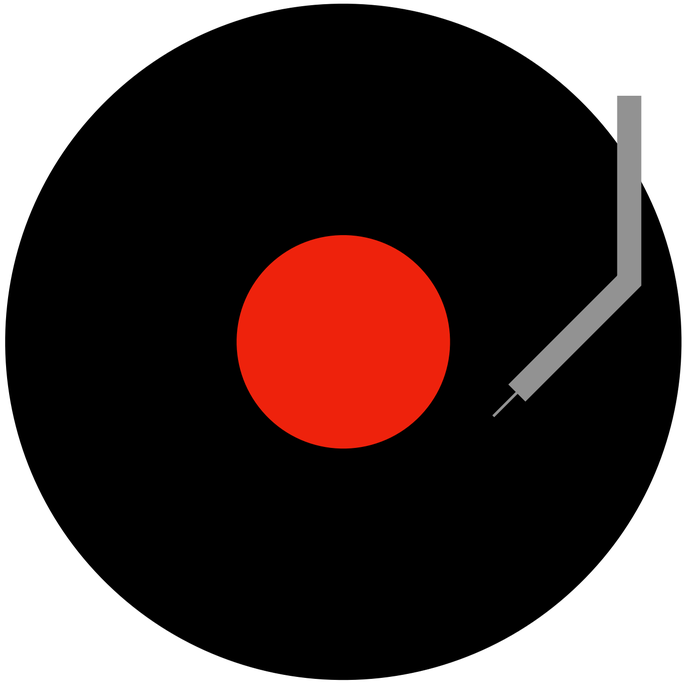


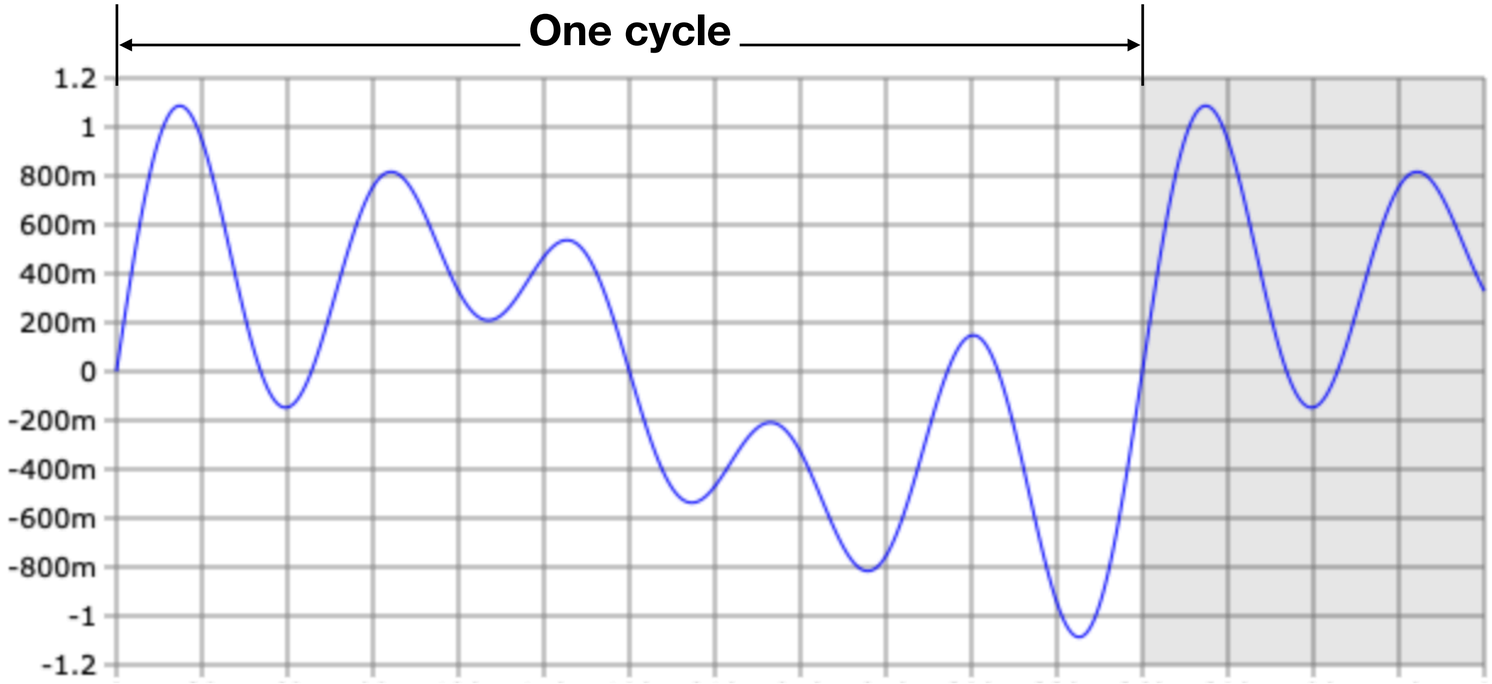
Case Study: Audio Signal Representation



P2
DA Conversion

