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CSCI 150
Introduction to Digital and Computer System Design
Lecture 4: Sequential Circuit III

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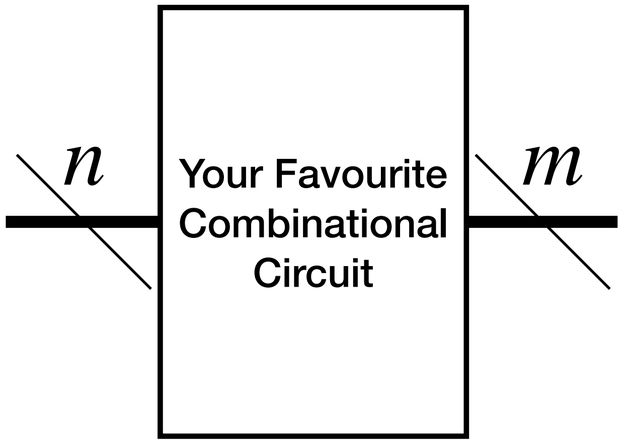
Overview

- Focus: Basic Information Retaining Blocks
- Architecture: Sequential Circuit
- Textbook v4: Ch5 5.3, 5.4; v5: Ch4 4.2 4.3
- Core Ideas:
    1. Latches and Flip-Flops (with Direct Input)
    2. Sequential Circuit Analysis

P0
Review

Combinational Logic Circuit Design

- Design Principles
  - Knows: fixed-Length input and output
  - Knows: input/output mapping relations
  - Optimisation: Minimise overall delay

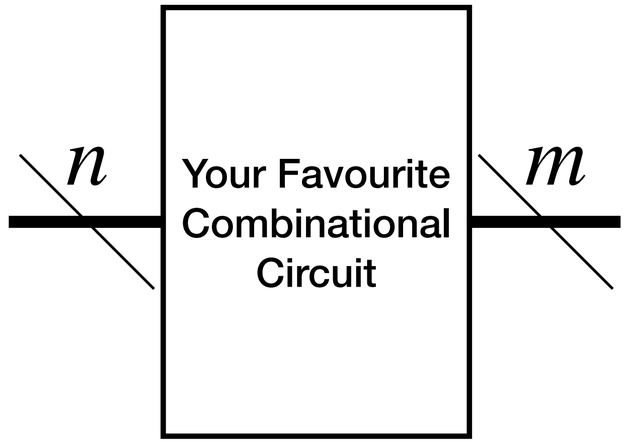


Review

P0
Review

Combinational Logic Circuit Design

- Cannot handle variable length input
- Cannot store information
- Cannot perform multi-step tasks



Review

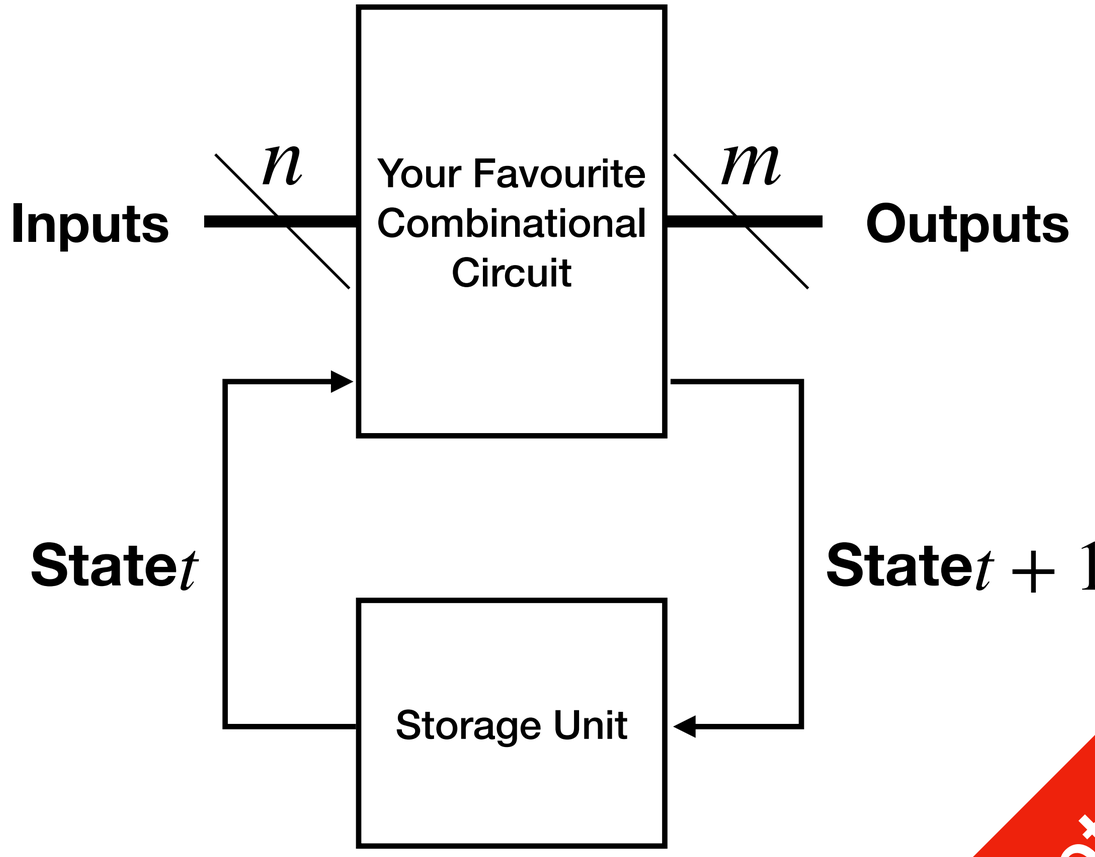
Definitions

1. **Storage Elements**
   circuits that can store binary information

2. **State**
   partial results, instructions, etc.

3. **Synchronous Sequential Circuit**
   Signals arrive at discrete instants of time, outputs at next time step

4. **Asynchronous Sequential Circuit**
   Signals arrive at any instant of time, outputs when ready



Concept

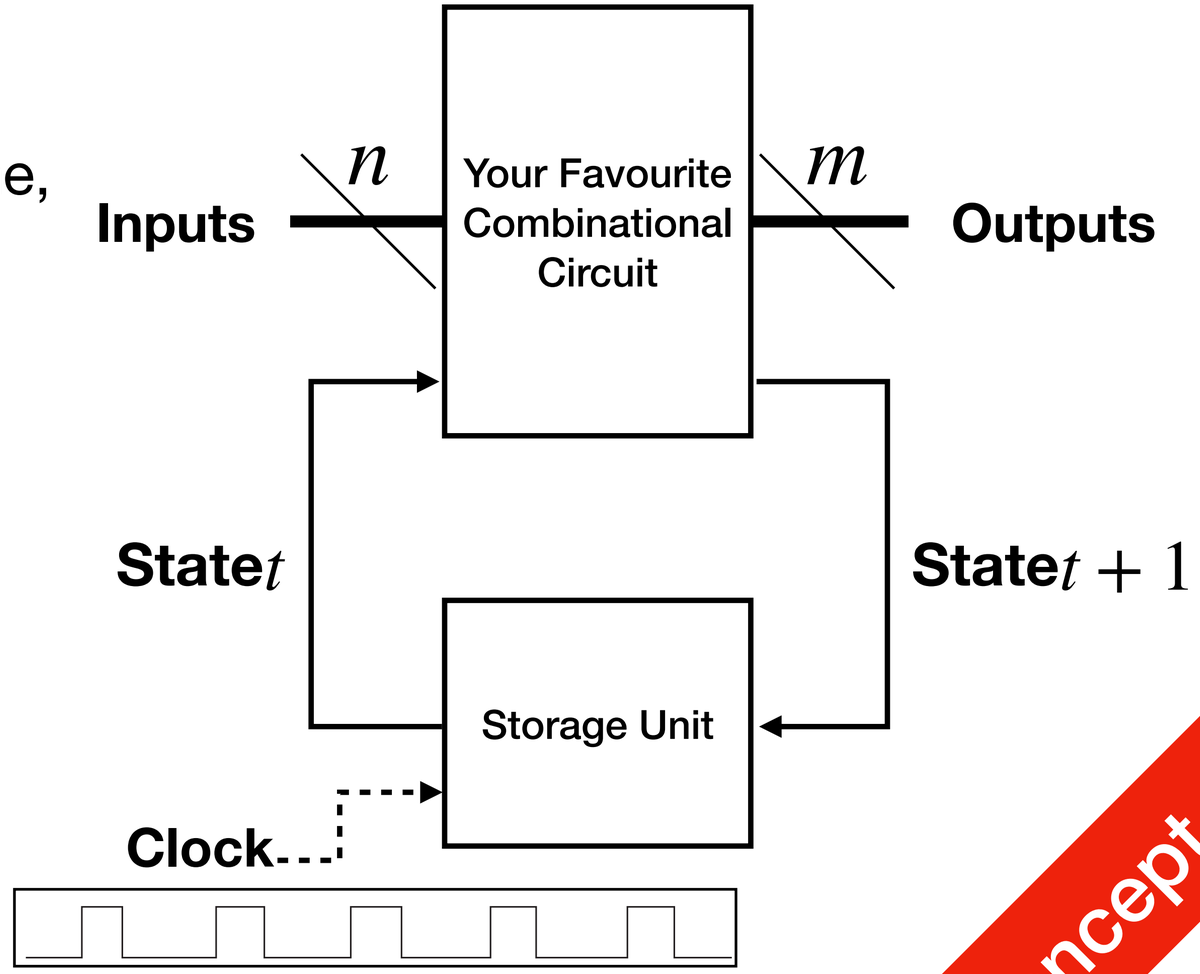
Definitions

3. **Synchronous Sequential Circuit**
   Signals arrive at discrete instants of time, outputs at next time step

   - Has Clock

4. **Asynchronous Sequential Circuit**
   Signals arrive at any instant of time, outputs when ready

   - May not have Clock



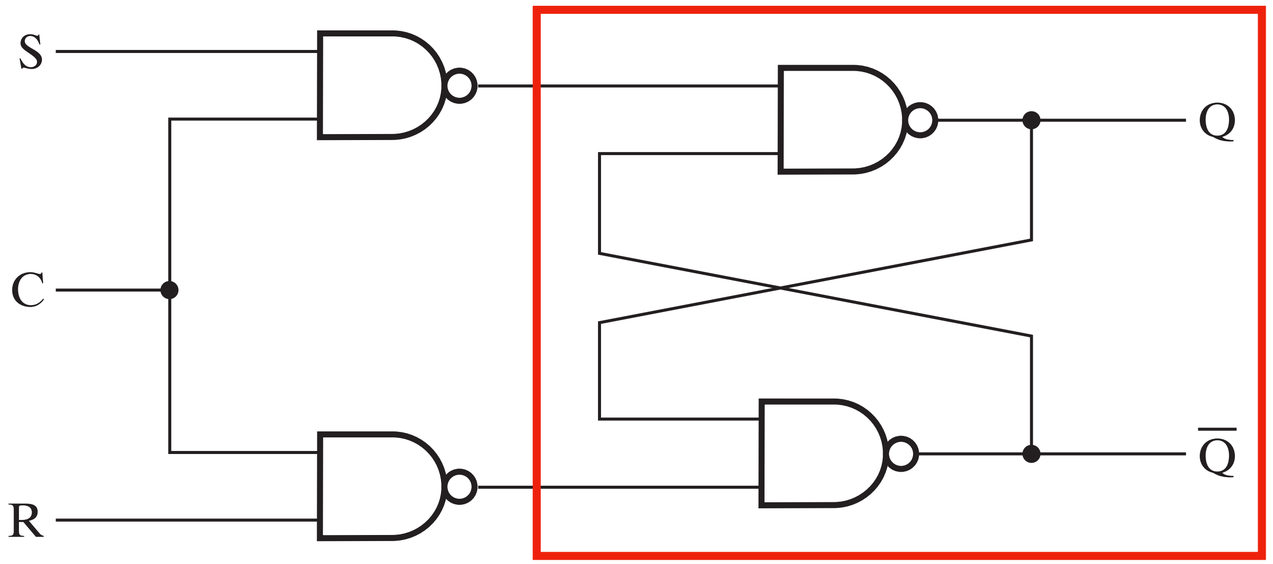
Concept

Review:
Latches and Flip-Flops

SR Latches, D Latches, D Flip-Flops

Summary

SR Latch with Control Input

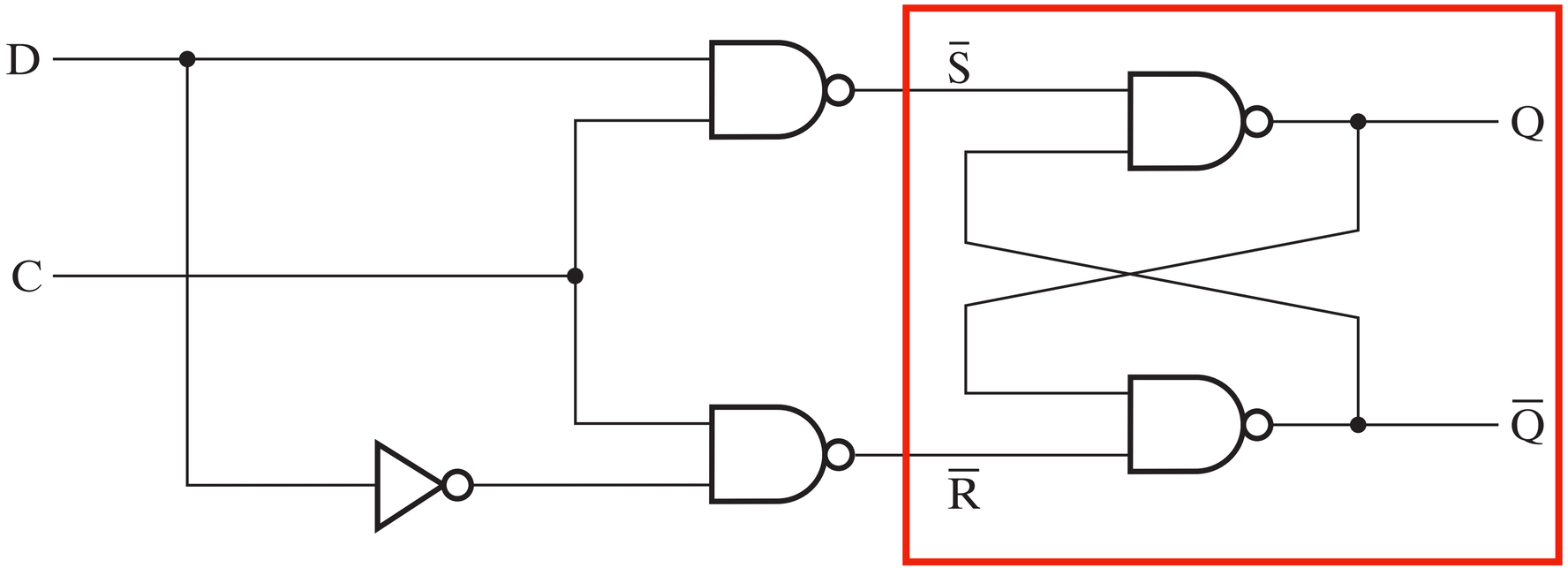


C	S	R	Next state of Q
0	X	X	No change
1	0	0	No change
1	0	1	Q = 0; Reset state
1	1	0	Q = 1; Set state
1	1	1	Undefined

- Implemented using $\overline{SR}$ latches
- C acts as an enabler; otherwise the entire circuit functions as an SR latch

Review

D Latch



C	D	Next state of Q
0	X	No change
1	0	Q = 0; Reset state
1	1	Q = 1; Set state

- Implemented using $\overline{SR}$ latches
- C: Signals changes to the stored states; D the value to change to

