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CSCI 150
Introduction to Digital and Computer System Design
Lecture 2: Combinational Logical Circuits III

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Overview

- Focus: Boolean Algebra
- Architecture: Combinatory Logical Circuits
- Textbook v4: Ch2 2.3; v5: Ch2 2.3
- Core Ideas:
    1. Boolean Algebra II: Standard Forms

P0
Review

Boolean Algebra I

- Boolean Algebra vs Physical Implementation of Logic Circuits
- AND, OR, NOT Operators and Gates
  - Simple digital circuit implementation
  - Algebraic manipulation using Binary Identities

Review

Boolean Algebra II: Standard Forms

Minterm/Maxterm;
Sum of Products; Product of Sums

Summary

Standard Forms

- Equivalent expressions can be written in a variety of ways
  Standard forms: typical such ways that incorporates some **unique characteristics** -> **simplify the implementation** of these designs

  - **Product terms** (AND terms): e.g. $\overline{X}YZ$
    Literals with inverts connected through only AND operators

  - **Sum terms** (OR terms): e.g. $X + \overline{Y} + Z$
    Literals with inverts connected through only OR operators

Concept

P1
Standard Forms

Minterms and Maxterms

- **Minterm**
  Product term; Contains **all variables**; Has only **one Positive row** in the truth table

	X	Y	$\overline{X}\overline{Y}$	$\overline{X}Y$	$X\overline{Y}$	XY
$(00)_2=0$	0	0	**1**	0	0	0
$(01)_2=1$	0	1	0	**1**	0	0
$(10)_2=2$	1	0	0	0	**1**	0
$(11)_2=3$	1	1	0	0	0	**1**

Concept

P1
Standard Forms

Minterms and Maxterms

- **Minterm**
  Product term; Contains **all variables**; Has only **one Positive row** in the truth table

	X	Y	$m_0 = \overline{X}\overline{Y}$	$m_1 = \overline{X}Y$	$m_2 = X\overline{Y}$	$m_3 = XY$
$(00)_2=0$	0	0	**1**	0	0	0
$(01)_2=1$	0	1	0	**1**	0	0
$(10)_2=2$	1	0	0	0	**1**	0
$(11)_2=3$	1	1	0	0	0	**1**

Concept

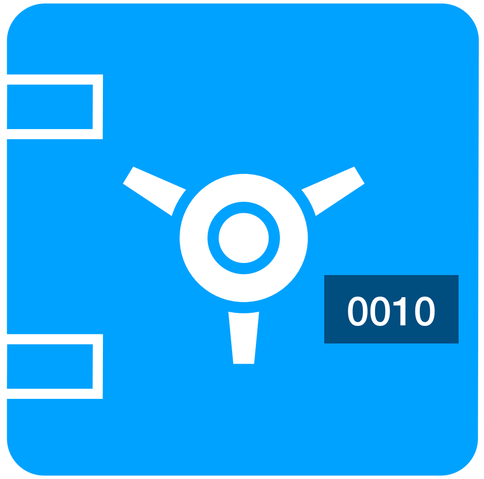
Minterms

X	Y	Z	m_0 $\overline{X}\overline{Y}\overline{Z}$	m_1 $\overline{X}\overline{Y}Z$	m_2 $\overline{X}Y\overline{Z}$	m_3 $\overline{X}YZ$	m_4 $X\overline{Y}\overline{Z}$	m_5 $X\overline{Y}Z$	m_6 $XY\overline{Z}$	m_7 XYZ
0	0	0	**1**	0	0	0	0	0	0	0
0	0	1	0	**1**	0	0	0	0	0	0
0	1	0	0	0	**1**	0	0	0	0	0
0	1	1	0	0	0	**1**	0	0	0	0
1	0	0	0	0	0	0	**1**	0	0	0
1	0	1	0	0	0	0	0	**1**	0	0
1	1	0	0	0	0	0	0	0	**1**	0
1	1	1	0	0	0	0	0	0	0	**1**

Example

P1
Standard Forms

Minterms



- Minterm
  - The output is 1 when a unique combination of input (condition) is met (like a combination lock)

Example

Minterms

- Minterms to Boolean conversions m_i
  - Write down i in binary $(\alpha_1\alpha_2\ldots\alpha_n)_2$
  - List all the variables, connect them with AND, if $\alpha_i = 0$ then invert the ith variable

Concept

Minterms

- With variables X, Y, Z, A, write down
  - m_4 $\quad m_4 = \overline{X}Y\overline{Z}\overline{A}$
  - m_{10} $\quad m_{10} = X\overline{Y}Z\overline{A}$

Example

P1
Standard Forms

Minterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

- Write down the Sum of Minterm of the truth table on the left

$$m_0 + m_2 + m_5$$

Example

Minterms

- With variables X, Y, Z, write down
  - $m_2 + m_3$ $\qquad$ $m_2 + m_3 = \overline{X}Y\overline{Z} + \overline{X}YZ$
  - $m_1 + m_3 + m_5 + m_7$ $\qquad$ $m_1 + m_3 + m_5 + m_7 = \overline{X}\overline{Y}Z + \overline{X}YZ + X\overline{Y}Z + XYZ$