Case Study: Audio Signal Representation

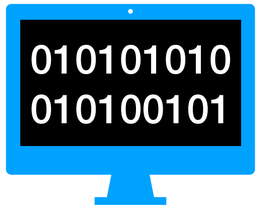


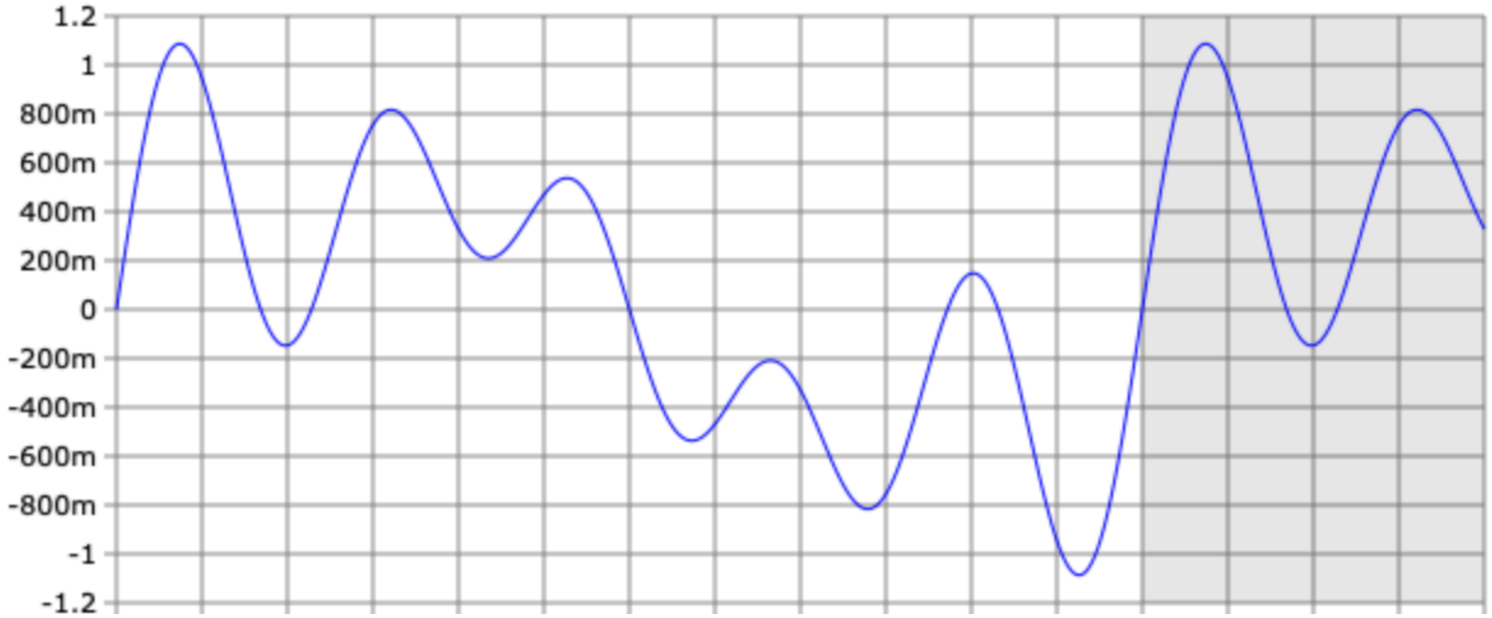


440 Hz = 440 cycles per sec

Technical