Review

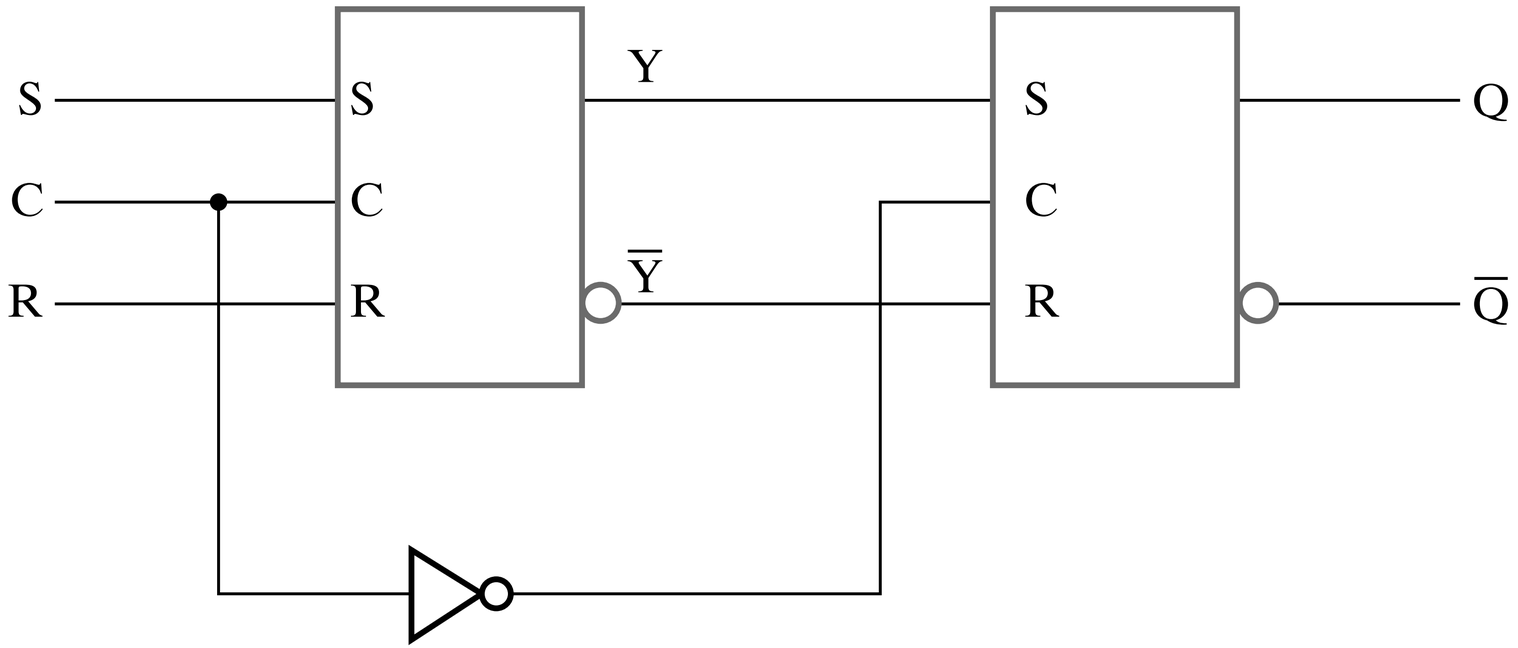
Flip-Flops

- Latches are **Transparent**
  input can be seen from outputs while control pulse is 1

- Flip-Flops are not **Transparent**
  Output state changes require changes of control signal

Review

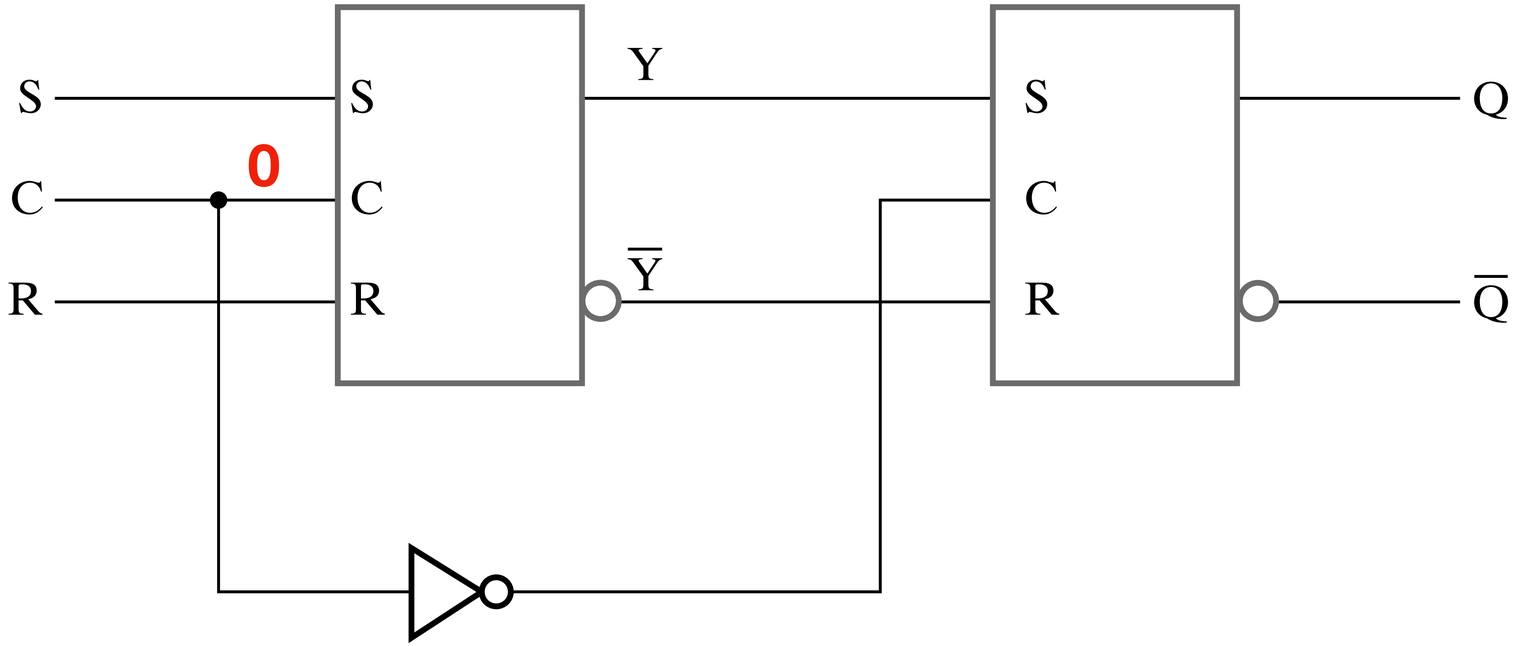
SR Master-Slave Flip-Flop



- Constructed using SR latches, left Master, right Slave
- Output state changes require $C = 0$ -> $C = 1$ -> $C = 0$ (Positive Pulse)

Review

SR Master-Slave Flip-Flop



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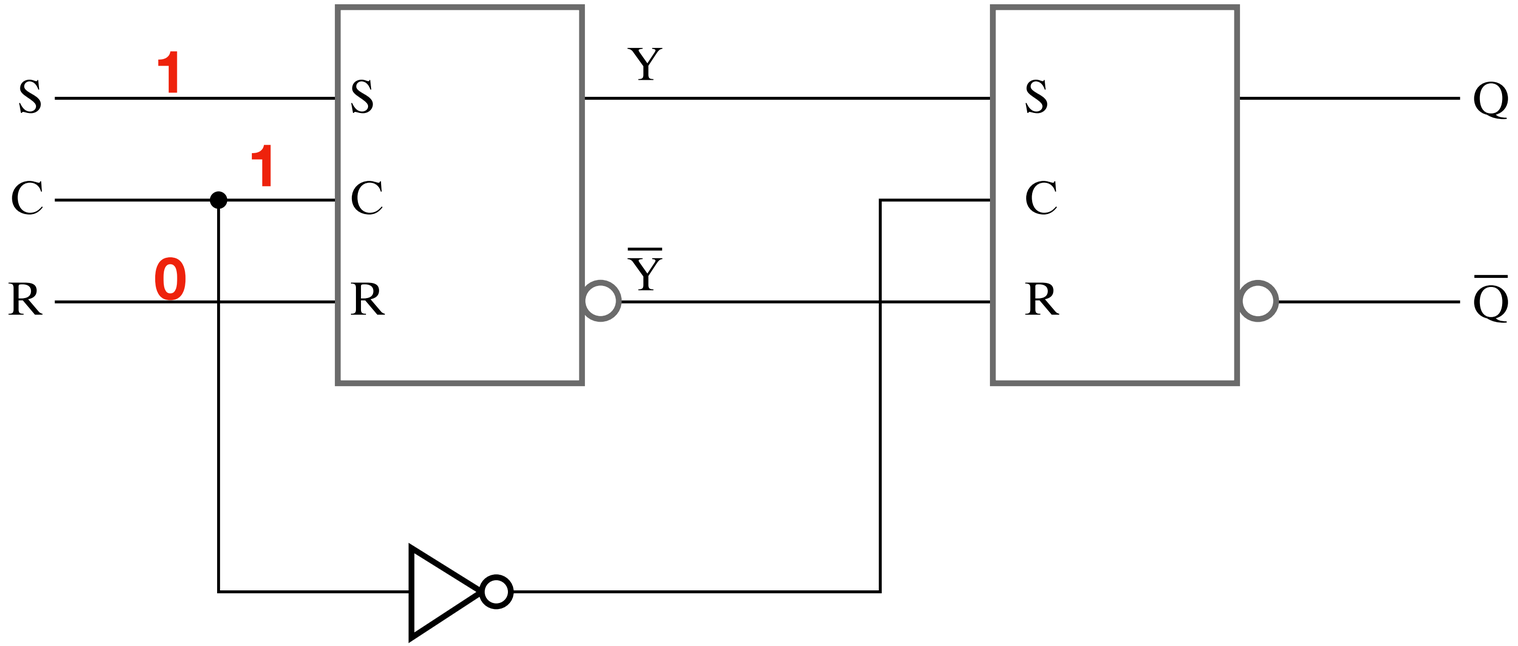

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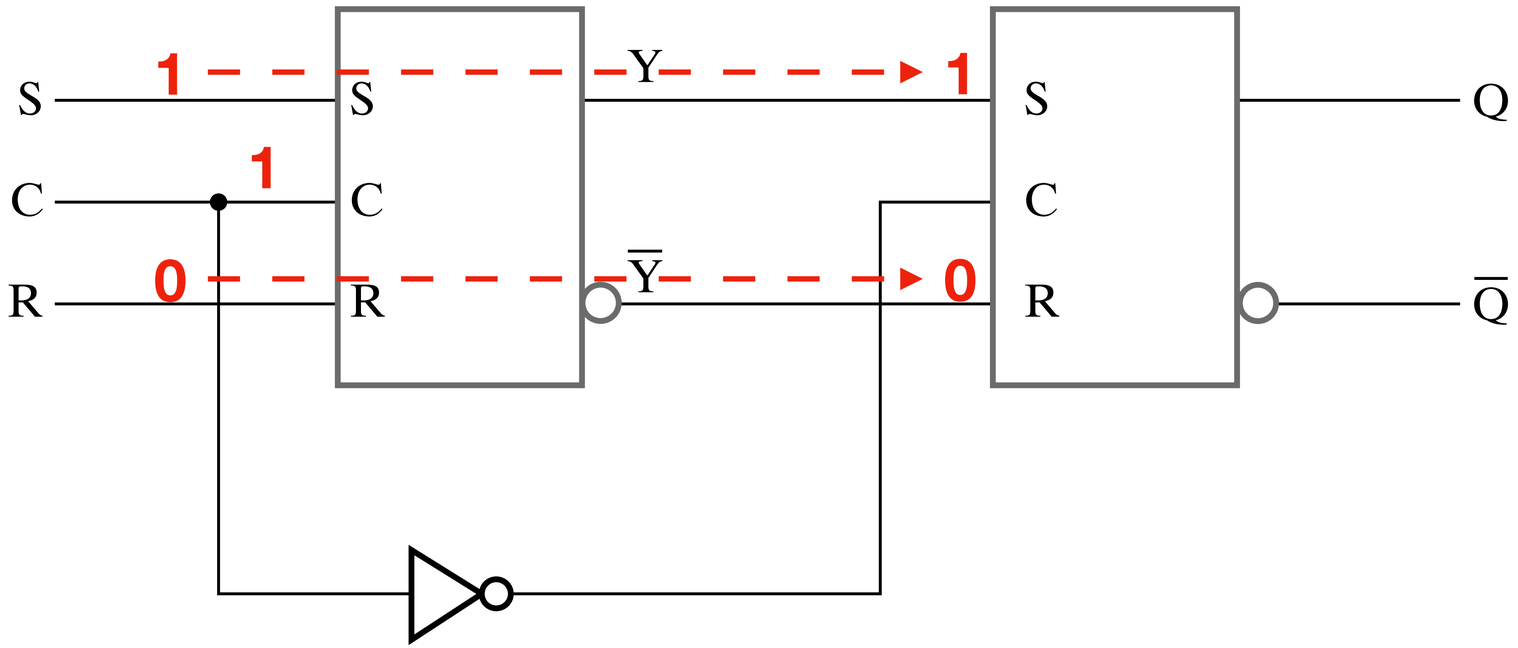

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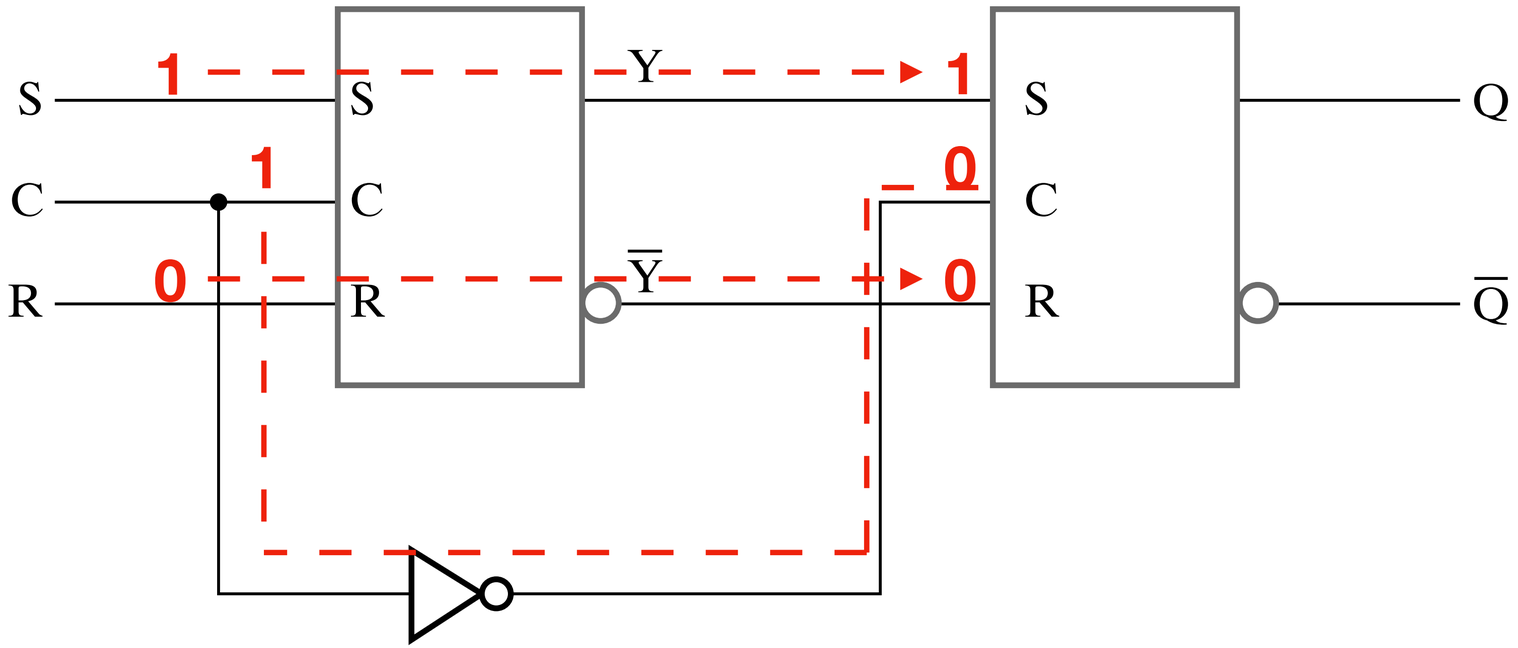