Exercise

Minterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

- Write down the Sum of Minterm of the truth table on the left

$$m_0 + m_3 + m_6$$

Exercise

P1
Standard Forms

Minterms and Maxterms

- **Maxterm**
  Sum term; Contains **all variables**; Has only **one Negative row** in the truth table

$$M_i = \overline{m_i}$$

	X	Y	$M_0 = X + Y$	$M_1 = X + \overline{Y}$	$M_2 = \overline{X} + Y$	$M_3 = \overline{X} + \overline{Y}$
$(00)_2$=0	0	0	**0**	1	1	1
$(01)_2$=1	0	1	1	**0**	1	1
$(10)_2$=2	1	0	1	1	**0**	1
$(11)_2$=3	1	1	1	1	1	**0**

Concept

Maxterms

- Maxterms to Boolean conversion M_i
  - Write down i in binary $(\alpha_1\alpha_2\ldots\alpha_n)_2$
  - List all the variables, connect them with OR, if $\alpha_i = 1$ then invert the ith variable

Example

Maxterms

- With variables X, Y, Z, A, write down
  - M_4 $$M_4 = (X + \overline{Y} + Z + A)$$
  - M_{10} $$M_{10} = (\overline{X} + Y + \overline{Z} + A)$$

Example

Maxterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

- Write down the Product of Maxterm of the truth table on the left

$$M_1 M_3 M_4 M_6$$

Example

Maxterms

- With variables X, Y, Z, write down
  - $M_2 \cdot M_3$ $(X + \overline{Y} + Z)(X + \overline{Y} + \overline{Z})$
  - $M_1 \cdot M_3 \cdot M_5 \cdot M_7$

$$(X + Y + \overline{Z})(X + \overline{Y} + \overline{Z})(\overline{X} + Y + \overline{Z})(\overline{X} + \overline{Y} + \overline{Z})$$

Exercise

Maxterms

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

- Write down the Product of Maxterm of the truth table on the left

$$M_1 M_2 M_4 M_7$$

Exercise

Minterms and Maxterms

- e.g. $M_3 = X + \overline{Y} + \overline{Z} = \overline{\overline{X}YZ} = \overline{m_3}$
- Sum of Minterms
  - e.g. $F = \overline{X}\overline{Y}\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}Z + XYZ = m_0 + m_2 + m_5 + m_7$
    $= \Sigma m(0,2,5,7)$
- Product of Maxterm
  - e.g. $F = (X + Y + Z)(X + \overline{Y} + Z)(\overline{X} + Y + \overline{Z})(\overline{X} + \overline{Y} + \overline{Z})$
    $= M_0 M_2 M_5 M_7$
    $= \Pi M(0,2,5,7)$

Concept

P1
Standard Forms

Minterms and Maxterms

- $F(X, Y, Z) = \Sigma m(1,4,5)$
  - Write down F in boolean expression $F = \overline{X}\overline{Y}Z + X\overline{Y}\overline{Z} + X\overline{Y}Z$
  - Write down $\overline{F}$ in Product of Maxterm form

$$\overline{F} = \Pi_M(1,4,5)$$

Example

Minterms and Maxterms

- $F(X, Y, Z) = \Pi M(2,3,7)$
  - Write down F in boolean expression

$$F = (X + \overline{Y} + Z)(X + \overline{Y} + \overline{Z})(\overline{X} + \overline{Y} + \overline{Z})$$

  - Write down $\overline{F}$ in Sum of Minterm form

$$\overline{F} = \Sigma_m(2,3,7)$$

Exercise

Sum of Products

- Sum of Minterm can come directly from the truth table
- Sum of Product: simplified version of Sum of Minterm
  - $F(X, Y, Z) = \Sigma m(0,1,2,6)$

Product of Sums

- Product of Maxterms can also come directly from the truth table
- Product of Sums: simplified version of Product of Maxterms
  - $F(X, Y, Z) = \Pi M(0,1,2,6)$

Concept

Summary

- Minterm/Maxterm
- Sum of Products
- Product of Sums

Review

P2
Exercise

Boolean Algebra II

Exercises!

Summary

P2
Exercise

Boolean Algebra

Difficulty: Simple

Obtain the truth table of the following function, and ex press each function in sum-of-minterms and product-of-maxterms form:

- $(XY + Z)(Y + XZ)$

Exercise

P2
Exercise

Boolean Algebra

Difficulty: Simple

For the Boolean functions E and F, as given in the following truth table:

- List the minterms and maxterms of each function
- List the minterms of $\overline{E}$ and $\overline{F}$
- List the minterms of $E + F$ and EF

X	Y	Z	E	F
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	0
1	1	1	0	1

Exercise

P2
Exercise

Boolean Algebra

Difficulty: Simple

For the Boolean functions E and F, as given in the following truth table:

- Express E and F in sum-of-minterms algebraic form
- Simplify E and F to expressions with a minimum of literals

X	Y	Z	E	F
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	0
1	1	1	0	1

Exercise

P2
Exercise

Boolean Algebra

Difficulty: Mid

Convert the following expressions into sum-of-products and product-of-sums form

- $(AB + C)(B + \overline{C}D)$
- $\overline{X} + X(X + \overline{Y})(Y + \overline{Z})$

Exercise