Case Study: Audio Signal Representation





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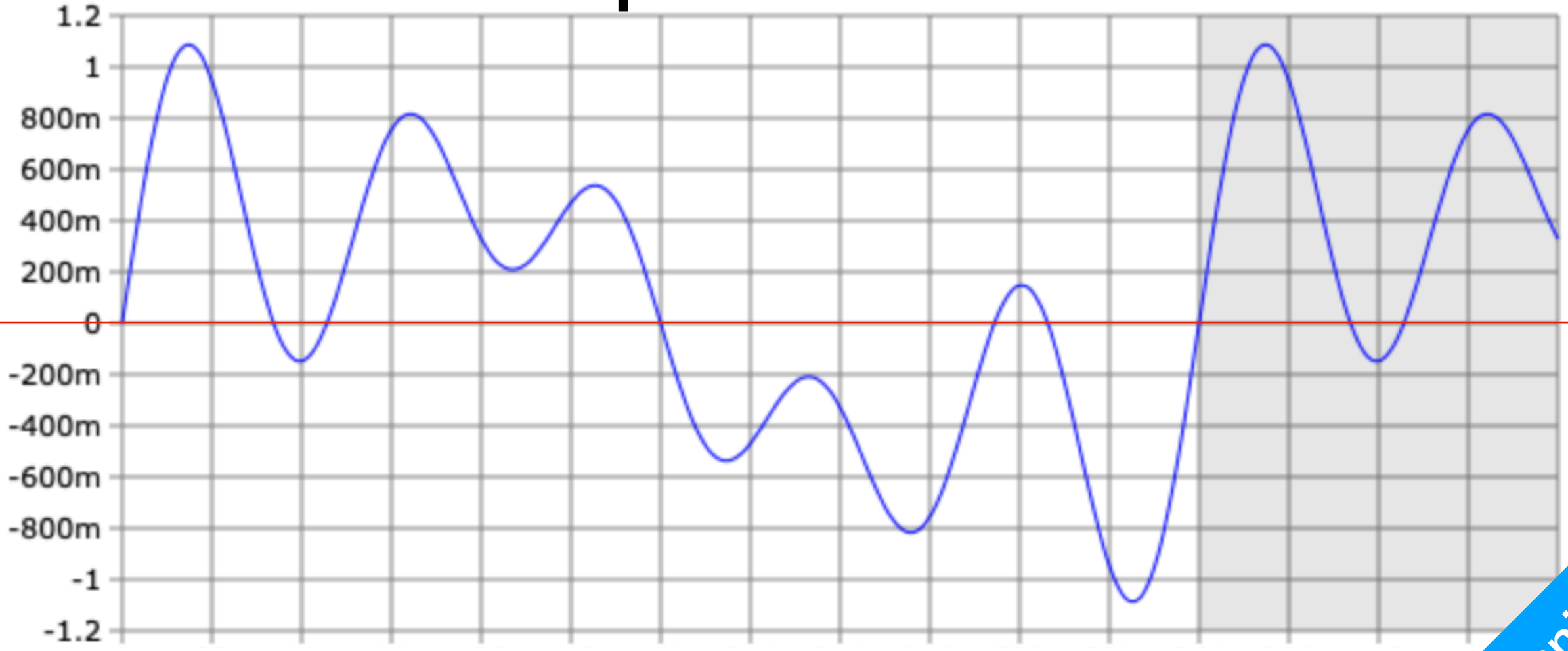
P2
DA Conversion