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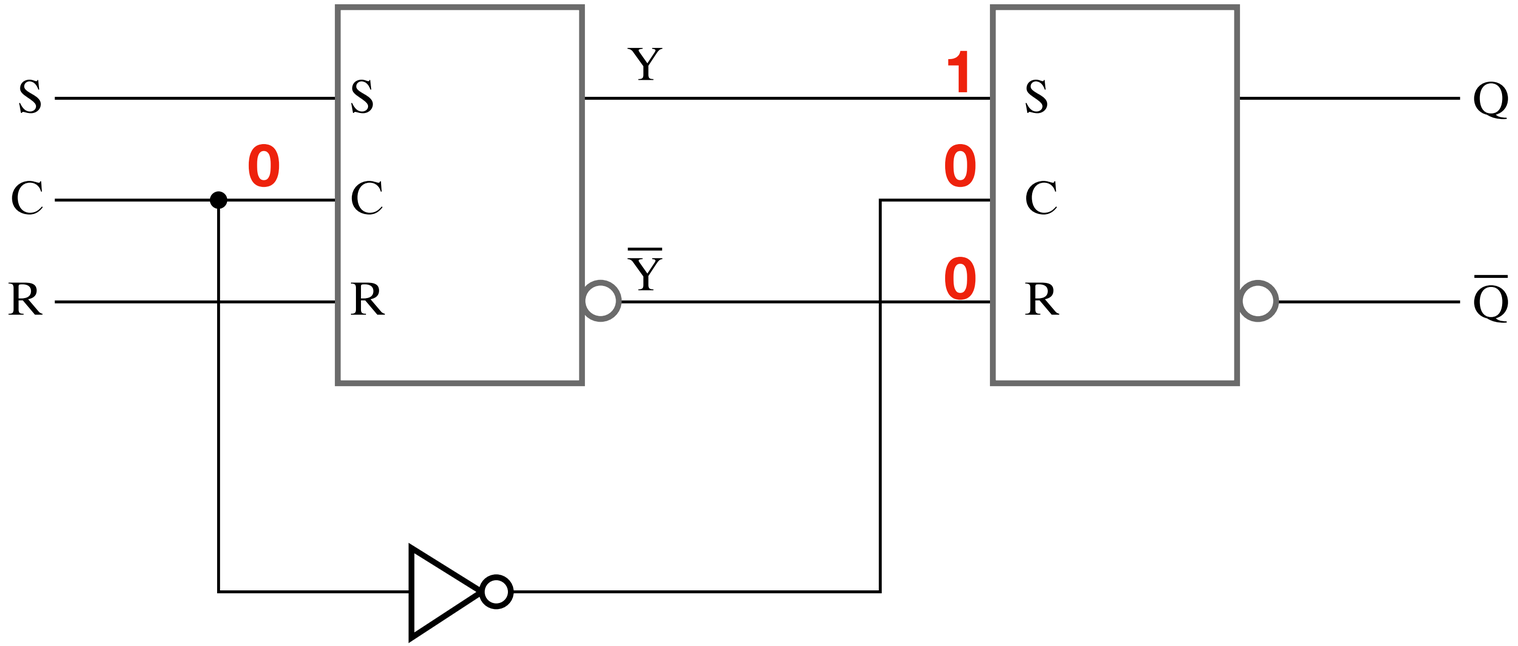

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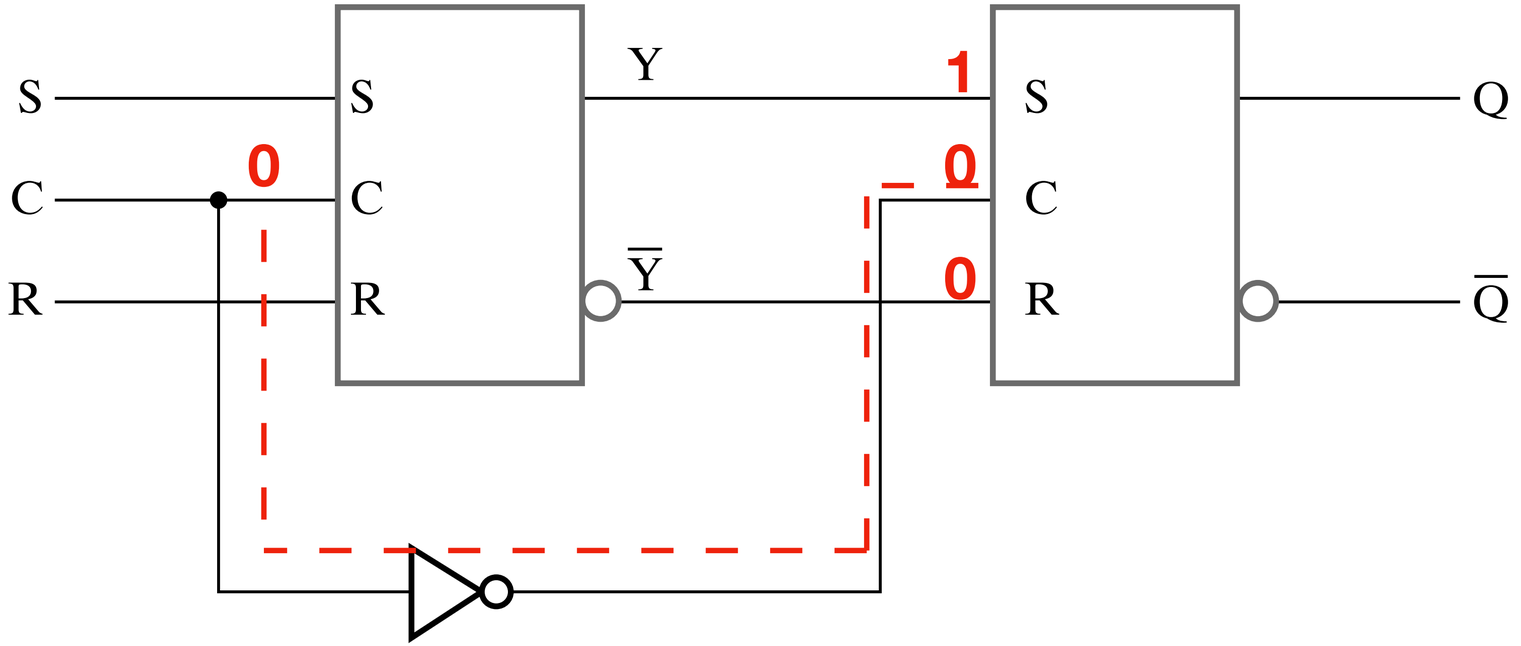

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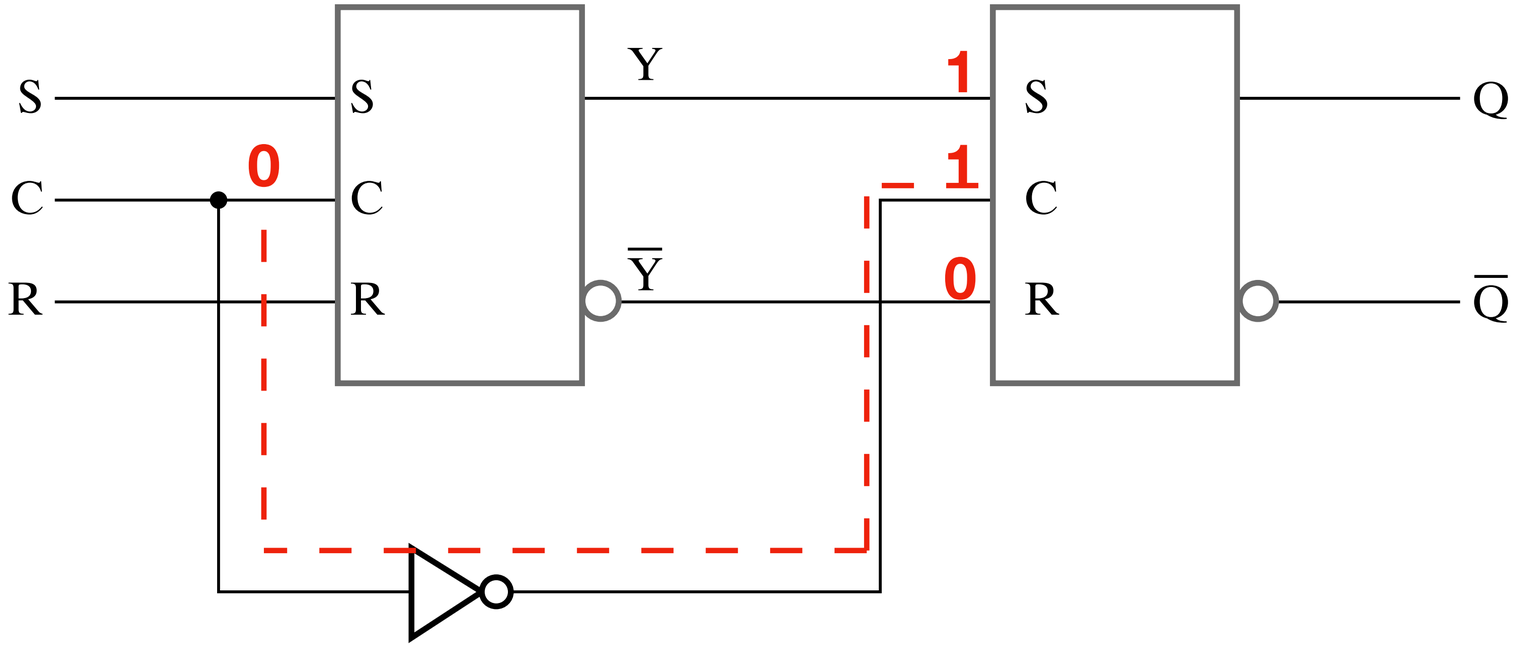

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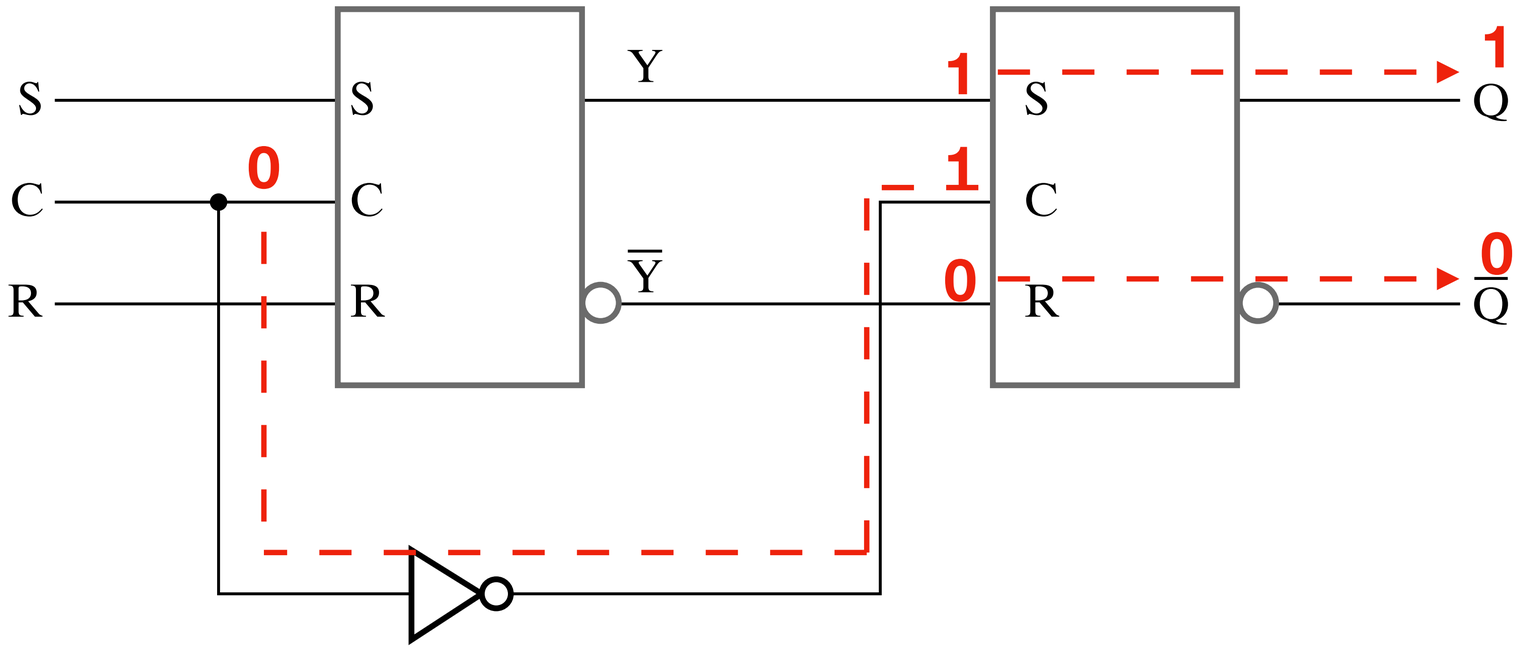

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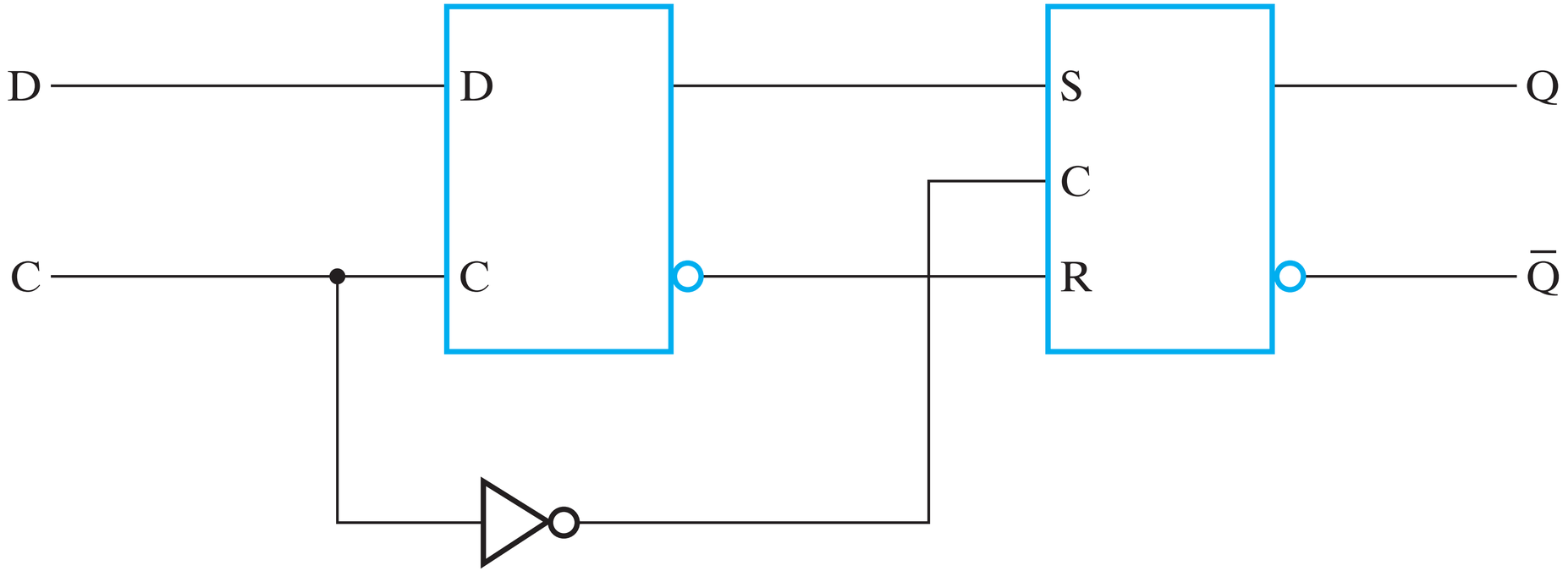
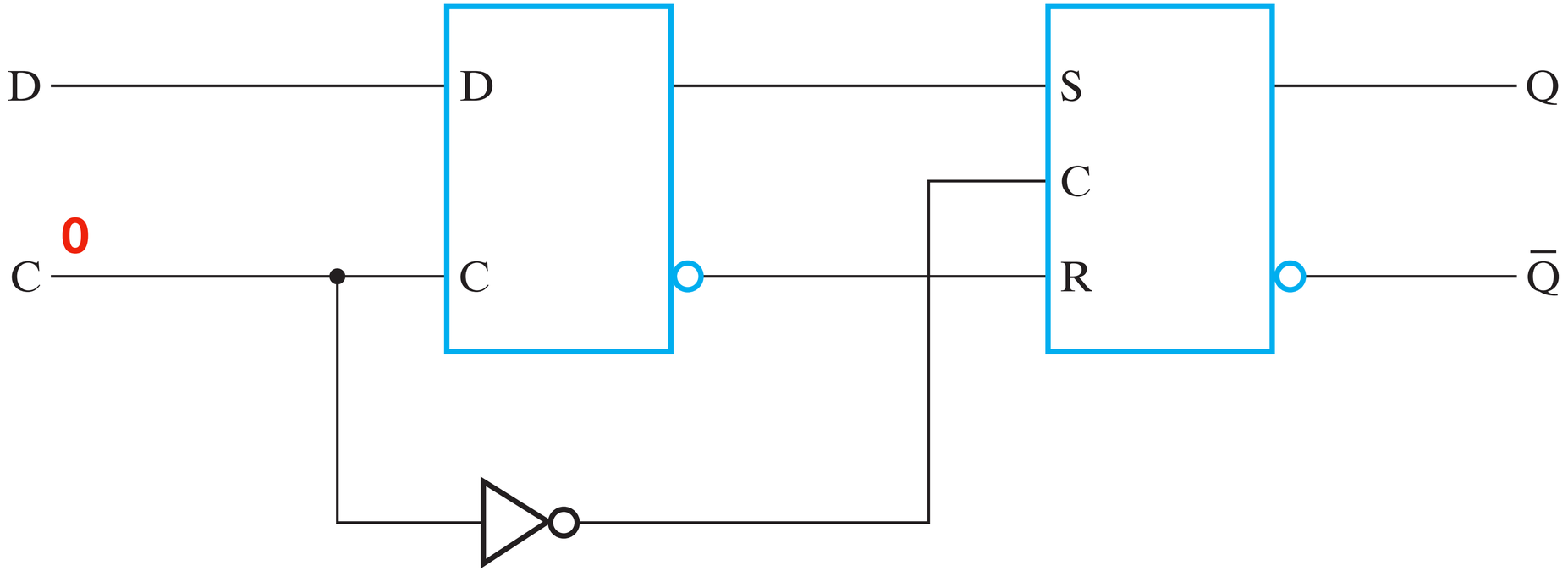
D Flip-Flop