Case Study: Audio Signal Representation

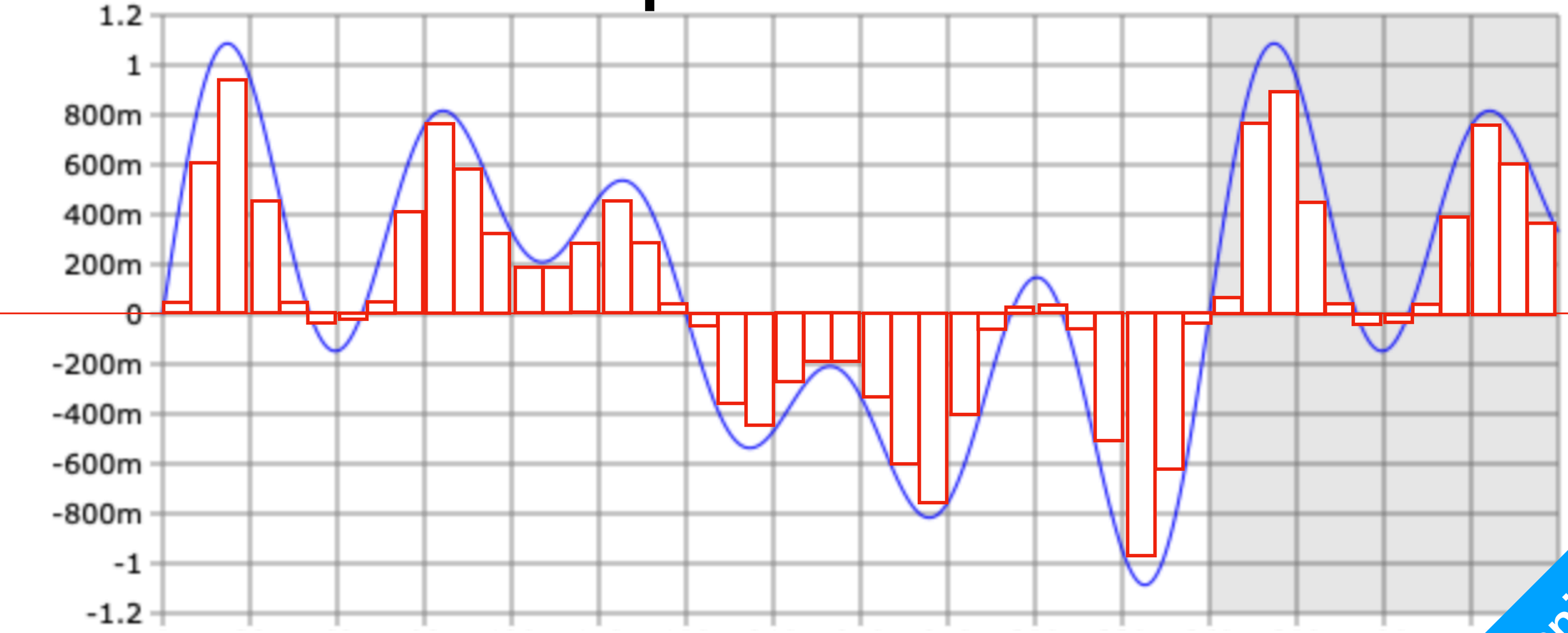


440 Hz = 440 cycles per sec, Sample rate 36 x 440 = 15,840 = 16kps

Technical

P2
DA Conversion

Case Study: Audio Signal Representation



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Technical

P2
DA Conversion

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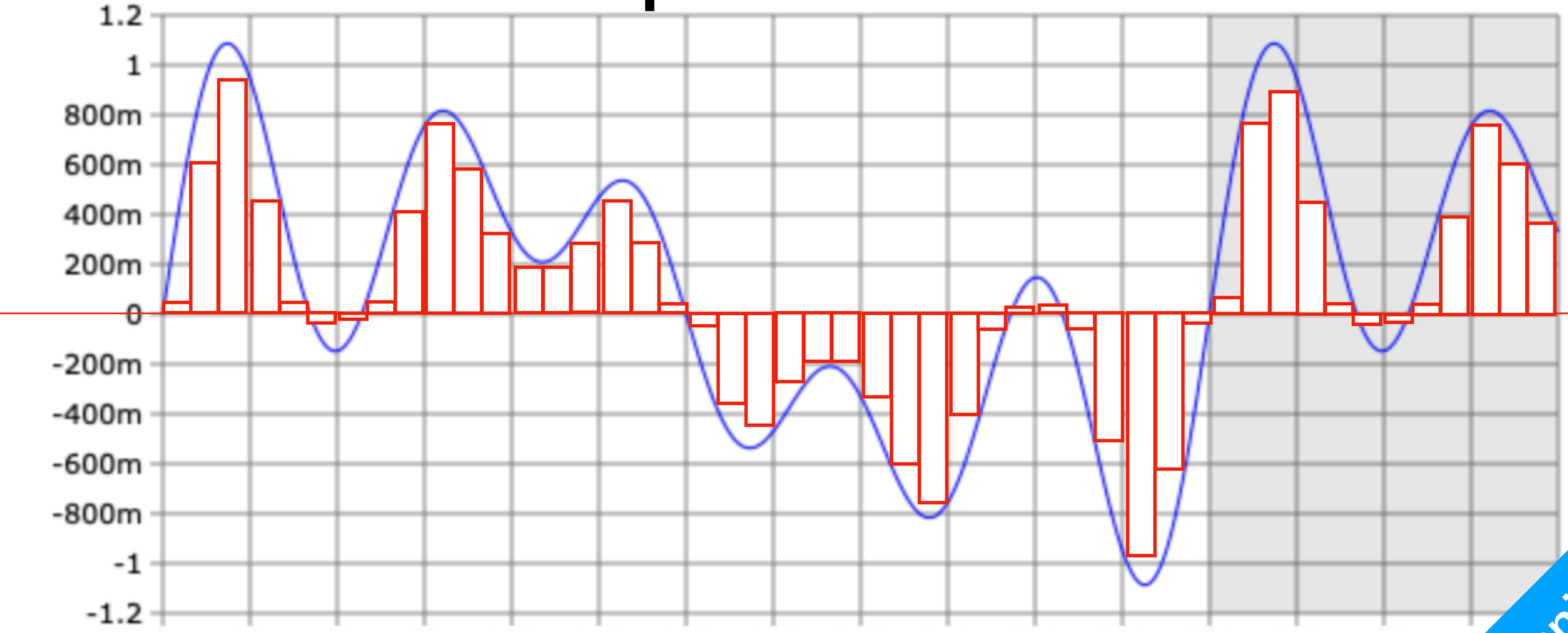

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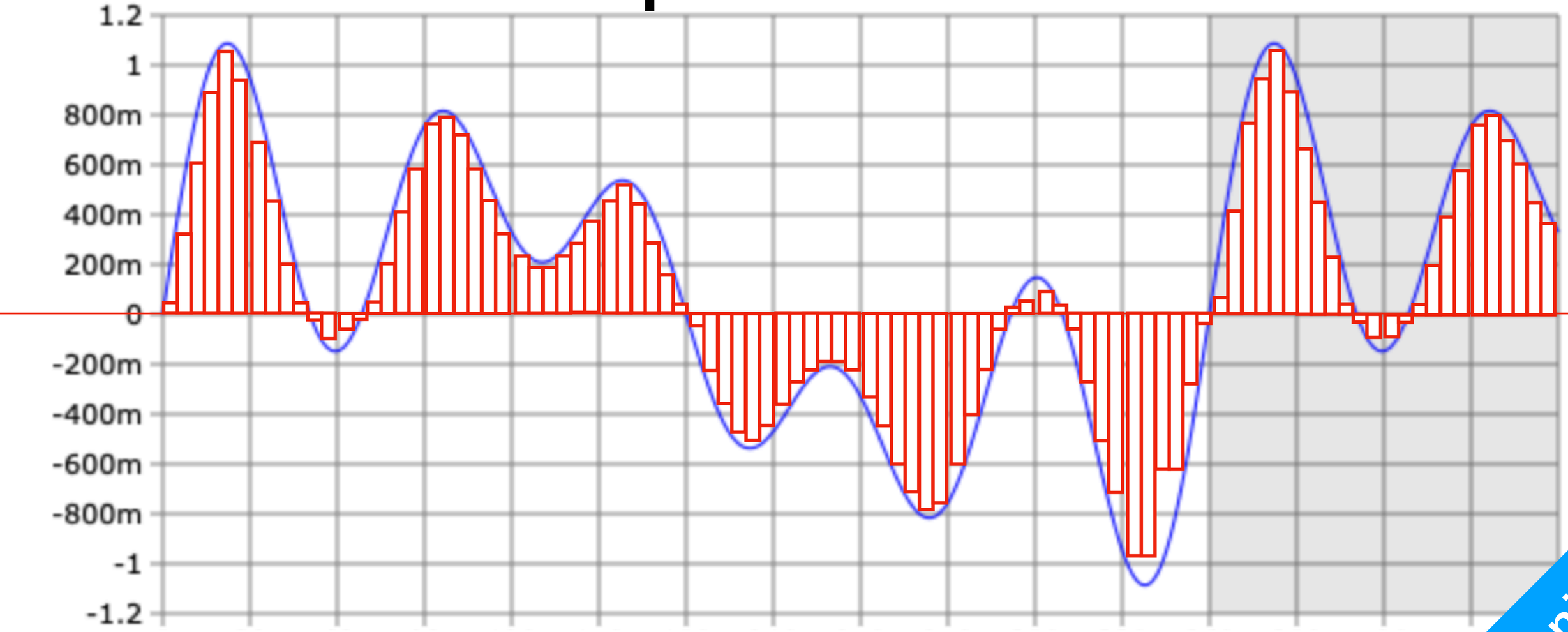
Technical