- Replaces SR master in SR Master-Slave with D master Latch
- **Negative Edge Triggered** D (Flip-Flop): $C = 1$ -> $C = 0$

Review

D Flip-Flop



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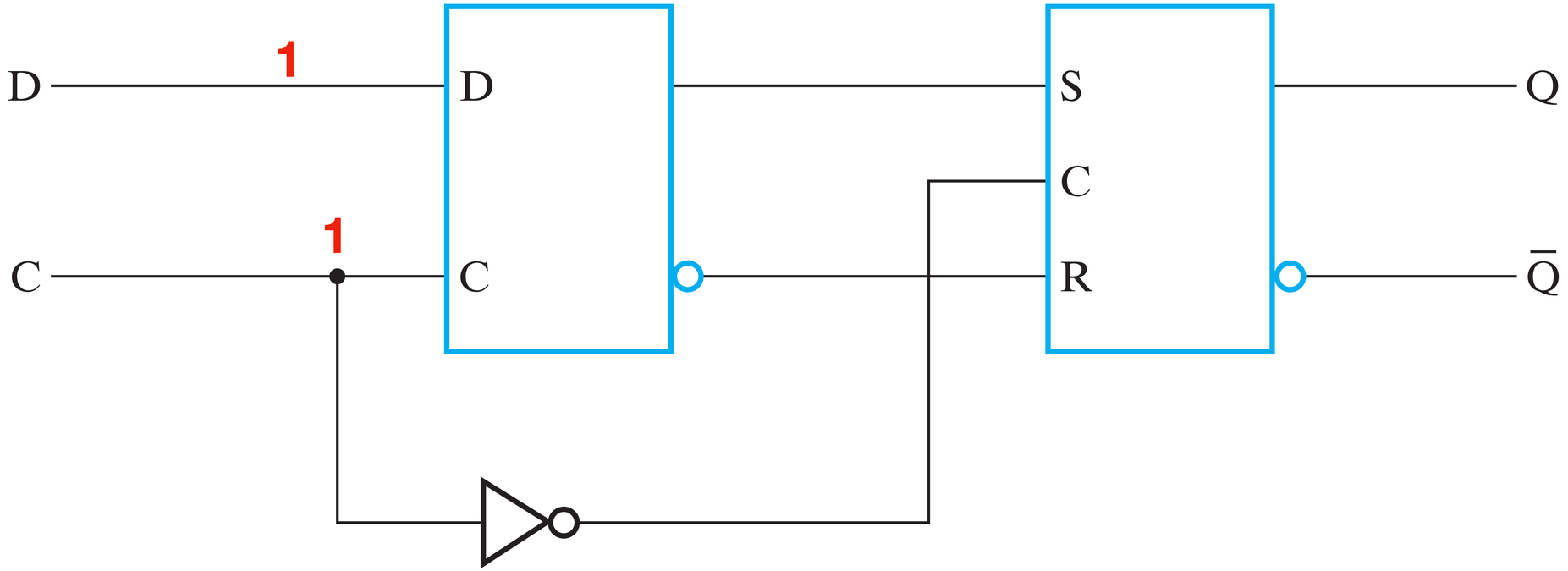
Demo

D Flip-Flop



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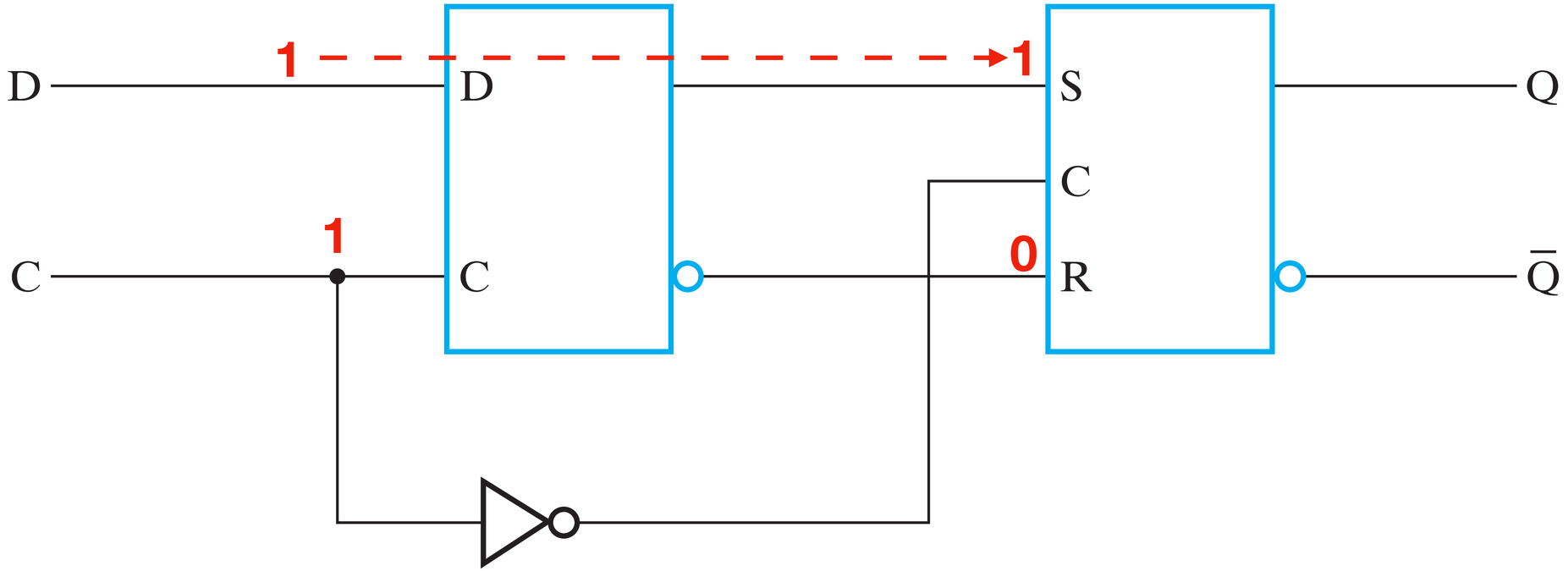

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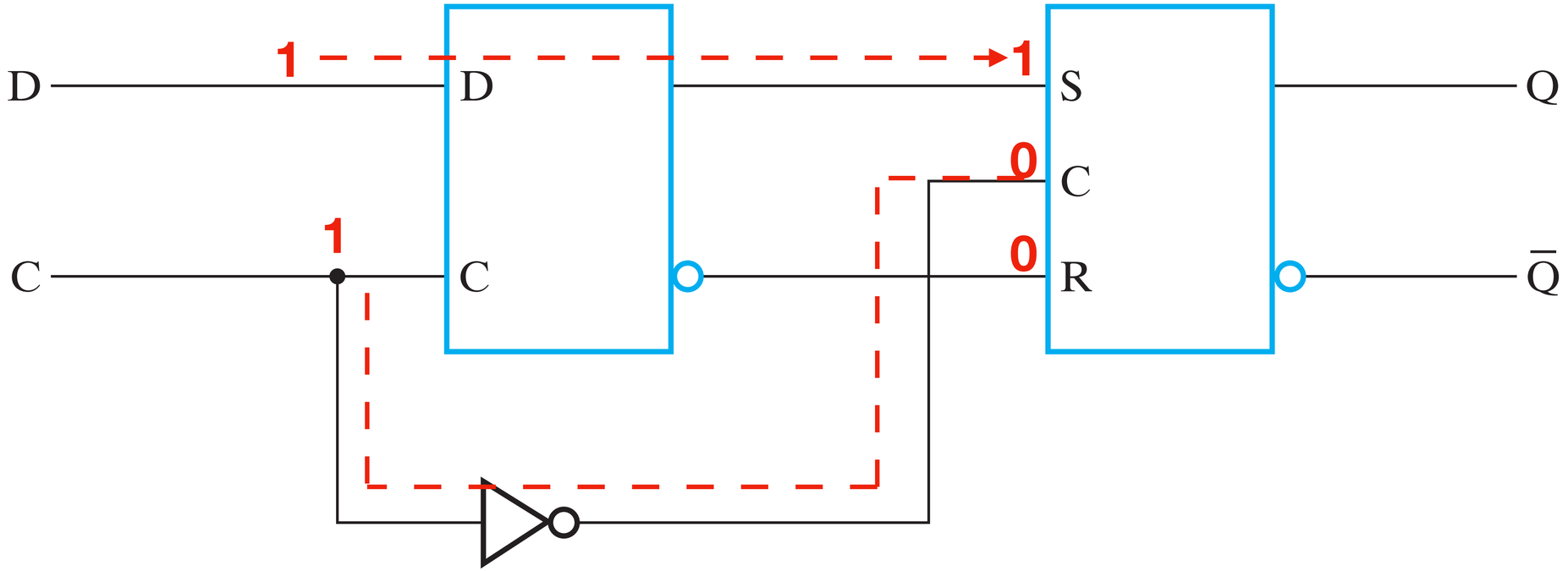

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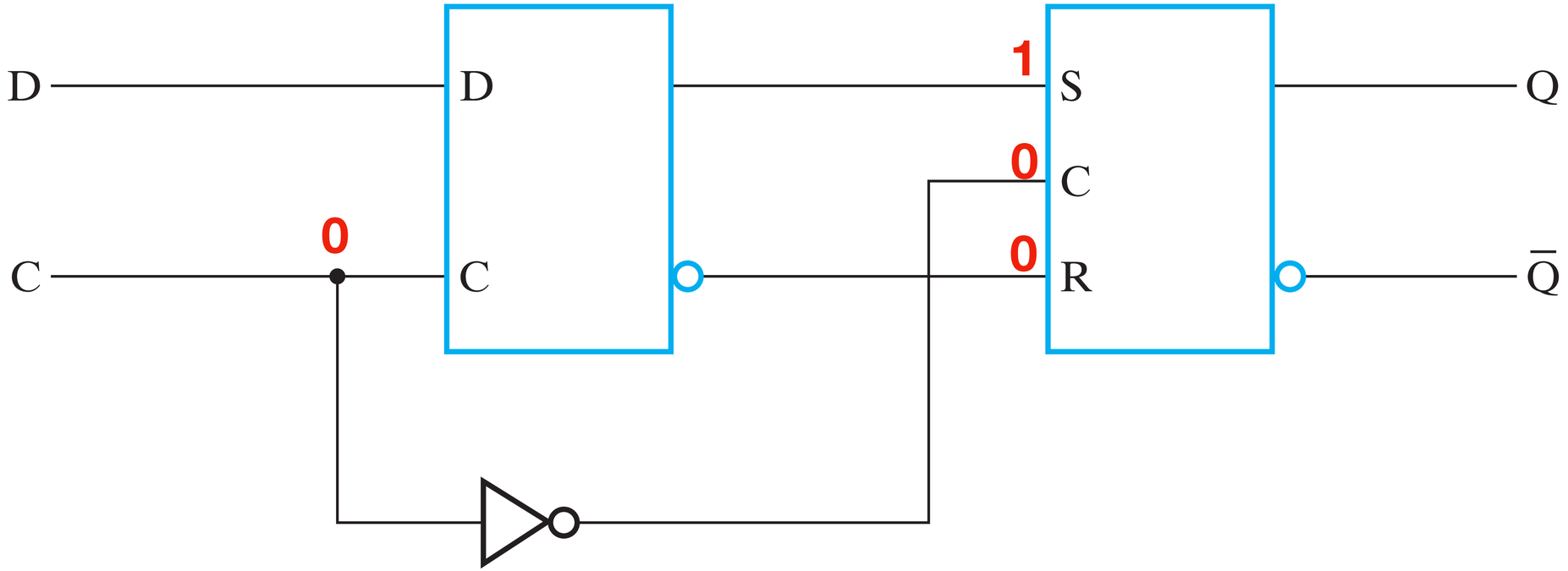

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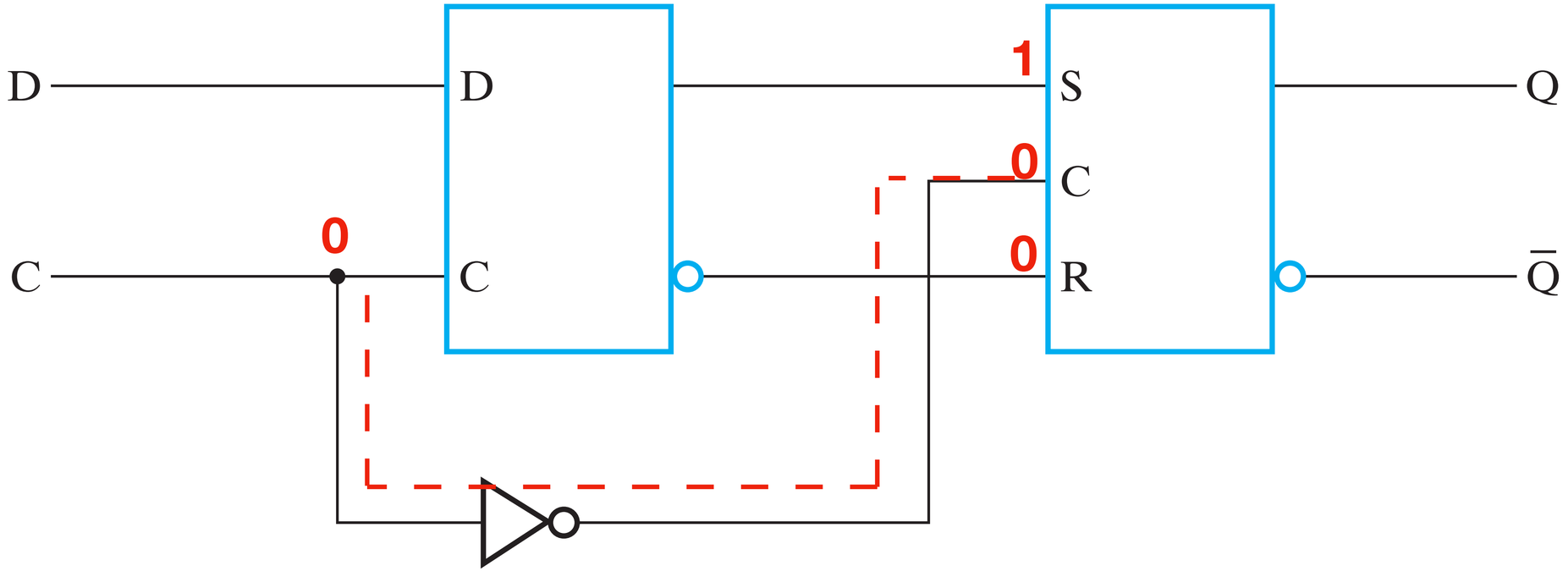

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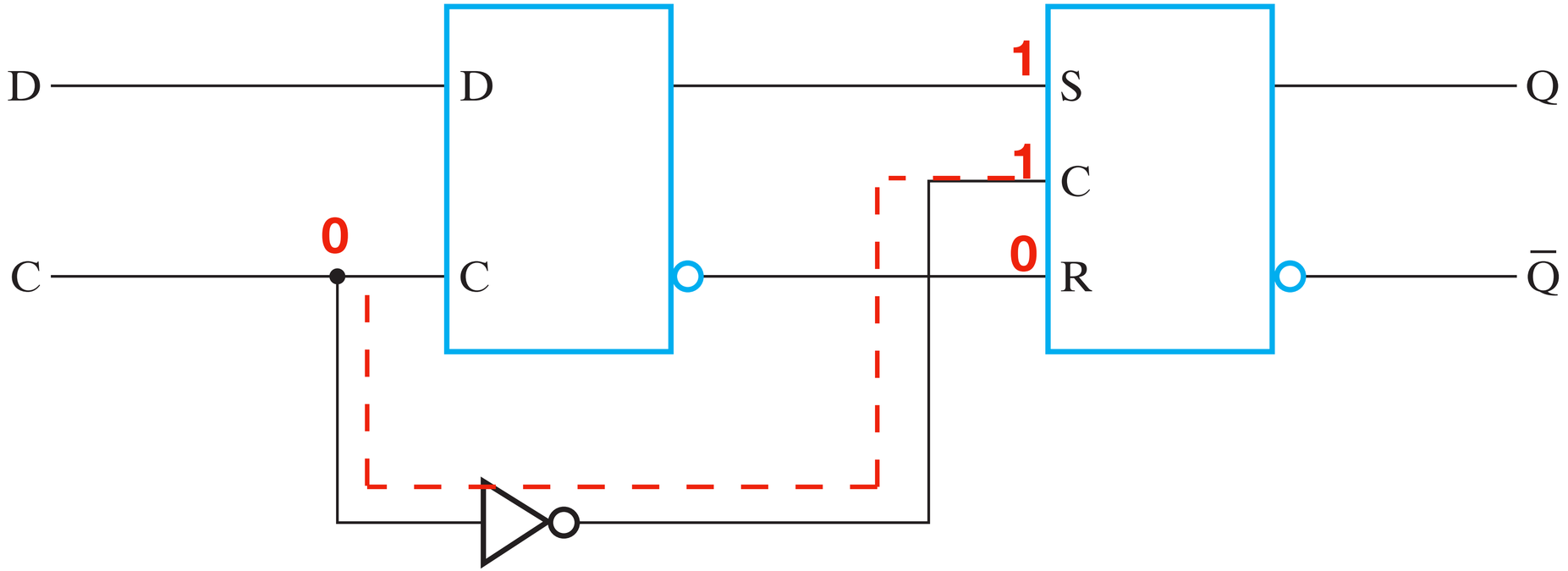