P2
DA Conversion

Case Study: Audio Signal Representation

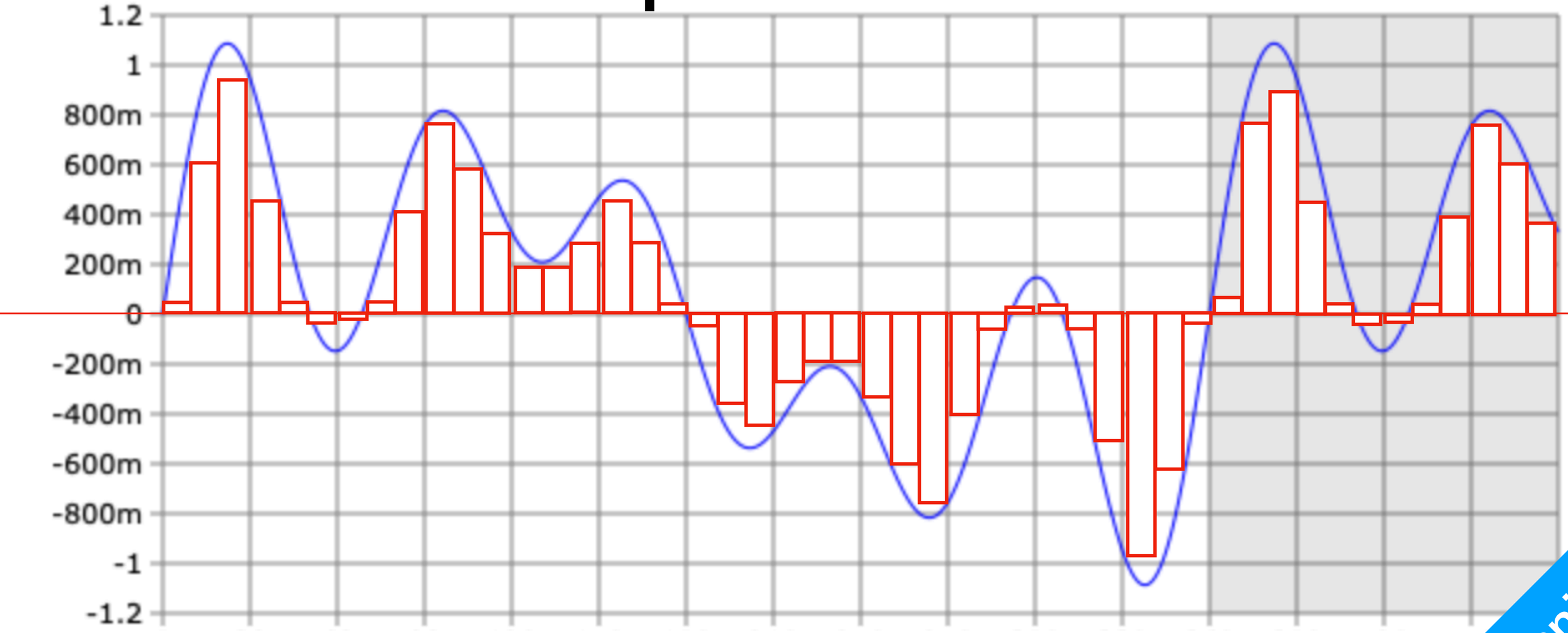


440 Hz = 440 cycles per sec, Sample rate 64 x 440 = 31,680 = 32kps

Technical

P2
DA Conversion

Case Study: Audio Signal Representation



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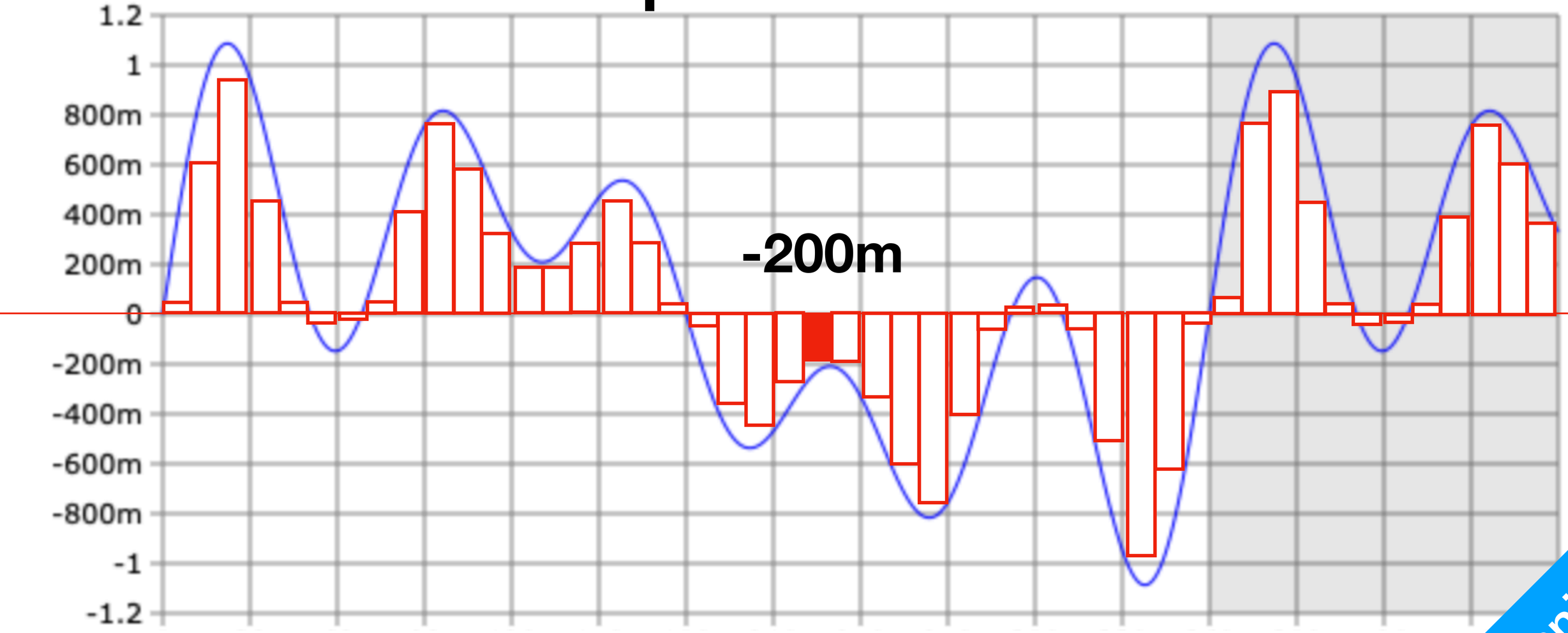
Technical