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Demo

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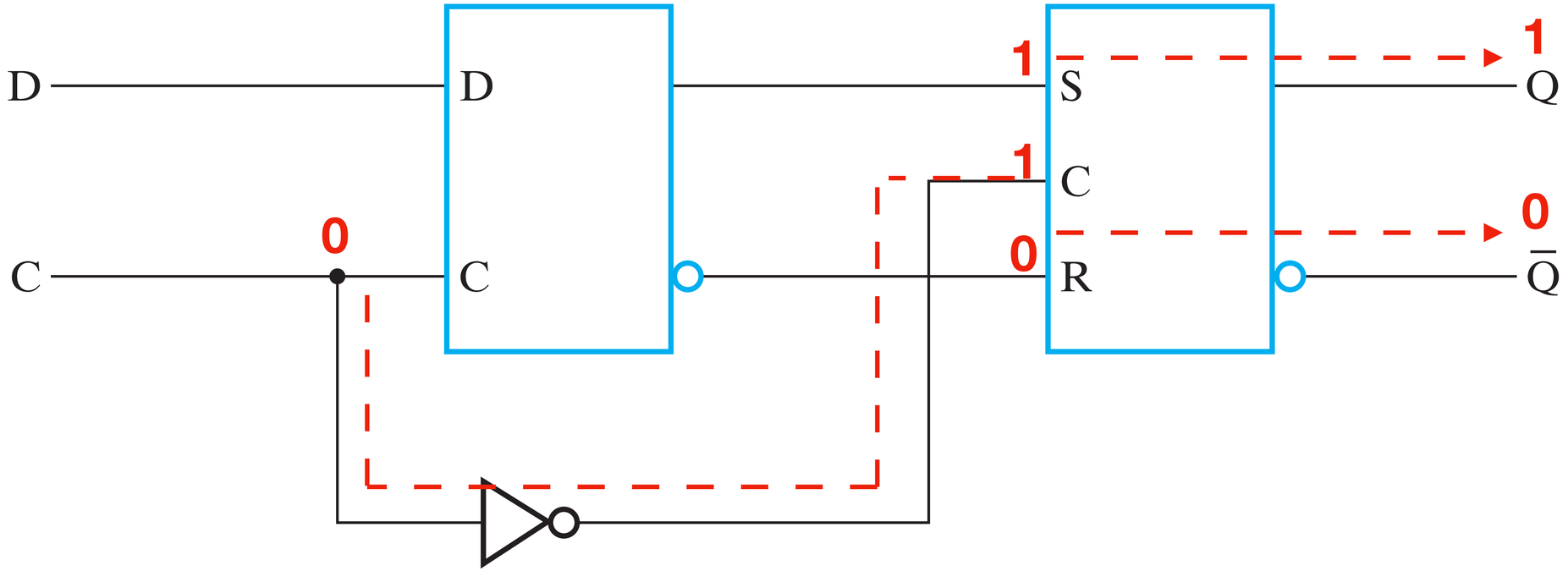

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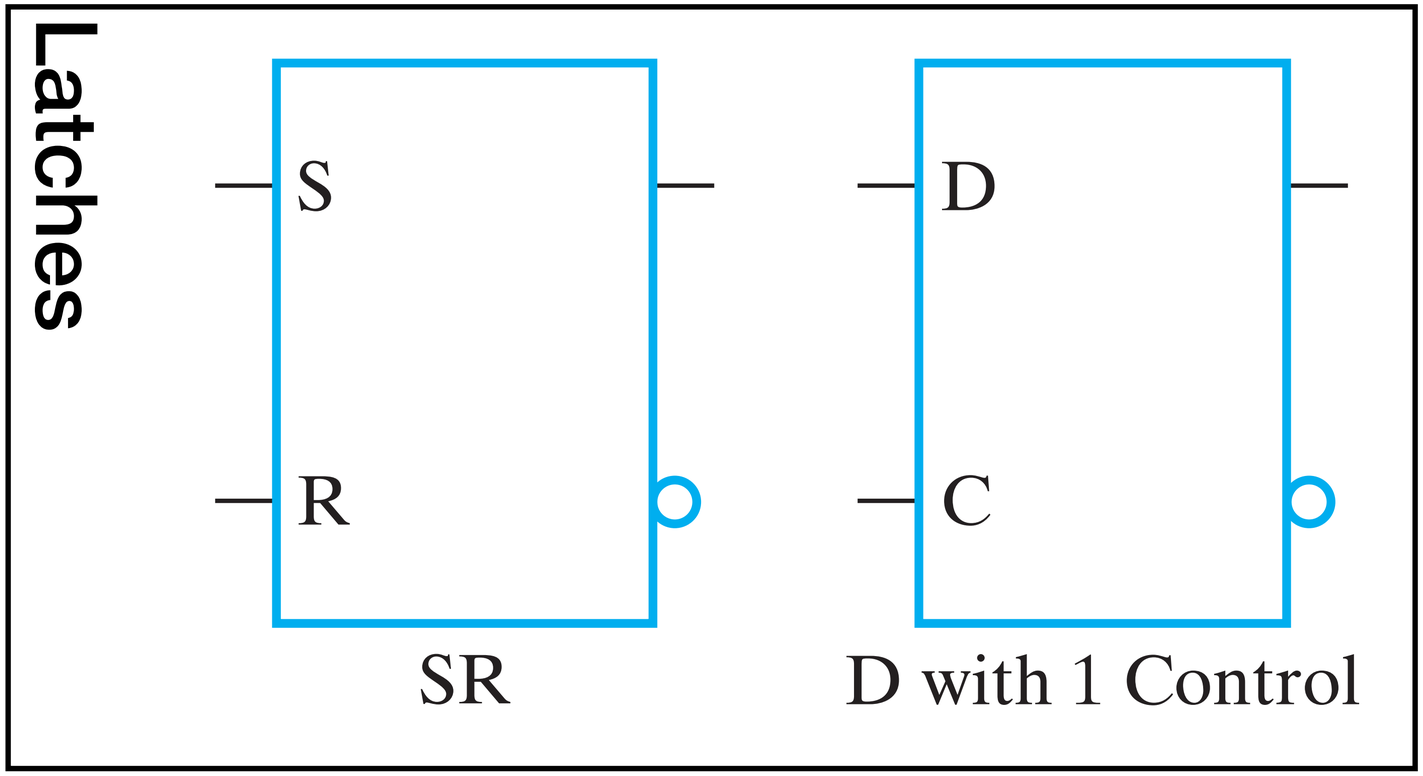
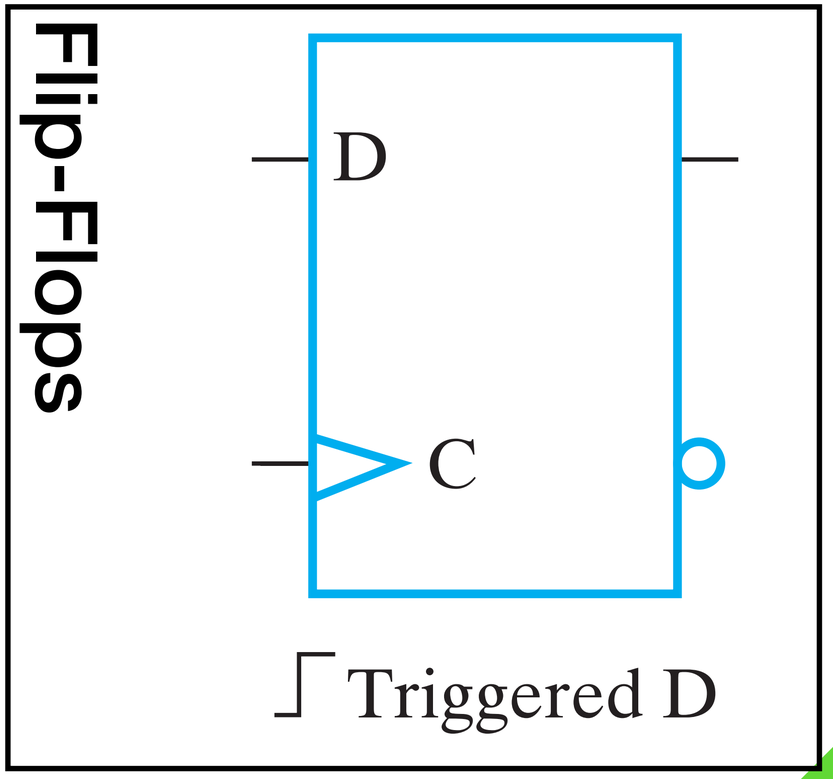
Demo

Summary





Demo

P2
Analysis

Sequential Circuit Analysis

State Table; State Diagram

Summary

P2
Analysis

State Table

- Similar to truth table
  - Input, and Current states
  - Output and Next states

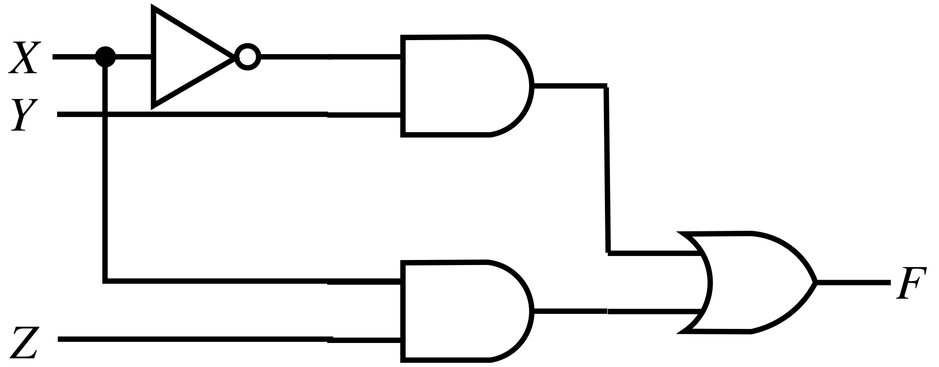
Concept

P2
Analysis

State Table

Truth Table

X	Y	Z	F
0	0	0	0
0	1	0	1
1	0	0	0
1	1	0	0
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1



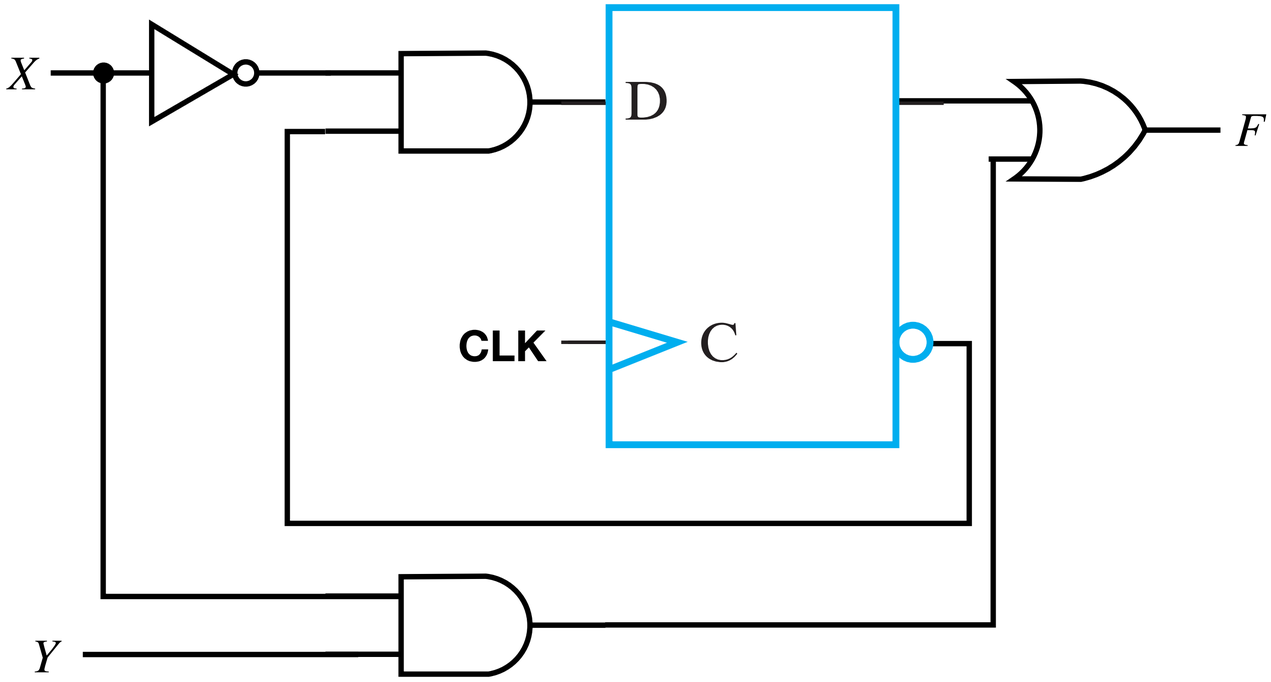
Review

P2
Analysis

State Table

State Table

Present State	X	Y	Next State	F
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



- When filling the state table, consider CLK = 0
- When CLK = 1, next state becomes current state

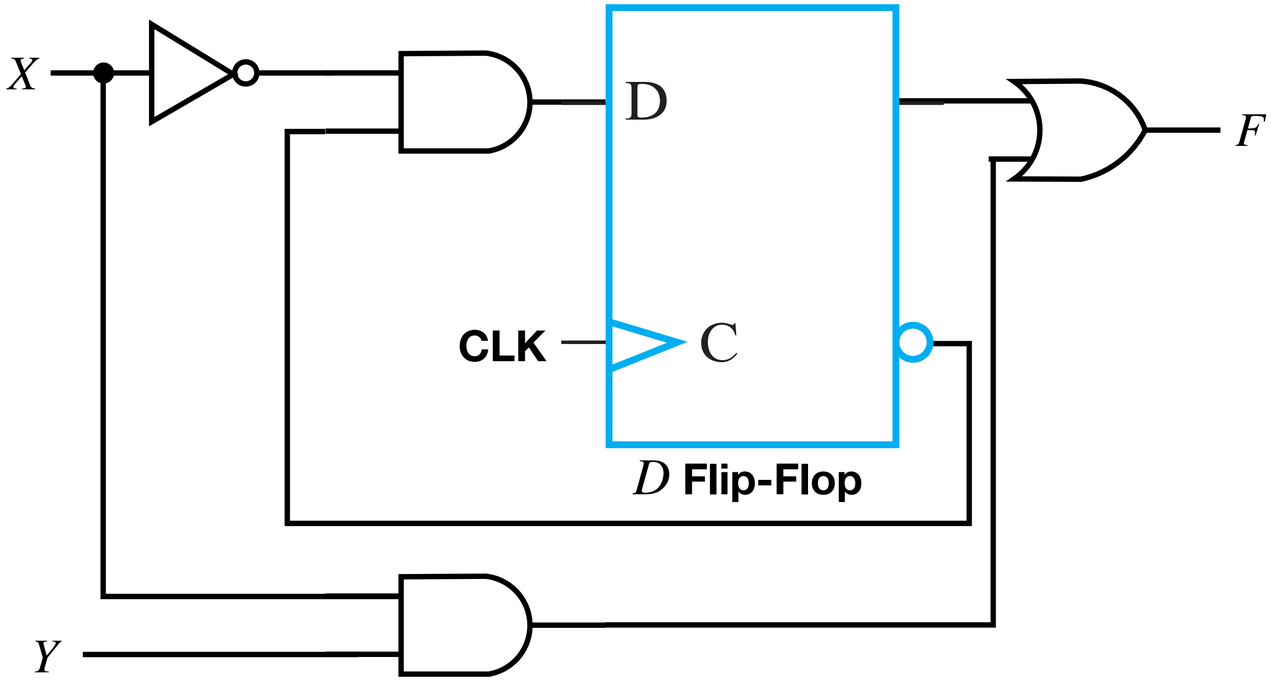
Example

P2
Analysis

State Table

State Table

Present State	X	Y	Next State	F
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