P2
DA Conversion

Case Study: Audio Signal Representation



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Technical

P2
DA Conversion

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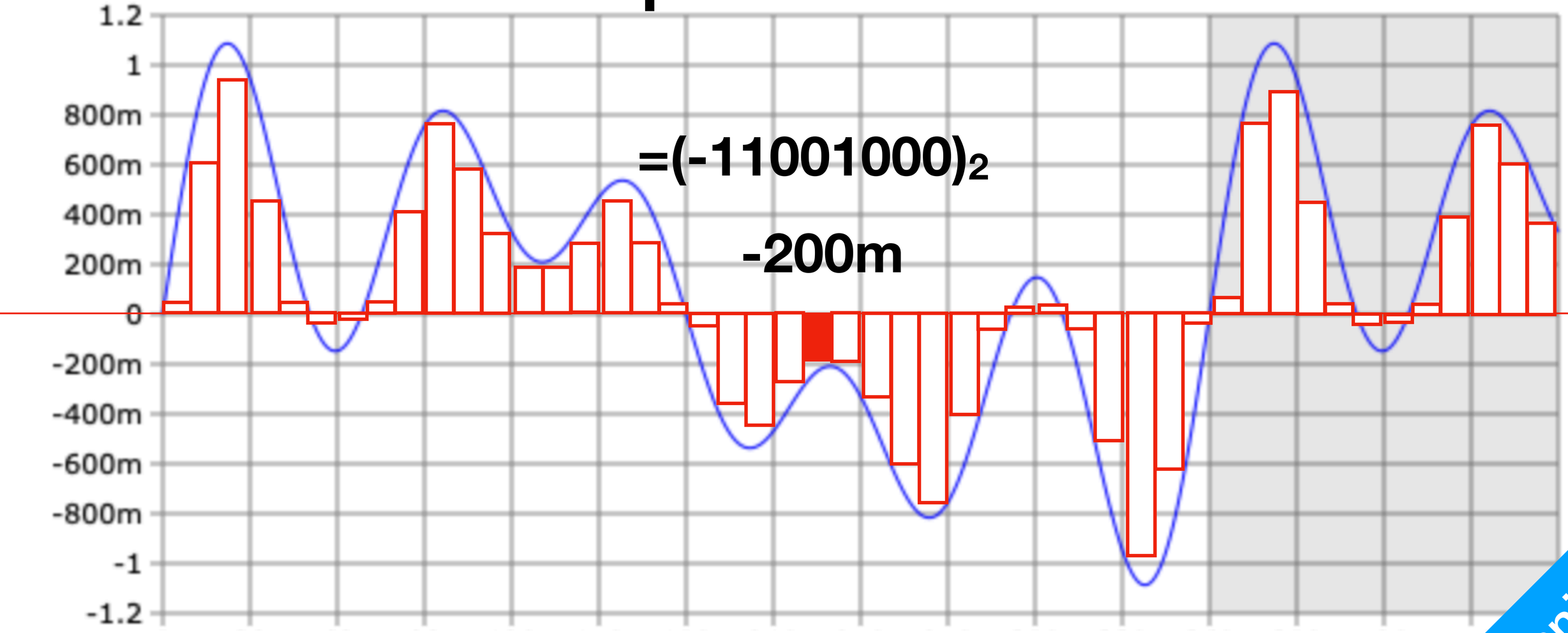

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Technical

P2
DA Conversion

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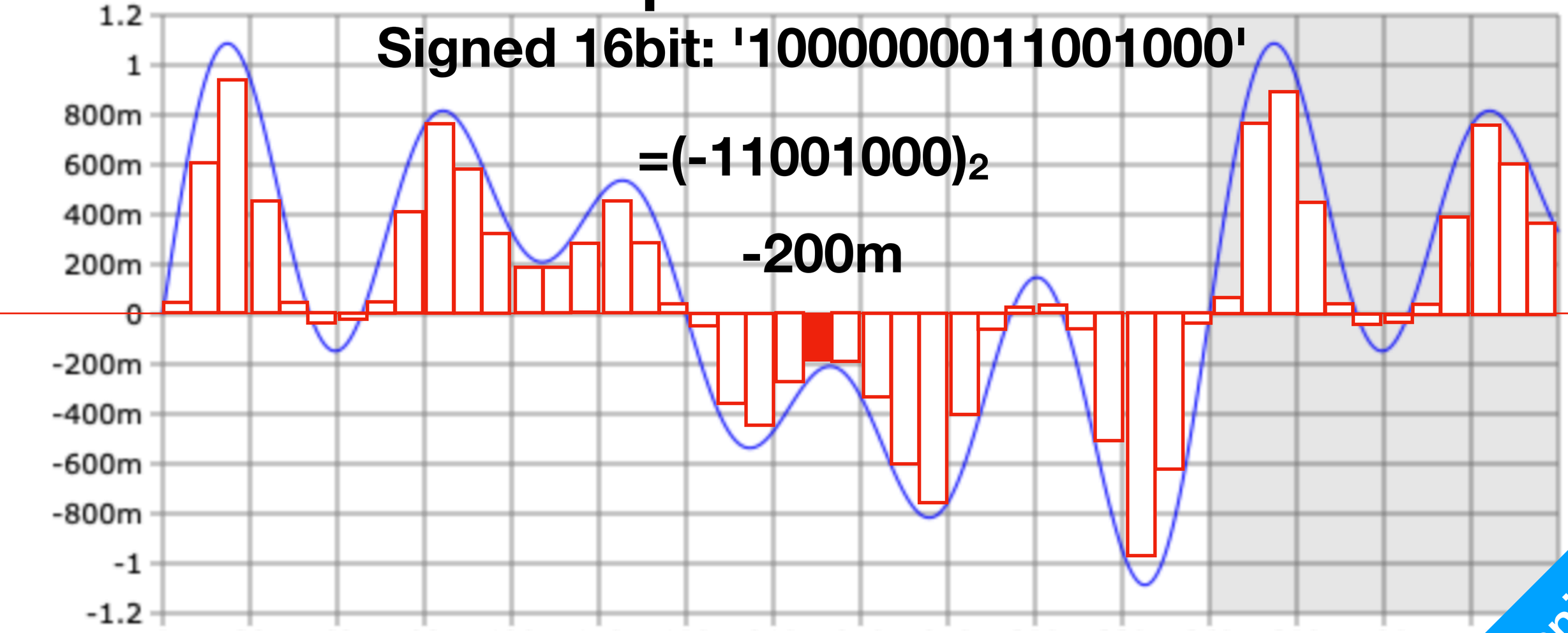