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Example

P2
Analysis

State Table

State Table

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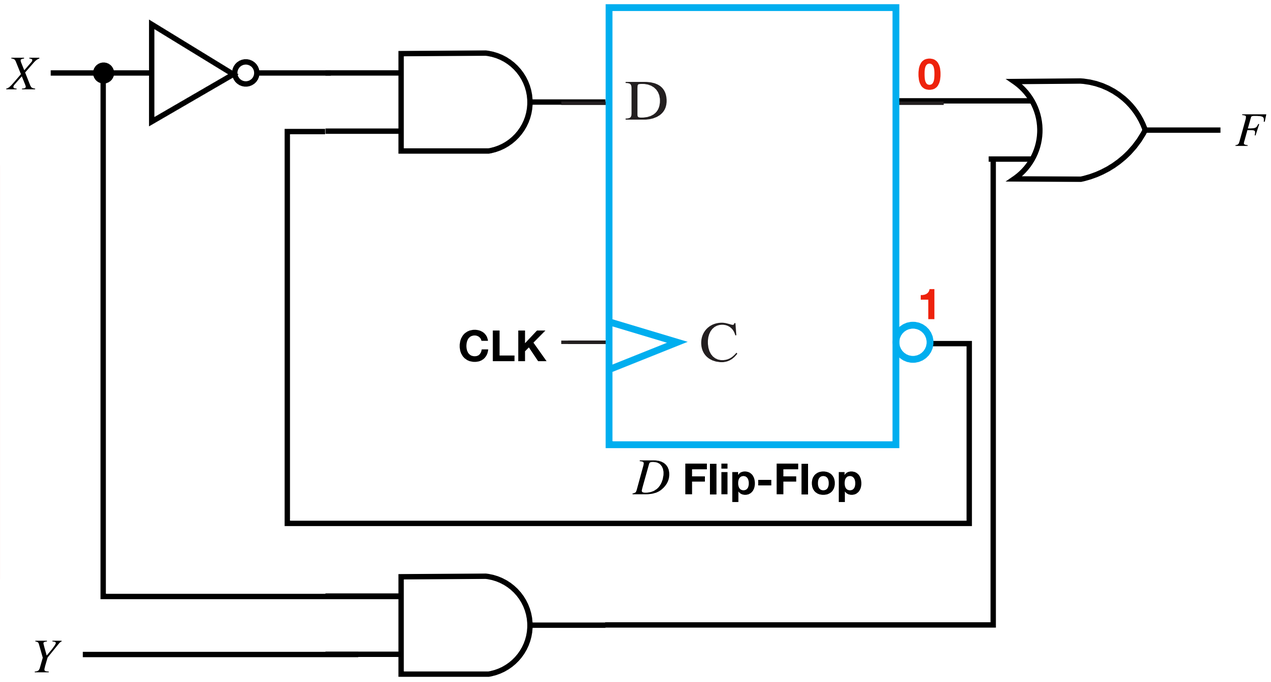
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



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Example

P2
Analysis

State Table

State Table

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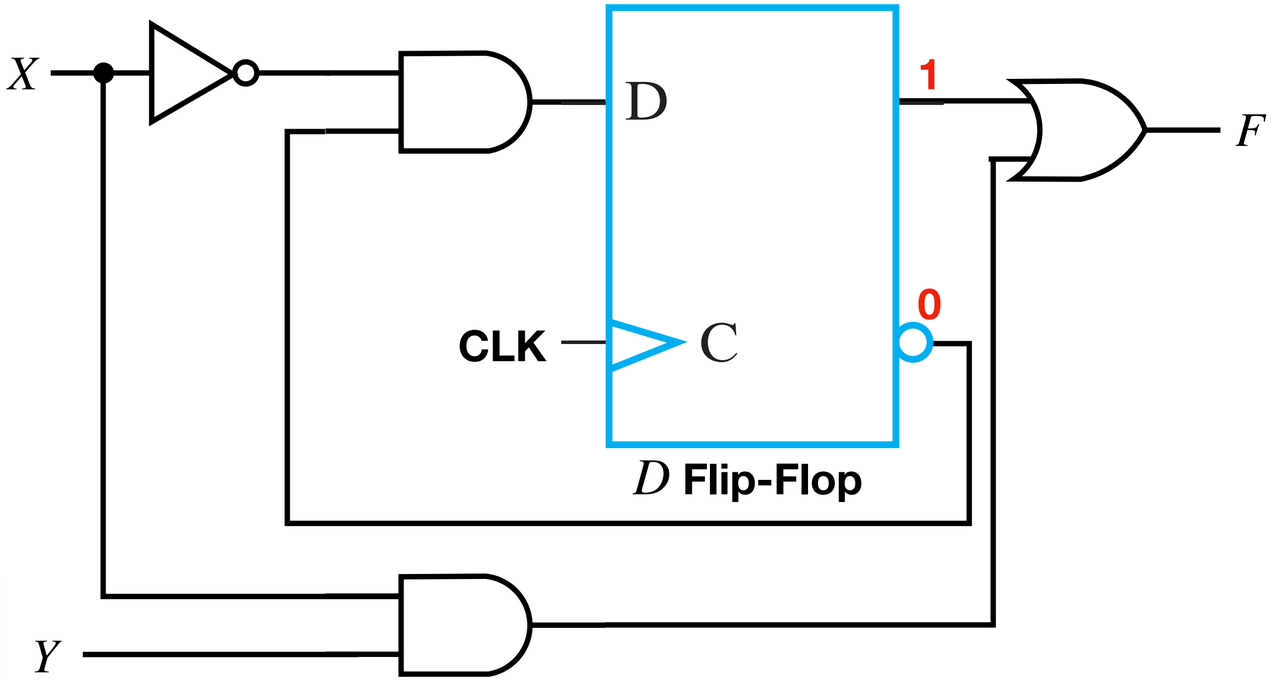
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



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Example

P2
Analysis

State Table

State Table

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0	1	0		
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1	0	0		
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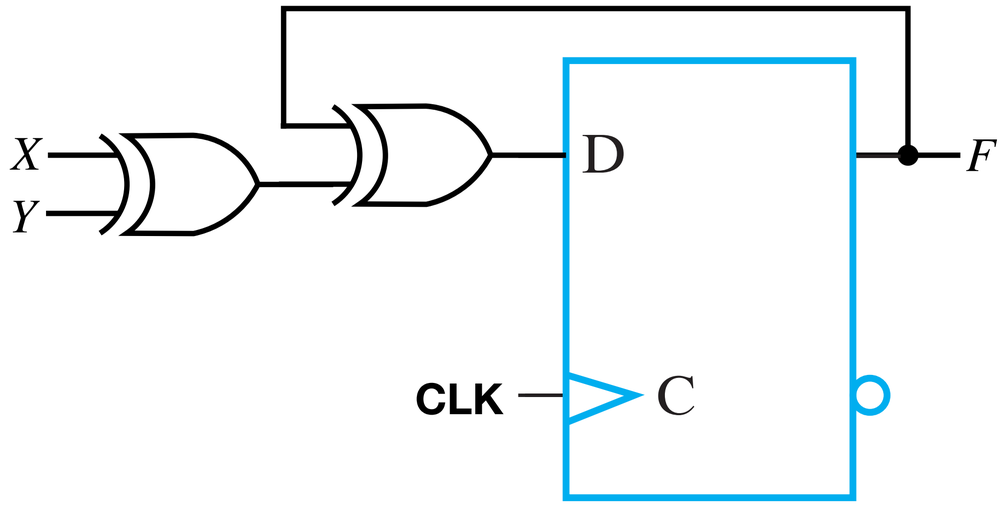
1	1	1		



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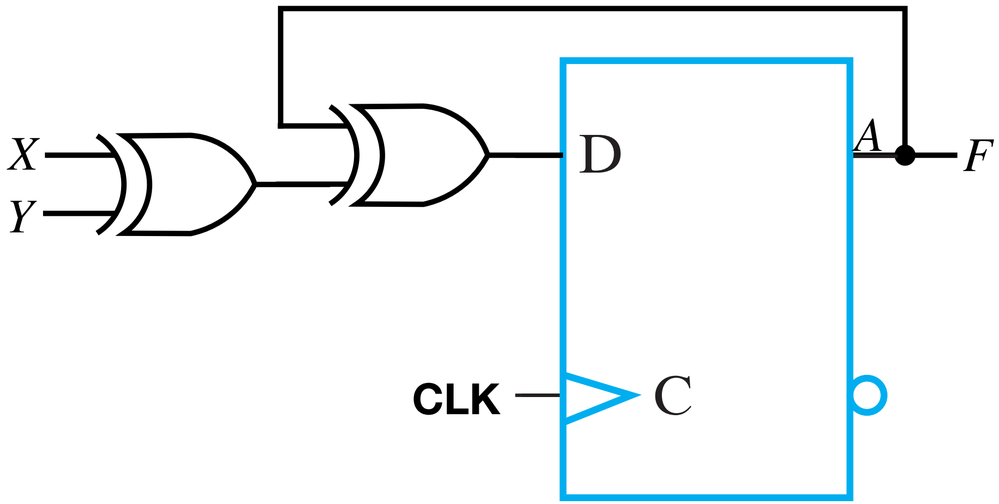
Exercise

P2
Analysis

State Table

State Table

Present State	X	Y	Next State	F
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



$$D_A = X \oplus Y \oplus A$$

- When filling the state table, consider CLK = 0
- When CLK = 1, next state becomes current state

Exercise

P2
Analysis

State Table

State Table

Present State	X	Y	Next State	F
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
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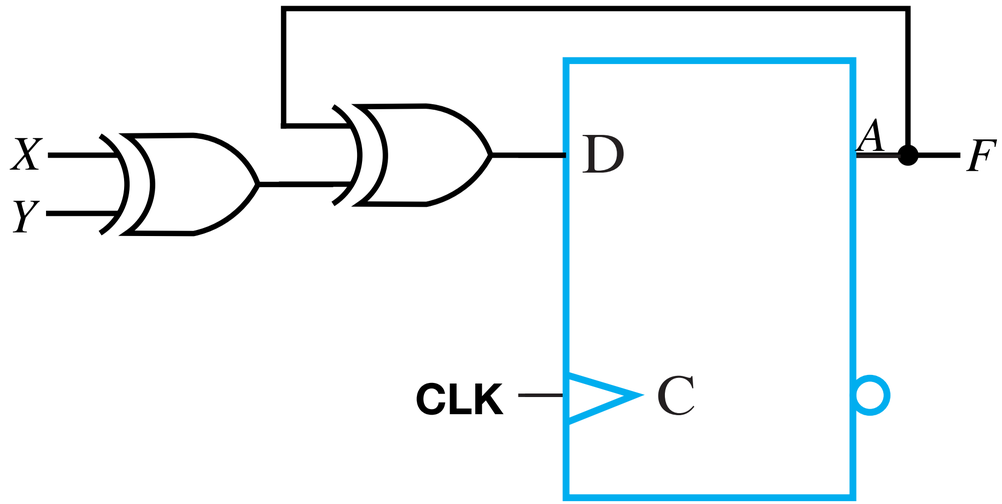

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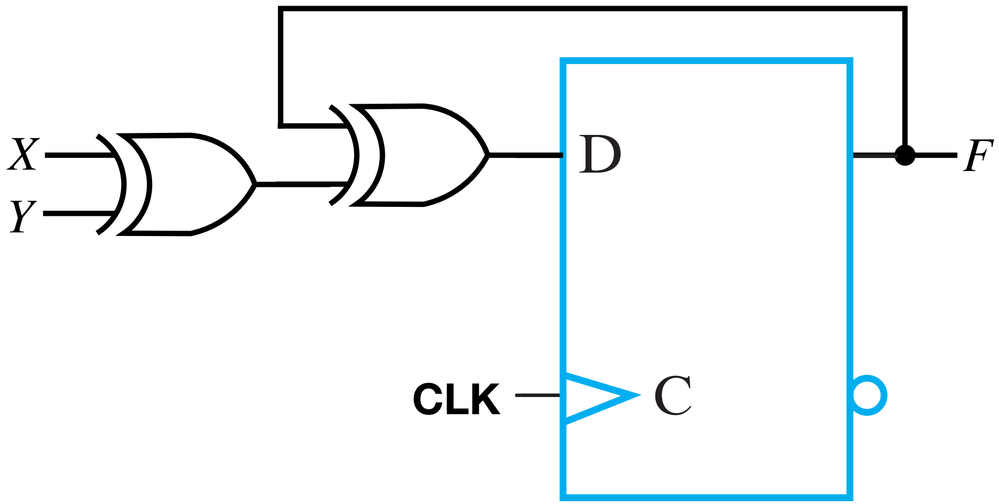
Exercise

P2
Analysis

State Table

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0		
1	0	1		
1	1	0		
1	1	1		



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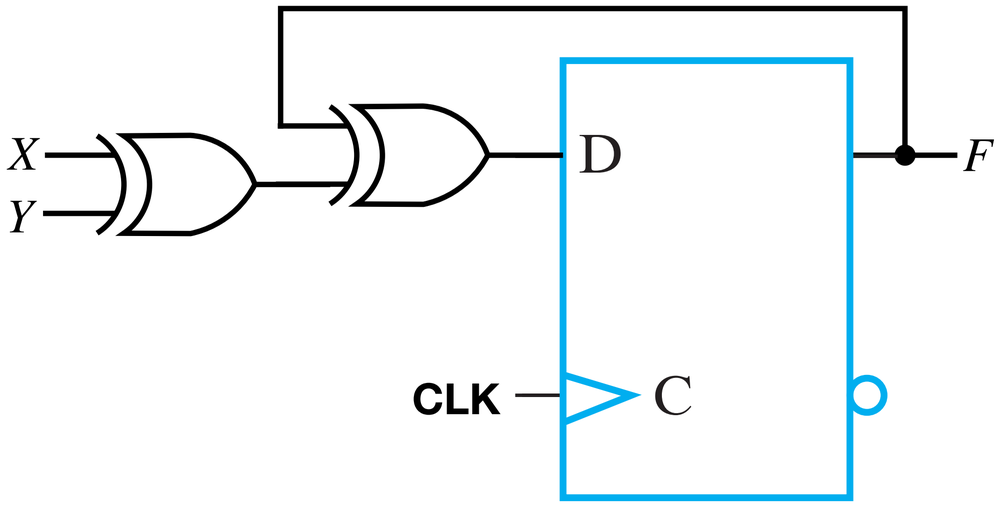
Exercise

P2
Analysis

State Table

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0		
1	0	1		
1	1	0		
1	1	1		



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Exercise

P2
Analysis

State Table

State Table

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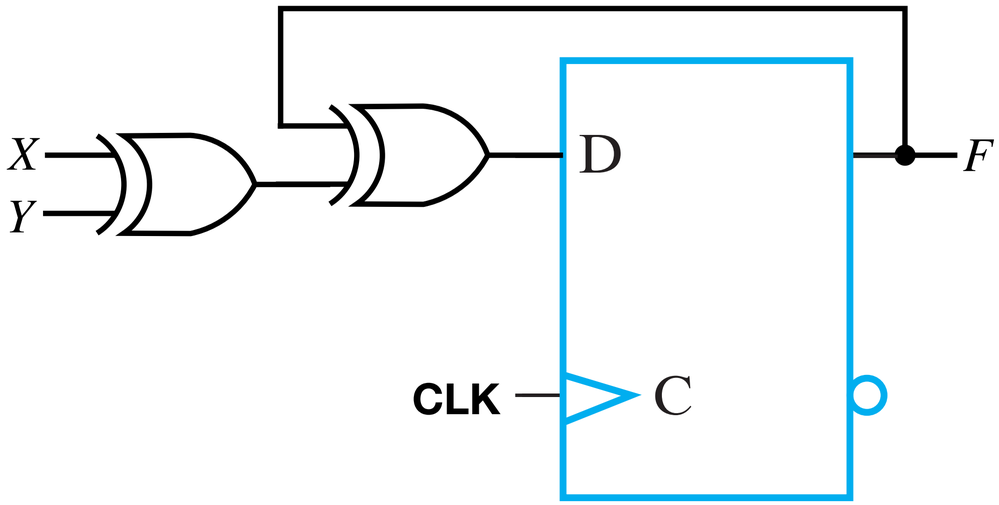
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



- When filling the state table, consider CLK = 0
- When CLK = 1, next state becomes current state

Exercise

State Table

State Table

Present State A	Present State B	X	Y	Next State A	Next State B	Z
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0			
0	1	1	1			

...

- Draw circuit diagram
- Fill the 8 rows

Exercise

State Table

State Table

Present State A	Present State B	X	Y	Next State A	Next State B	Z
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0			
0	1	1	1			

...

$$D_A = \overline{X}A + XY$$
$$D_B = \overline{X}B + XA$$
$$Z = XB$$

- Draw circuit diagram
- Fill the 8 rows

Exercise

P2
Analysis

State Table

Think

P2
Analysis

State Table

- What happens when there are multiple flip-flops?

Think

P2
Analysis

State Table

- What happens when there are multiple flip-flops?
- Does it work with Latches?

Think

In Class Exercise 1

- A circuit with one D flip-flop: $D_A = A \oplus X$
- Draw the circuit diagram
- Do the state table

Exercise

P2
Analysis

In Class Exercise 1

- A circuit with one D flip-flop: $D_A = A \oplus X$
- Draw the circuit diagram
- Do the state table

State Table

Present State	X	Next State
0	0	
0	1	
1	1	
1	0	

Exercise

P2
Analysis

In Class Exercise 2

- A circuit with 2 D flip-flops: $D_A = A \oplus B$, $D_B = \overline{B} \cdot X$, $F = \overline{A}B$
- Do the state table

State Table

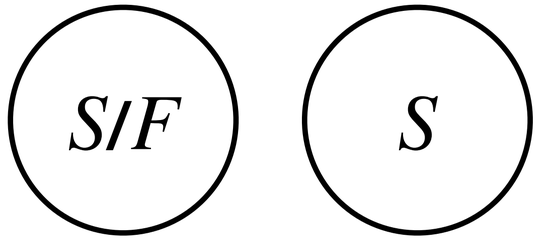
Present State		X	Next State		F
A	B		A	B	
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Exercise

P1
Analysis

State Diagram

- Similar to state table
  - Models state transitions
  - A state is represented in a bubble
  - Directed links between bubbles: the input used to perform transition
    Source: **present** state, Target: **next** state (next **CLK**); (optional output F)
  - State bubble with state as S (optional output F)



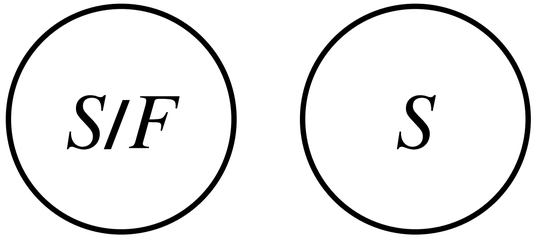
Concept

P1
Analysis

State Diagram

- Similar to state table
  - Models state transitions
  - A state is represented in a bubble

  - Directed links between bubbles: the input used to perform transition
    Source: **present** state, Target: **next** state (next **CLK**); (optional output F)
  - State bubble with state as S (optional output F)

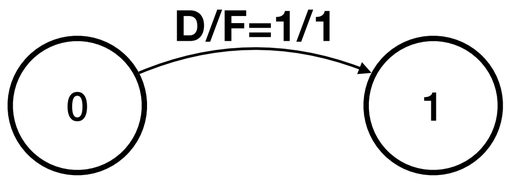


Concept

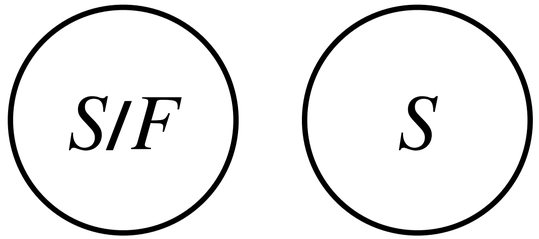
P1
Analysis

State Diagram

- Similar to state table
  - Models state transitions
  - A state is represented in a bubble



  - Directed links between bubbles: the input used to perform transition
    Source: **present** state, Target: **next** state (next **CLK**); (optional output F)
  - State bubble with state as S (optional output F)

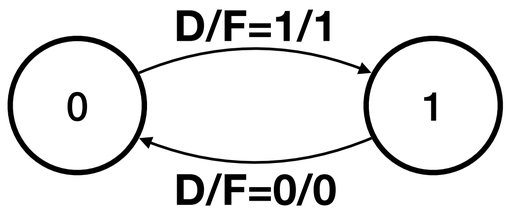


Concept

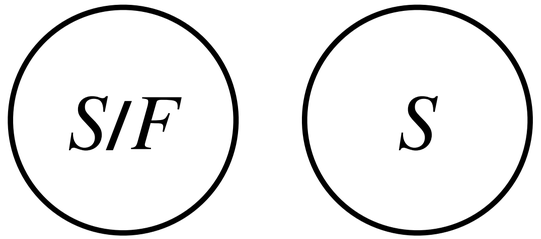
P1
Analysis

State Diagram

- Similar to state table
  - Models state transitions
  - A state is represented in a bubble



  - Directed links between bubbles: the input used to perform transition
    Source: **present** state, Target: **next** state (next **CLK**); (optional output F)
  - State bubble with state as S (optional output F)