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Technical

P2
DA Conversion

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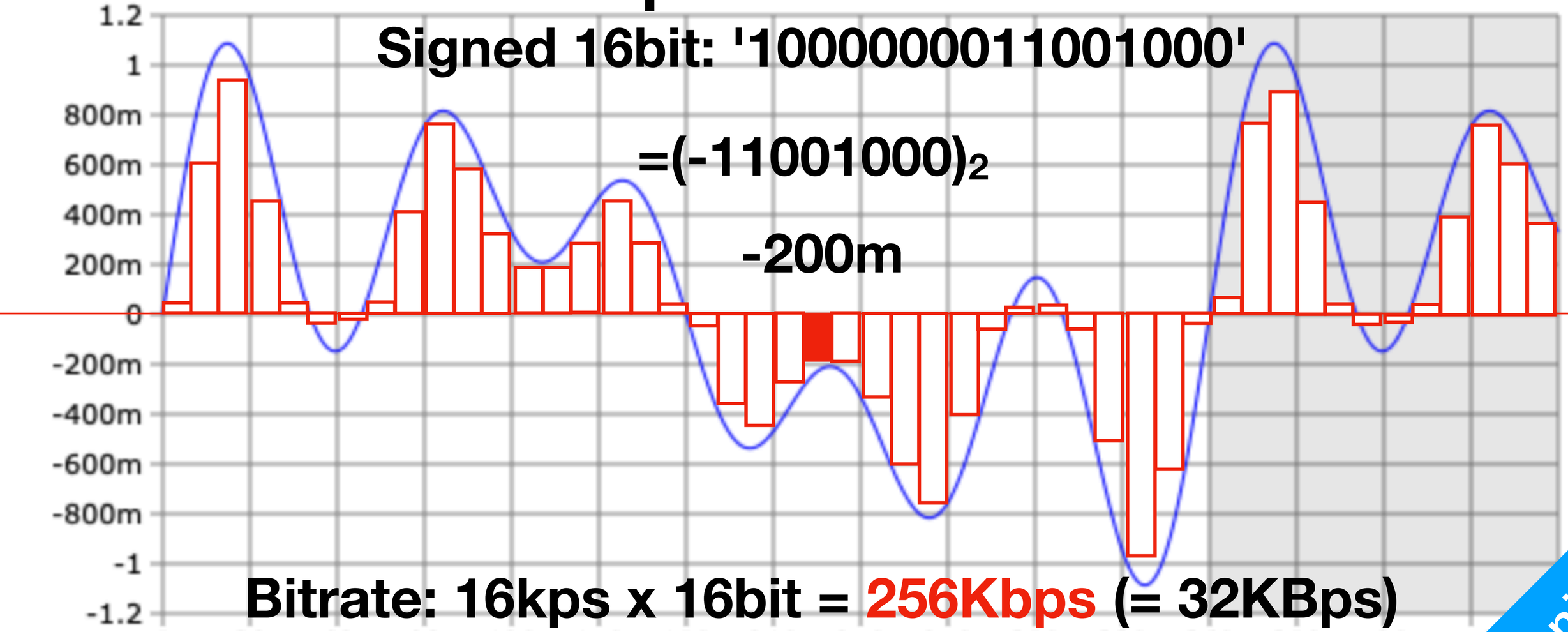

Bitrate: 16kps x 16bit = 256Kbps (= 32KBps)

440 Hz = 440 cycles per sec, Sample rate 36 x 440 = 15,840 = 16kps

Technical

Case Study: Audio Signal Representation

- Standard Mp3 audio file
  - Sampling rate: 44,100
  - Bitrate: 256 kBit/s (Kbps)
  - 4 min audio file size? $> 4 \times 60 \times 256 \div 8 = 7680\text{KB}$

Summary

- Digital to Analog Conversion
  - Frequency: number of cycles per second
  - Sample rate: number of samples per unit time
  - Bitrate: number of bits per second

Review