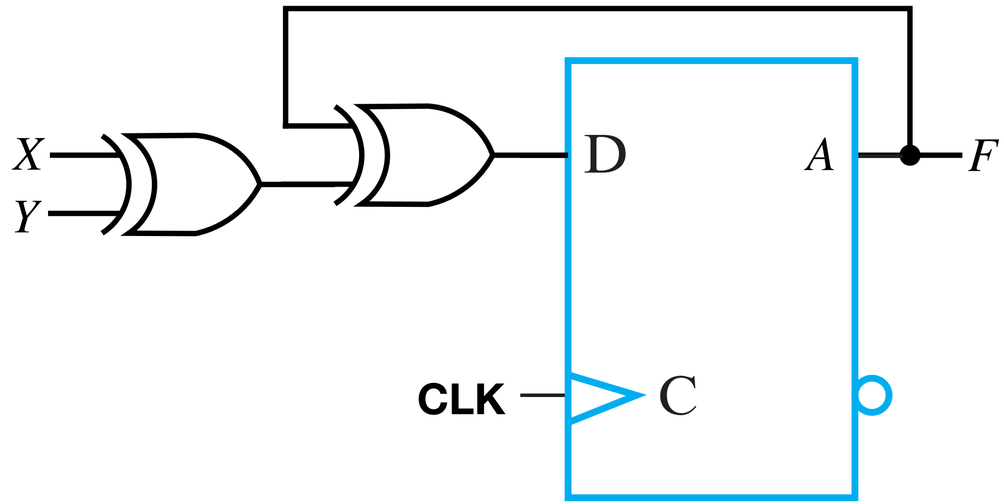
Concept

P1
Analysis

State Diagram

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



A single state bubble

A single link

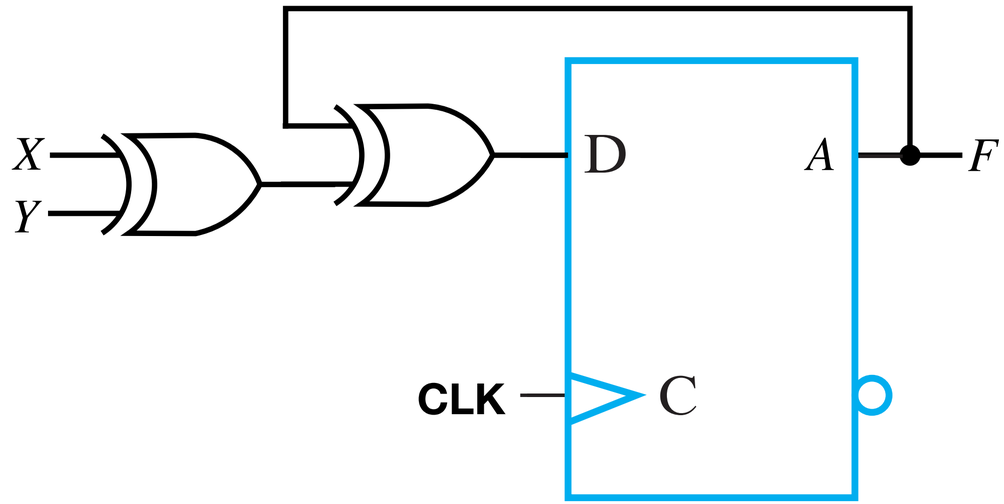
Concept

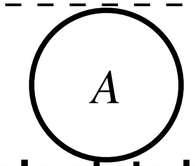
P1
Analysis

State Diagram

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1





A single state bubble

A single link

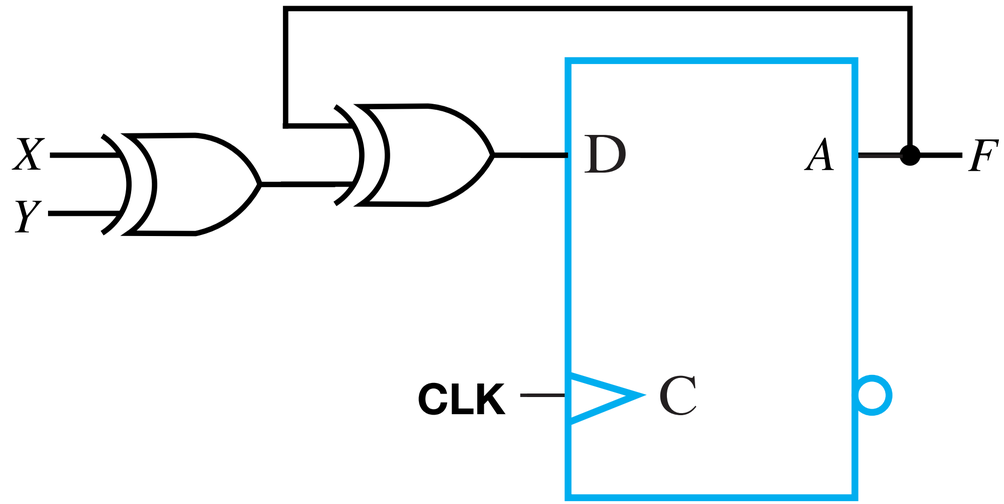
Concept

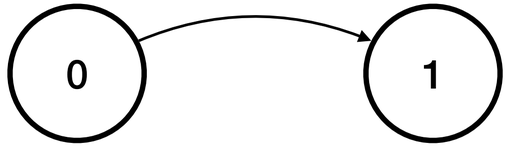
P1
Analysis

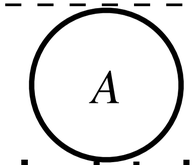
State Diagram

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1







A single state bubble

A single link

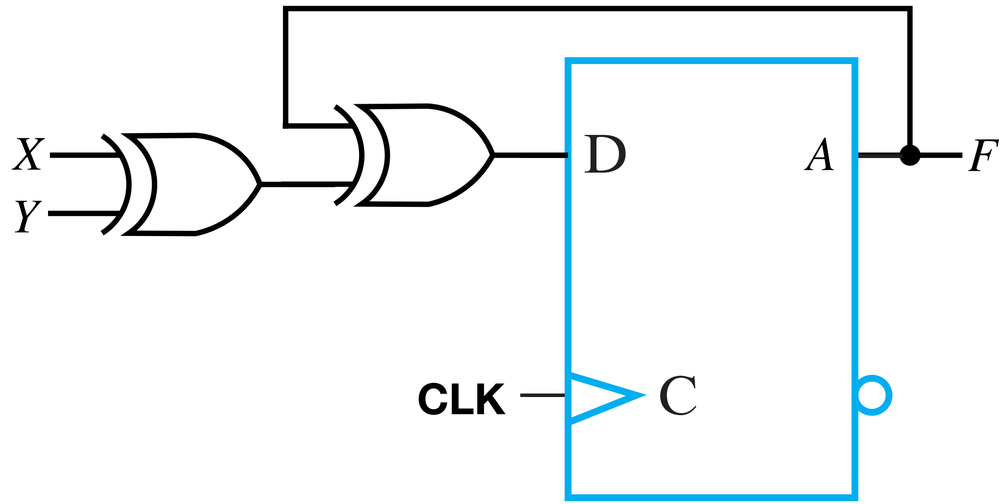
Concept

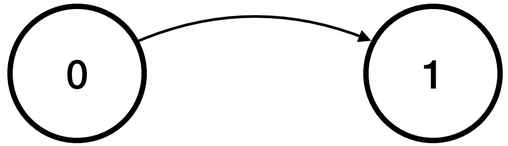
P1
Analysis

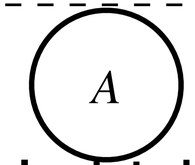
State Diagram

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1







A single state bubble

A single link

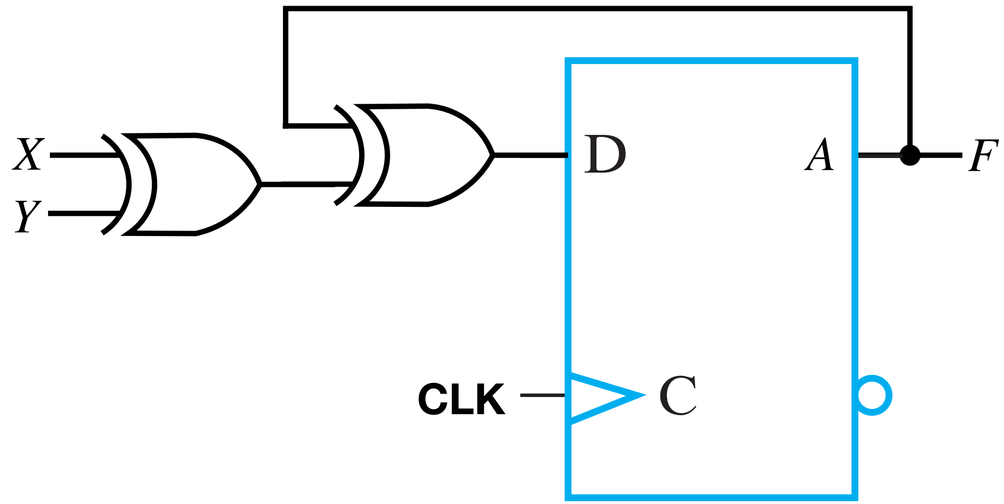
Concept

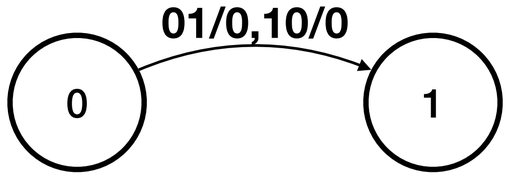
P1
Analysis

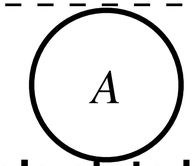
State Diagram

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1







A single state bubble

A single link

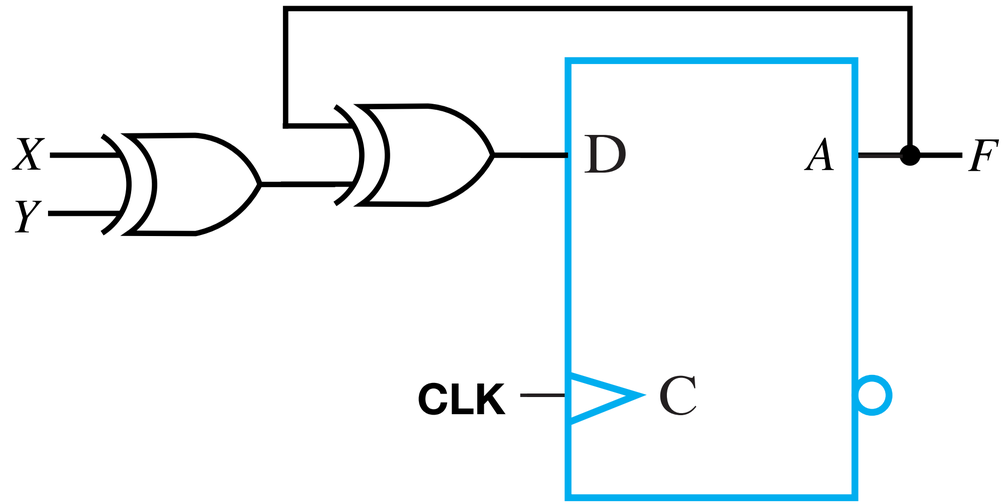
Concept

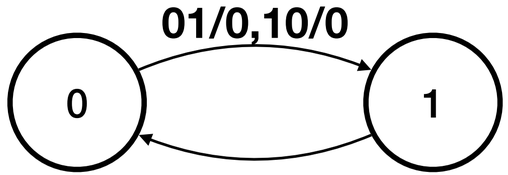
P1
Analysis

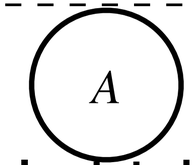
State Diagram

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1







A single state bubble

A single link

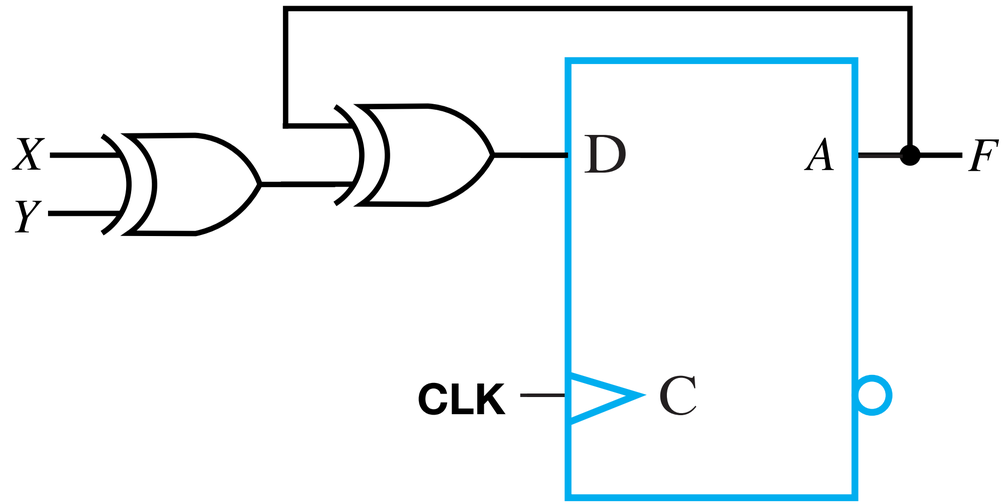
Concept

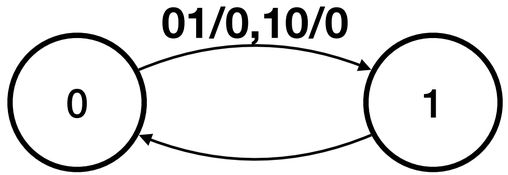
P1
Analysis

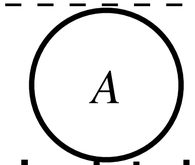
State Diagram

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1







A single state bubble

A single link

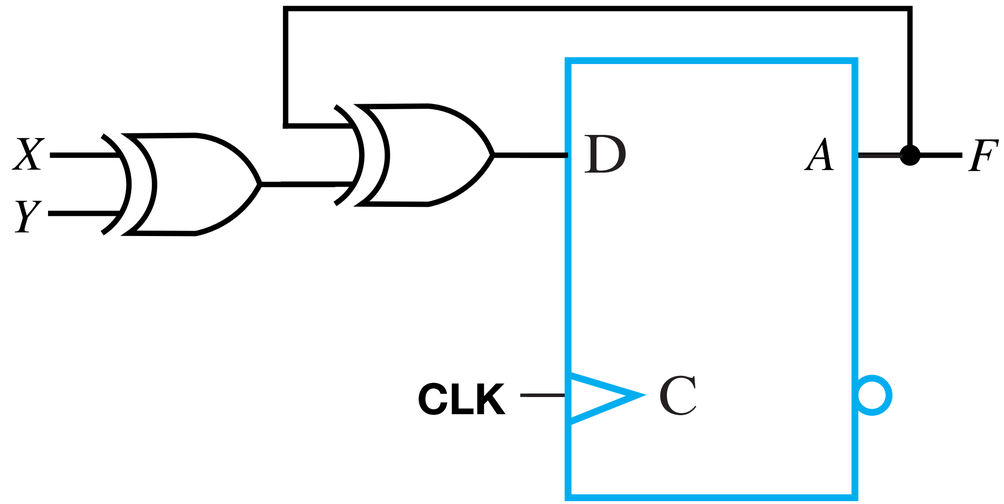
Concept

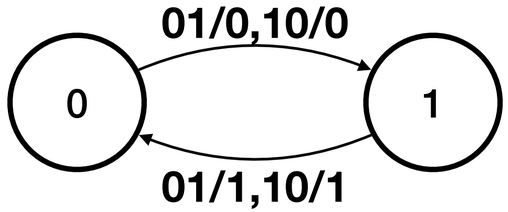
P1
Analysis

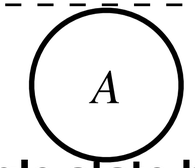
State Diagram

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1







A single state bubble

A single link

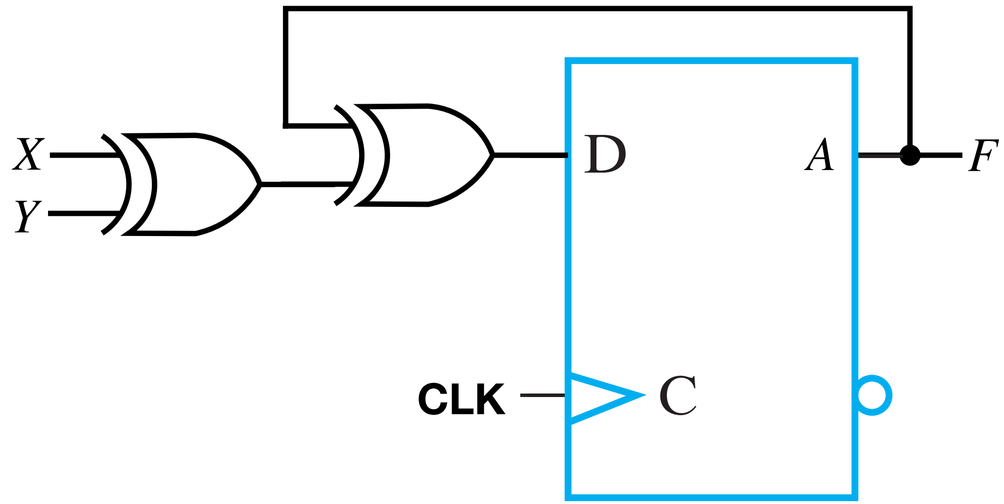
Concept

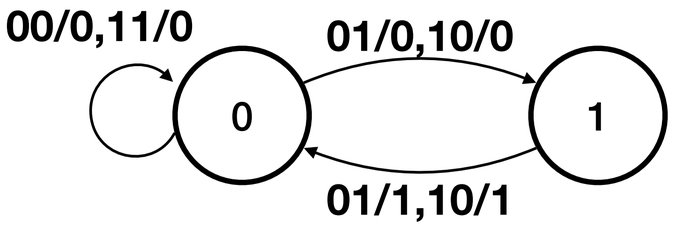
P1
Analysis

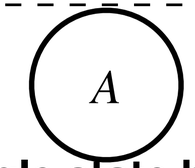
State Diagram

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1







A single state bubble

A single link

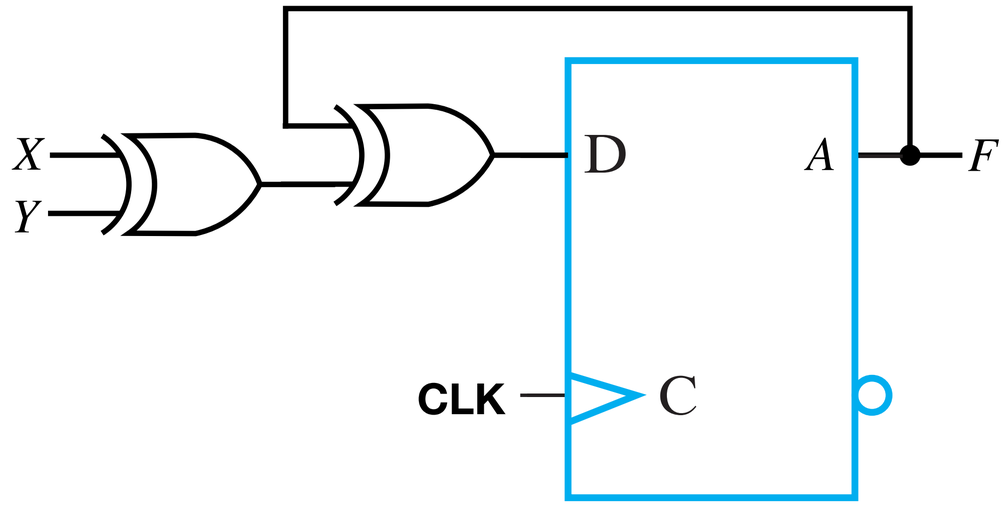
Concept

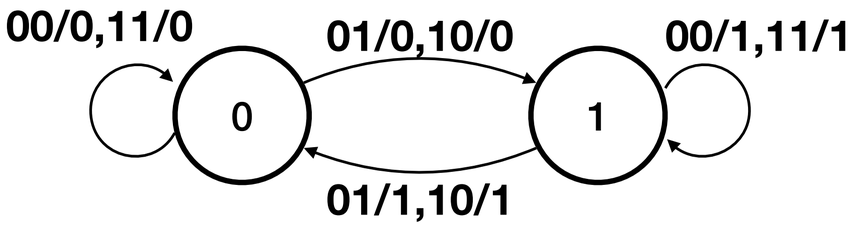
P1
Analysis

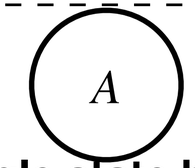
State Diagram

State Table

Present State	X	Y	Next State	F
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1







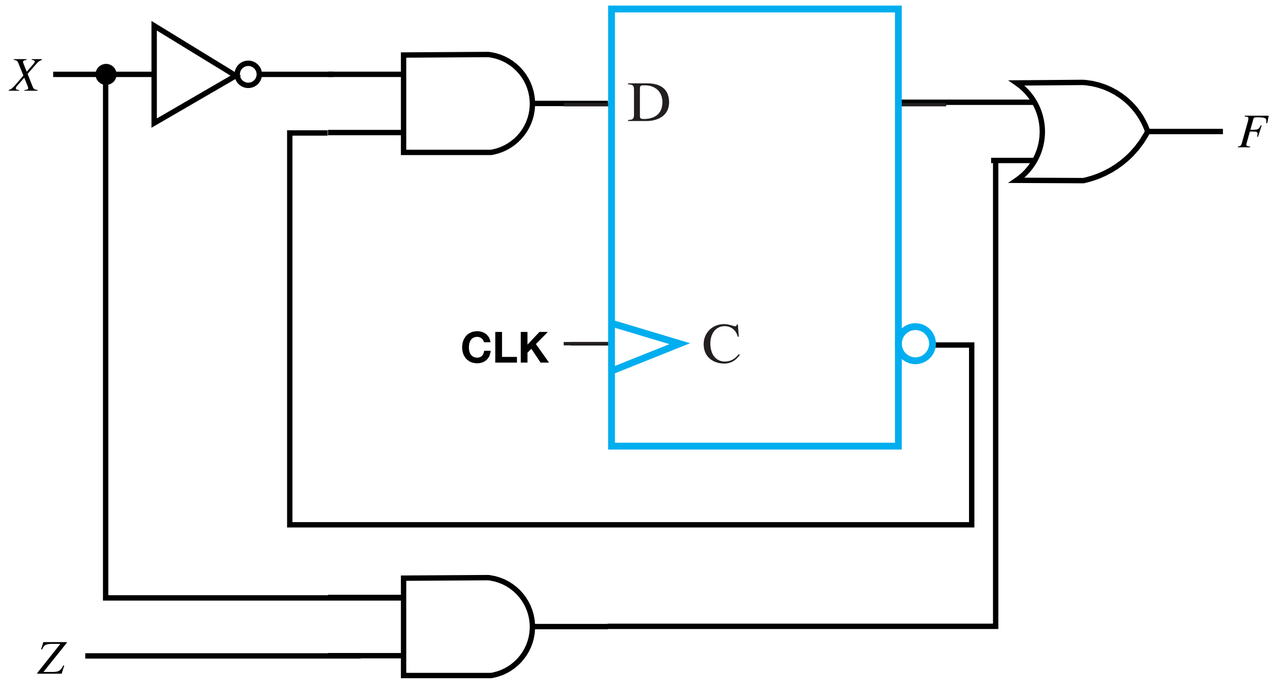
A single state bubble

A single link

Concept

P1
Analysis

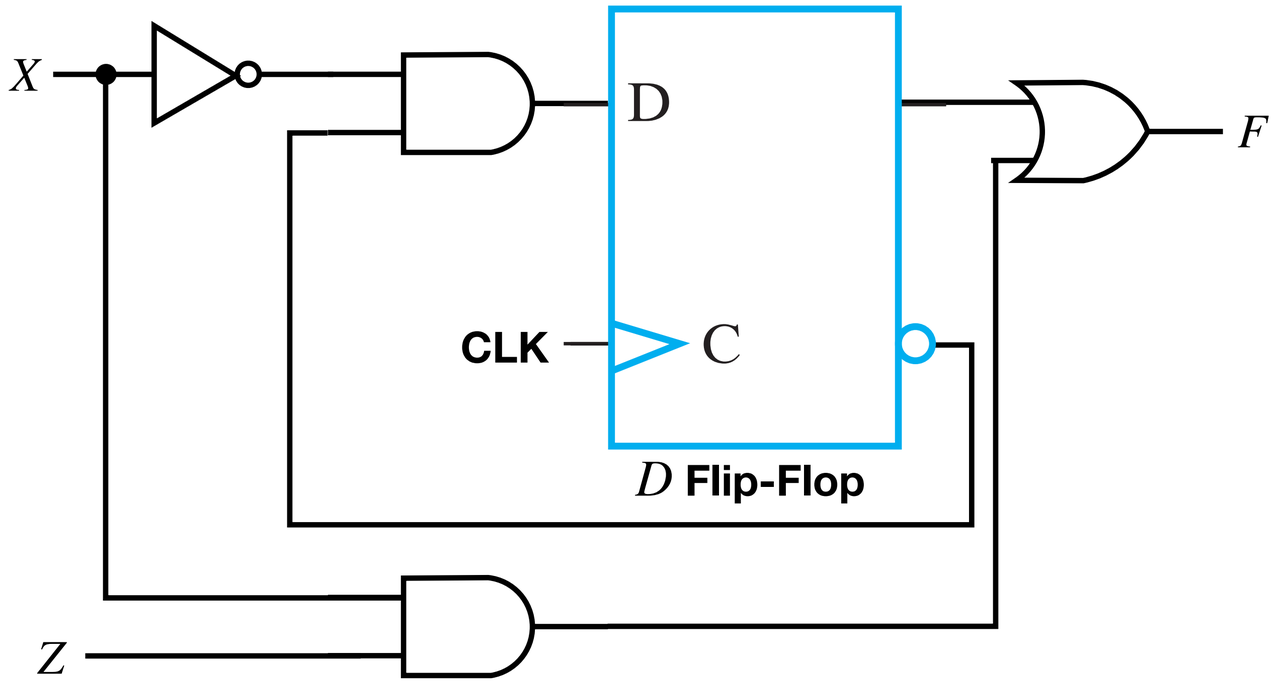
State Diagram



Example

P1
Analysis

State Diagram



Example

P1
Analysis

State Diagram

- Draw the state diagram for: $D_A = \overline{X}A + XY$, $D_B = \overline{X}B + XA$, $Z = XB$

Exercise

P1
Analysis

In Class Exercise 1

- A circuit with one D flip-flop: $D_A = A \oplus X$
- Draw the state diagram

Exercise

P1
Analysis

In Class Exercise 1

- A circuit with one D flip-flop: $D_A = A \oplus X$
- Draw the state diagram

State Table

Present State	X	Next State
0	0	0
0	1	1
1	0	1
1	1	0

Exercise

P1
Analysis

In Class Exercise 2

- A circuit with 2 D flip-flops: $D_A = A \oplus B$, $D_B = \overline{B} \cdot X$, $F = \overline{A}B$
- Do the state diagram

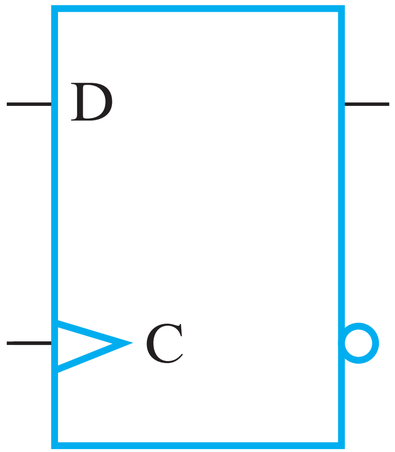
State Table

Present State		X	Next State		F
A	B		A	B	
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	1
0	1	1	1	0	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	1	0	0	0

Exercise

LogicWorks Exercise

- Implement D flip flop using D latch and SR latch
  Save it as a component in your library
- Implement circuit $D_S = X \oplus Y \oplus S$, where D_S is a D flip flop
- Implement $D_A = \overline{X}A + XY$, $D_B = \overline{X}B + XA$, $Z = XB$
- Draw the state table and diagram, and verify your table with LogicWorks



Exercise